

# RBF Neural Network based Trajectory Control and Impedance Control of a Upper Limb Tele-rehabilitation Process

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**Abstract:** In the passive tele-rehabilitation process, the safety is the most important thing for patients avoiding the secondary damage of the impaired upper limb. Aiming at adjusting the appropriated contact force in time during the training exercises, an adaptive impedance control is proposed for the slave side. At the same time, the trajectory control based on the Hamilton-Jacobi-Inequality theory and the RBF Neural network is performed for the master manipulator operated by therapists. The stability is analyzed and numerical simulations show the efficiencies and high performances of the proposed method.

## 1 INTRODUCTION

Recently, researchers publish their study in the New England Journal of Medicine (NEJM) based on the GBD data in 2016 (GBDStrokeCollaborators, 2018), which calculates the lifetime stroke risk in various countries and regions from 1990 to 2016. Results demonstrate that the risk of the lifetime stroke in adults aged 25 increases by 8.9% to 24.9% in the past 26 years. What is more, the overall risk of the lifetime stroke in China and Chinese men both reach 40%. Scientists continue to point out that the number of stroke in China accounts for the first rank in the world (GBDStrokeCollaborators, 2019)(Gorelick, 2019). After the participation of the Chinese researchers, the latest research evidence and the specific information in the field of stroke prevention and treatment in China are published in(S Wu, 2019). It reveals that the incidence of stroke in China is promoted in the past 20 years. The prevention and treatment of stroke in China is not balanced among regions. The risk factors of stroke are higher in the countryside than in the city (Brainin, 2019). With the continuous improvement of the stroke treatment in China in recent years, the mortality of stroke patients are not enhanced significantly in the past 20 years. However, the incidence of stroke is still rising. That is, the bur-

den of the stroke is still heavy. In recent 10 years, the prevalence of stroke among urban and rural residents has a stable trend, while the prevalence in rural areas has increased significantly (Z Li, 2019). Since the China has the vast territory, many rural stroke patients live far away from the city, so that it is more difficult for them to go to hospital for rehabilitation treatment than urban residents. Therefore, the tele-rehabilitation training is a good way to solve the problem. In the meantime, it may decrease the cost of expensive therapies, and it also raise the efficiency of the medical therapists.

The upper limb tele-rehabilitation refers that a therapist operates a rehabilitative manipulator in the master side, while another rehabilitation manipulator simultaneously assists patients in the slave side so as to achieve training exercises in the remote place. Via the internet vision and the communication on both sides, the therapist may connivently guide and adjust the patient's training exercises. Since it is an interesting and a new active issue, many researchers devote various study for the purpose of spreading the tele-rehabilitation mode rehabilitation to remote passive therapists. On the basis of the wave variable theorem, Mendoza and his colleagues present a novel bilateral tele-rehabilitation method in (M Mendoza, 2016). In the tele-rehabilitation system, the motion-based adaptive impedance control is exerted on both the master and slave robot manipulators for robot assisted passive rehabilitation. Thinking of the time de-

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lay, the stability of the tele-rehabilitation system is analyzed. Numerical simulations are performed and results demonstrate that the proposed method may ensure the stable humanrobot interaction as well as compensate the position drift. The paper developed a novel control method based on the estimation of the forces on both the master and slave robots so as to replace the expensive force sensor (F Azimifar, 2017). The stability is analyzed with the consideration of the time delay. The proposed method is verified by position tracking experiments. In order to enhance the patienttherapist interaction, Mojtaba and his colleagues introduce a adaptive bilateral impedance controller for the upper limb tele-rehabilitation training process (M Sharifi, 2017). The proposed method is verified by experiments on a nonlinear multi-DOF manipulators. Thus, they continue to study the impedance control of robot assisted tele-operation process (M Sharifi, 2018). They tune an adaptive law with bilateral impedance control for the purpose of changing impedance model parameters during the tele-surgery of a beating heart. The advantage of their proposed method is that it avoids affording expensive force sensor’s device as well estimating the heart’s motion. After the discuss of the stability, experimental results demonstrate that the proposed bilateral impedance control may increase the safety of patients and compensate the motion of the beating heart.

In this paper, we focus on the bilateral impedance control for the upper limb tele-rehabilitation training process so as to achieve the protection of the safety of impaired patients. Simultaneously, a trajectory controller is proposed also on the basis of the Hamilton-Jacobi-Inequality (HIJ) theory and the RBF Neural network to ensure the normal training exercises. The therapist may change the motion through regulating parameters of the impedance model from the contact force in the slave side. Rest of the paper is organized as follows. The dynamic model and the impedance model is introduced in section 2. Section 3 explains the trajectory control and the impedance control in the master side and the adaptive neural fuzzy impedance control in the slave side. Numerical simulations are demonstrated in section 4. Some conclusions are given in the conclusion part.

## 2 THE DYNAMIC MODEL AND THE IMPEDANCE MODEL OF THE TELE-REHABILITATION SYSTEM

The upper limb tele-rehabilitation process may be described in the following picture as shown in Figure.1.

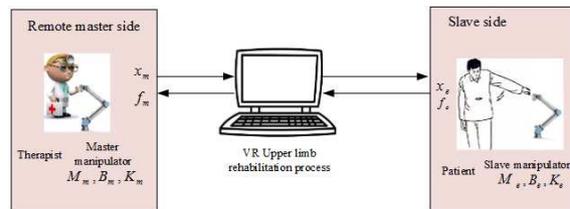


Figure 1: The upper limb tele-rehabilitation process.

The integral system may be regarded as a bilateral tele-operation system involving (1)the therapist and the master manipulator in the master side, (2) the patient and the slave manipulator in the slave side and (3) a VR screen which exhibits the training process by the internet. At the beginning of our study, the time delay is simplified to zero in this paper although it cannot be ignored in the reality. Therefore, the upper limb tele-rehabilitation system can be separated into two subsystems as shown in Figure. 2. The first subsystem is the master trajectory control loop. As the therapist designs the desired training exercises trajectory, the master manipulator tracks it by the master trajectory control. In the initial training exercises, the slave manipulator follows the master manipulator to push the upper limb of the patient. Since patients have different degree of disability, they can bear different contact forces. The second subsystem is the slave impedance control. With the changes of the contact forces, the desired trajectory is adjusted through the slave impedance control. In fact, the role of the slave impedance control is a forward feedback using to modify the training exercises trajectory depending on the affordable conditions of patients.

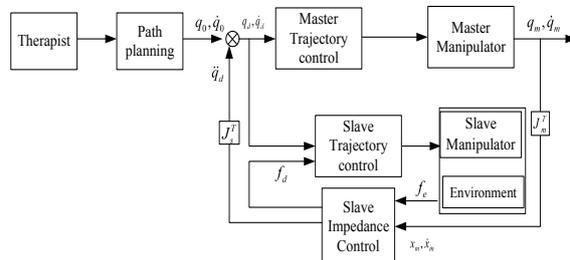


Figure 2: The composition of the tele-rehabilitation system.

Assuming that both the master and the slave take the n DOF manipulator system, the dynamic model is

expressed as follows.

$$M_i(q_i(t))\ddot{q}_i(t) + C_i(q_m(t), \dot{q}_i(t))\dot{q}_i(t) + G_i(q_i) + \Delta(q_i, \dot{q}_i) + d_i(t) = T_i(t) \quad (1)$$

where  $M_i \in \mathbb{R}^{m \times n}$ ,  $C_i \in \mathbb{R}^{m \times n}$ ,  $G_i \in \mathbb{R}^{m \times n}$ ,  $i = m, s$  stand for the inertial items, the Coriolis and centrifugal effects items and the gravitational items. The subscripts  $m$  and  $s$  represent respectively the master and the slave. State variables in the joint space of the master and the slave manipulators are described as  $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^{n \times n}$ .  $T_i$  indicates the control inputs.  $\Delta(q_i, \dot{q}_i)$  and  $d_i(t)$  represent the model uncertainties and the external disturbances.

The impedance model of the slave system is written as follow.

$$H_s \ddot{x}_s(t) + B_s \dot{x}_s(t) + K_s x_s = f_e, \quad (2)$$

where  $H_i, B_i$  and  $K_i$  are respectively inertial, damping and stiffness matrices.  $x_s, \dot{x}_s, \ddot{x}_s \in \mathbb{R}^{n \times n}$  are state variables in the work space. The state variables in the work space may transferred to the joint space via following nonsingular Jacobian matrices.

$$\begin{aligned} \dot{x}_i &= J(\dot{q}_i) \\ \ddot{x}_i &= \dot{J}(q_i) + J\ddot{q}_i \end{aligned} \quad (3)$$

where

$$J = \begin{bmatrix} -l_1 \sin(q_{i1}) - l_2 \sin(q_{i1} + q_{i2}) & -l_2 \sin(q_{i1} + q_{i2}) \\ l_1 \cos(q_{i1}) + l_2 \cos(q_{i1} + q_{i2}) & l_2 \cos(q_{i1} + q_{i2}) \end{bmatrix}$$

The tele-rehabilitation system has following properties (J Zhang, 2018).

- The inertia matrices  $M_i$  are symmetric positive definite matrices,  $M_i = M_i^T$ , and there exist the upper and lower boundedness,  $M_{down}I \leq |M_i| \leq M_{up}I$  ( $M_{down}$  and  $M_{up}$  are two positive constants)
- $\dot{M}_i(q_i) - 2C_i(q_i, \dot{q}_i)$  are skew-symmetric matrices.
- For all  $q_i(t), \dot{q}_i(t) \in \mathbb{R}^{n \times 1}$ , there exists a positive scalar  $c_i$  may render  $C_i(q_i(t), \dot{q}_i(t)) \leq c_3 |\dot{q}_i| (1 + |q_i|) \leq c_3 |\dot{q}_i|$ , in which  $c_1, c_2, c_3 > 0$  and  $|\cdot|$  represents the Euclidean matrix norm.
- The linear parameterizable dynamic model of the tele-rehabilitation system (in Eq.(1)) may be expressed as  $M_i(q_i(t))\ddot{q}_i(t) + C_i(q_i(t), \dot{q}_i(t))\dot{q}_i(t) + G_i(q_i(t)) = Y_i(q_i(t), \dot{q}_i(t), \ddot{q}_i(t))\theta_i$ , where  $Y_i(q_i(t), \dot{q}_i(t), \ddot{q}_i(t)) \in \mathbb{R}^{n \times p}$  are a certain function and  $\theta_i \in \mathbb{R}^p$  are physical parameter vectors of the master and the slave manipulators.

### 3 THE TRAJECTORY CONTROL BASED ON THE HJI THEORY AND RBF NEURAL NETWORK

#### 3.1 The HJI Theory

The bilateral tele-rehabilitation system (in Eq.1) may be rewritten as the following form.

$$\begin{cases} \dot{x}_i(t) = f(x_i) + g(x)d(t) \\ z(t) = h(x_i) \end{cases} \quad (4)$$

where  $x(t), u(t), z(t)$  are respectively the state variable, the input and the system evaluation index.  $d$  is the external disturbances causing by communication time delay, noises and so on.

**Definition** For the disturbance signal, its norm is defined as  $\|d(t)\|_2 = \{\int_0^\infty d^T(t)d(t)dt\}^{\frac{1}{2}}$ , which may measure the magnitude of the  $d(t)$  energy. In order to evaluate the disturbance suppression ability of the system, the performance index is defined as follow

$$J = \sup_{\|d(t)\| \neq 0} \frac{\|z\|_2}{\|d\|_2}, \quad (5)$$

where  $J$  is the  $L_2$  gain of the system, indicating the robust performance. The smaller  $J$  results better robust performance of the system.

According to (Schaft, 1992), the Hamilton-Jacobian-Inequality theorem can be described as follow. For a positive number  $\gamma$ , if there exists a positive definite and differentiable function  $L(x) \geq 0$ , and

$$\dot{L} \leq \frac{1}{2} \{\gamma^2 \|d\|^2 - \|z\|^2\}, (\forall d), \quad (6)$$

then  $J \leq \gamma$ .

#### 3.2 The Trajectory Control and Its Analysis

Assuming there is no time delay, the master is the same as the slave manipulator, and both of manipulators have the same initial state variables. Due to the limitation of the paper, we merely introduce the master trajectory control, and the slave adopts the same trajectory control method. The desired trajectory is noted by  $q_d$ , and the trajectory tracking error is defined as  $e = q - q_d$ . The forwards feedback control law is designed as

$$T = u + M\ddot{q}_d + C\dot{q}_d + G, \quad (7)$$

where  $u$  is the feedback control law.

Substituting the Eq.(7) into the Eq. (1), the close

tele-rehabilitation system may change to the following form,

$$M\ddot{e} + C\dot{e} + \Delta(q, \dot{q}) + d = u. \quad (8)$$

Ordering  $\Delta f(q, \dot{q}) + d$ , we may get  $M\ddot{e} + C\dot{e} + \Delta f = u$ . Taking the RBF neural network (RBFNN) to approximate the  $\Delta f$ , the  $\Delta f$  is expressed as

$$\Delta f = W_f^* \sigma_f + \varepsilon_f, \quad (9)$$

where  $\varepsilon_f$  stands for the approximation error.  $\sigma_f$  is the RBF Gaussian function and  $W_f^*$  is the ideal weights of the RBFNN. Combing Eq.(8) and Eq.(9), we may acquire  $M\ddot{e} + C\dot{e} + W_f^* \sigma_f + \varepsilon_f = u$ .

Define

$$\begin{cases} x_1 = e \\ x_2 = \dot{e} + \alpha e, \alpha > 0 \end{cases} \quad (10)$$

then,

$$\begin{cases} \dot{x}_1 = x_2 - \alpha x_1 \\ M\dot{x}_2 = -Cx_2 + \omega - W_f^* \sigma_f - \varepsilon_f + u \\ \omega = M\alpha \dot{e} + C\alpha e \end{cases} \quad (11)$$

Using the HJI inequality, rewritten the Eq. (11) to the state space form as follow,

$$\begin{cases} \dot{x} = f(x) + g(x)d \\ z = h(x) \end{cases} \quad (12)$$

where  $f(x) = \begin{bmatrix} x_2 - \alpha x_1 \\ \frac{1}{M}(-Cx_2 + \omega - W_f^* \sigma_f + u) \end{bmatrix}$ ,  $g(x) = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$ ,  $d = \varepsilon_f$ . The evaluation index is defined as follow. Due to  $d = \varepsilon$ , the approximation error may be regarded as the external disturbance. Therefore, the evaluation index is denoted by  $z = x_2 = \dot{e} + \alpha e$ , and its  $L_2$  gain calculated by  $J = \sup_{\|\varepsilon_f\| \neq 0} \frac{\|z\|_2}{\|\varepsilon_f\|_2}$ .

The adaptive law of the tele-rehabilitation system is designed as

$$\dot{W}_f = -\eta x_2 \sigma_f^T. \quad (13)$$

Thus, the feedback control is expressed as

$$u_i = -\omega - \frac{1}{2\gamma^2} x_2 + \hat{W}_f \sigma_f - \frac{1}{2} x_2, \quad (14)$$

where  $\hat{W}_f$  and  $\sigma_f$  respectively note the weight of the RBFNN and the output of the Gaussian function. That is, the tele-rehabilitation system satisfies  $J \leq \gamma$ . According to (Y wang, 2009), the stability is analyzed as follow. The Lyapunov function is selected as

$$L = \frac{1}{2} x_2^T M x_2 + \frac{1}{2\eta} \text{tr}(\tilde{W}_f^T \tilde{W}_f),$$

where  $\tilde{W}_f = \hat{W}_f - W_f^*$ . Due to the properties of the tele-rehabilitation system and the proposed feedback control law, we may get following relations.

$$\begin{aligned} \dot{L} &= x_2^T M \dot{x}_2 + \frac{1}{2} x_2^T \dot{M} x_2 + \frac{1}{\eta} \text{tr}(\dot{\tilde{W}}_f^T \tilde{W}_f) \\ &= x_2^T (-Cx_2 + \omega - W_f^* \sigma_f - \varepsilon_f + u) \\ &\quad + \frac{1}{2} x_2^T \dot{M} x_2 + \frac{1}{\eta} \text{tr}(\dot{\tilde{W}}_f^T \tilde{W}_f) \\ &= x_2^T (-Cx_2 - W_f^* \sigma_f - \varepsilon_f - \frac{1}{2\gamma^2} x_2 + \hat{W}_f \sigma_f - \frac{1}{2} x_2) \\ &\quad + \frac{1}{2} x_2^T \dot{M} x_2 + \frac{1}{\eta} \text{tr}(\dot{\tilde{W}}_f^T \tilde{W}_f) \\ &= x_2^T (-\varepsilon_f - \frac{1}{2\gamma^2} x_2 + \tilde{W}_f \sigma_f - \frac{1}{2} x_2) \\ &\quad + \frac{1}{2} x_2^T (\dot{M} - 2C)x_2 + \frac{1}{\eta} \text{tr}(\dot{\tilde{W}}_f^T \tilde{W}_f) \\ &= -x_2^T \varepsilon_f - \frac{1}{2\gamma^2} x_2^T x_2 + x_2^T \tilde{W}_f \sigma_f - \frac{1}{2} x_2^T x_2 + \frac{1}{\eta} \text{tr}(\dot{\tilde{W}}_f^T \tilde{W}_f) \end{aligned}$$

Order

$$H = \dot{L} - \frac{1}{2} \gamma^2 \|\varepsilon_f\|^2 + \frac{1}{2} \|z\|^2, \quad (15)$$

then,

$$\begin{aligned} H &= -x_2^T \varepsilon_f - \frac{1}{2\gamma^2} x_2^T x_2 + x_2^T \tilde{W}_f \sigma_f - \frac{1}{2} x_2^T x_2 + \frac{1}{\eta} \text{tr}(\dot{\tilde{W}}_f^T \tilde{W}_f) \\ &\quad - \frac{1}{2} \gamma^2 \|\varepsilon_f\|^2 + \frac{1}{2} \|z\|^2 \end{aligned}$$

Considering the following conditions,

- $-x_2^T \varepsilon_f - \frac{1}{2\gamma^2} x_2^T x_2 - \frac{1}{2} \gamma^2 \|\varepsilon_f\|^2 = -\frac{1}{2} \|\frac{1}{\gamma} x_2 + \gamma \varepsilon_f\|^2 \leq 0$ ,
- $x_2^T \tilde{W}_f \sigma_f + \frac{1}{\eta} \text{tr}(\dot{\tilde{W}}_f^T \tilde{W}_f) = 0$ ,
- $-\frac{1}{2} x_2^T x_2 + \frac{1}{2} \|z\|^2$ ,

we may easily get  $H \leq 0$ . Depending on the definition of  $H$ , we have  $\dot{L} \leq \frac{1}{2} \gamma^2 \|\varepsilon_f\|^2 - \frac{1}{2} \|z\|^2$ . Due to Eq.(6) of the HJI theorem, we may deduce  $J \leq \gamma$ , so that  $\|z\|$  satisfies the performance index. That is, the trajectory tracking error  $e_i$  and  $\dot{e}$  conform to the convergency requirement.

### 3.3 Adaptive Slave Impedance Control

The impedance model and the contact force between the patient and the slave manipulator can be written as follow.

$$H(\ddot{x} - \ddot{x}_m) + B(\dot{x} - \dot{x}_m) + K(x - x_m) = f_e, \quad (16)$$

where  $f_e$  is the desired contact force between the impaired upper limb and the slave manipulator.. Since we assume that there is not the time delay, the angle and the angular velocity of the master manipulator  $q, \dot{q}, \ddot{q}$  may lossless send to the slave manipulator. Via the transfer of the Jacobian matrix, we may acquire the modified angular accelerator of the slave manipulator. H, B and K are the inertia, the stiffness

and damping matrices parameters of the impedance model.  $x, \dot{x}, \ddot{x}$  are the position under the appropriate contact force of patients.

Assuming that the tracking error is defined as  $\tilde{x} = x - x_m$ , the reference model is expressed as follow,

$$\ddot{x}_m + \lambda_1 \dot{x}_m + \lambda_2 x_m = \lambda^2 f_e, \quad (17)$$

where  $\lambda_1$  and  $\lambda_2$  are positive numbers. For an uncertain  $H$ , the adaptive law is set as follow,

$$U = \hat{H}(\ddot{x}_m - 2\lambda\dot{\tilde{x}} - \lambda^2\tilde{x}), \quad (18)$$

where  $\lambda$  is strictly positive and  $\hat{H}$  is the estimation of the inertial matrix.

Defining  $v = \ddot{x}_m - 2\lambda\dot{\tilde{x}} - \lambda^2\tilde{x}$ , and substituting the Eq.(18) into Eq.(16), we may get the following relation,

$$H\ddot{x} = \hat{H}(\ddot{x}_m - 2\lambda\dot{\tilde{x}} - \lambda^2\tilde{x}) = \hat{H}v. \quad (19)$$

Setting  $\tilde{H} = \hat{H} - H$ , the Eq.(19) may be transferred to

$$H(\ddot{x} - v) = \tilde{H}v. \quad (20)$$

Define the tracking error function  $s$  as

$$s = \tilde{x} + \lambda\dot{\tilde{x}} \quad (21)$$

From Eq. (19), it can be seen that the convergence of  $s$  implies the the convergence of the position tracking error  $\tilde{x}$  and the velocity tracking error  $\dot{\tilde{x}}$ .

Due to

$$\begin{aligned} \ddot{x} &= v + \ddot{x} - \ddot{x}_m + 2\lambda\dot{\tilde{x}} + \lambda^2\tilde{x} \\ &= \ddot{x} + \lambda\dot{\tilde{x}} + \lambda(\dot{\tilde{x}} + \lambda\tilde{x}) \\ &= \dot{s} + \lambda s \end{aligned}$$

, the Eq.(20) changes into

$$H(\dot{s} + \lambda s) = \tilde{H}v. \quad (22)$$

That is,  $Hs\dot{s} = -\lambda Hs^2 + \tilde{H}vs$ . Defining the Lyapunov function as

$$V = \frac{1}{2}(Hs + \frac{1}{\gamma}\dot{m}^2), \gamma > 0. \quad (23)$$

The derivation of  $V$  is as follows,

$$\begin{aligned} \dot{V} &= Hs\dot{s} + \frac{1}{\gamma}\dot{H}\dot{H} \\ &= -\lambda Hs^2 + \tilde{H}vs + \frac{1}{\gamma}\dot{H}\dot{H} \\ &= -\lambda Hs^2 + \tilde{H}(vs + \frac{1}{\gamma}\dot{H}) \end{aligned}$$

The adaptive law of the  $\hat{H}$  is defined as  $\dot{\hat{H}} = -\gamma vs$ . Thus, it is obvious that  $\dot{V} = -\lambda Hs^2 \leq 0$ . Due to  $V \geq 0, \dot{V} \leq 0$ ,  $S$  and  $\hat{H}$  are bounded depending on (J LaSalle, 1961)(Hassan, 2002). As  $\dot{V} \equiv 0$ ,  $s = 0$ . According to LaSalle invariance principle, the close system are asymptotically stable. As  $t \rightarrow 0$ ,  $s \rightarrow 0$ ,  $\tilde{x} \rightarrow 0$ ,  $\dot{\tilde{x}} \rightarrow 0$ .

## 4 NUMERICAL SIMULATION

In the simulation, the dynamic model set as follow,

$$\begin{aligned} M_{11}(q_i) &= (m_1 + m_2)r_1^2 + m_2r_2^2 + 2m_2r_1r_2 \cos(q_{i2}), \\ M_{22}(q_i) &= m_2r_2^2 \\ M_{12}(q_i) &= M_{21}(q_i) = m_2r_1r_2 \cos(q_{i2}), \\ C_{11}(q_i, \dot{q}_i) &= -m_2r_1 \sin(q_{i2})\dot{q}_2, \\ C_{22}(q_i, \dot{q}_i) &= 0 \\ C_{12}(q_i, \dot{q}_i) &= -m_2r_1r_2 \sin(q_{i2})(\dot{q}_1 + \dot{q}_2) \\ C_{21}(q_i, \dot{q}_i) &= m_2r_1r_2 \sin(q_{i2})\dot{q}_2 \\ g_1(q_{i1}) &= (m_1 + m_2)r_1 \cos(q_{i2}) + m_2r_2 \cos(q_1 + q_2), \\ g_2(q_{i2}) &= m_2r_2 \cos(q_1 + q_2), \end{aligned}$$

where  $m_1$  and  $m_2$  are respectively masses of the upper limb and the forearm.  $r_1$  and  $r_2$  are the lengths of the two links (the upper limb and the forearm). The model uncertainties  $\Delta(q_i, \dot{q}_i)$  is the uncertainty part of the tele-rehabilitation dynamic model while  $d_i$  represents the external disturbances. Set  $D = \Delta(q_i, \dot{q}_i) + d_i$ , where

$$d_i = \begin{bmatrix} 30 \text{sgn} q_1 \\ 30 \text{sgn} q_2 \end{bmatrix}.$$

Real values in the numerical simulations are used as follows:  $m_1 = m_2 = 1.5 \text{kg}$ ,  $r_1 = 1 \text{m}$ ,  $r_2 = 0.8 \text{m}$ . Ideal tracking signals are selected as  $q_{1d} = \sin t$ ,  $q_{2d} = \sin t$ ,  $\gamma = 0.05$ . The with and the center of the RBF Gaussian function choose  $c_i = [-1.5, -1.0, -0.5, 0, 0.5, 1.0, 1.5]$  and  $b_i = 10$ . Other parameters list as follows:  $\lambda_1 = 10$ ,  $\lambda_2 = 25$ ,  $\lambda = 6$ . Initial conditions are set as  $\dot{x}_i(0) = 0$ ,  $x_i = 0.5$ .

Applying the proposed trajectory tracking control and impedance control to the master and the slave manipulators of the tele-rehabilitation system, results are illustrated in the following Figures. from 3 to 12. In the Figure. 3, the position signal and the speed tracking result are displayed. The ideal position signal and the ideal speed signal mark with red solid lines while the position tracking results and the speed tracking result are used by black dotted lines.

The details of the position tracking show in the Figure.4. The ideal position of the upper limb and the forearm represent by red solid lines while tracking results are marked with black dotted lines. The speed tracking results are displayed in Figure. 5. The ideal speed of the upper limb and the forearm represent by red solid lines while tracking results are marked with black dotted lines. The position tracking error and the speed tracking error are showed in Figure. 6. The control inputs of the upper limb and the forearm are

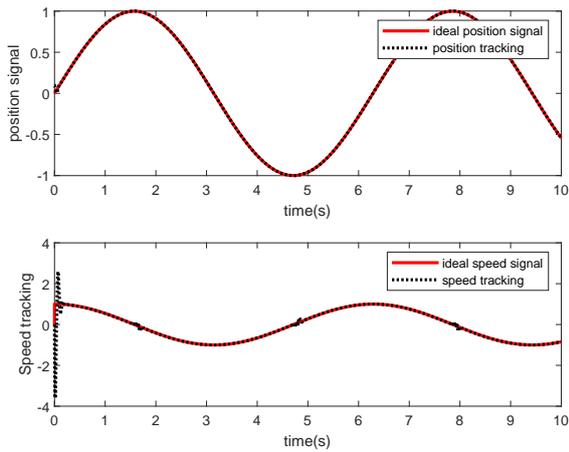


Figure 3: The upper limb tele-rehabilitation process.

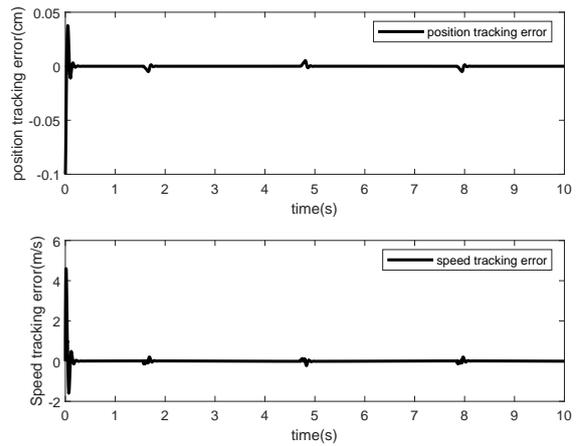


Figure 6: The tracking error of the tele-rehabilitation process.

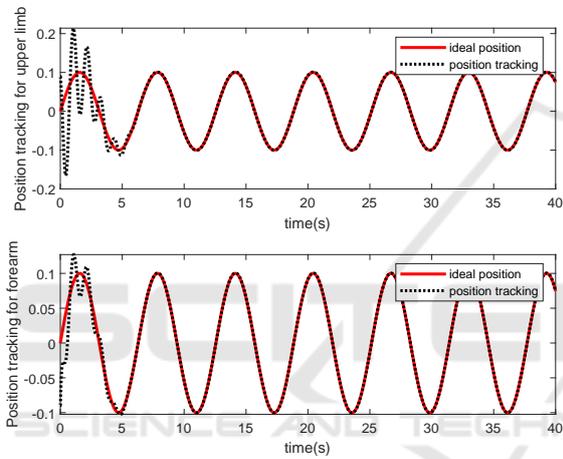


Figure 4: The position tracking of the tele-rehabilitation process.

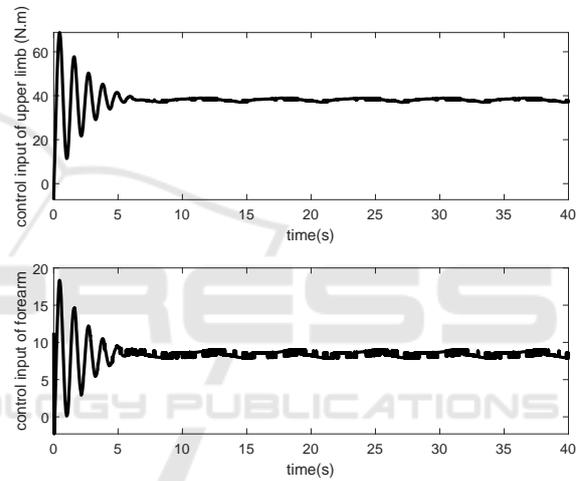


Figure 7: The control input of the tele-rehabilitation process.

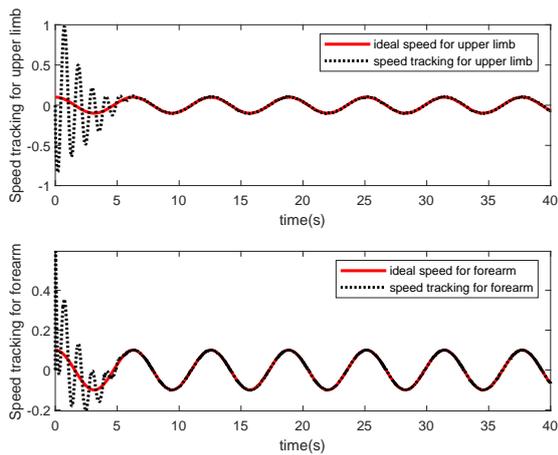


Figure 5: The speed tracking of the tele-rehabilitation process.

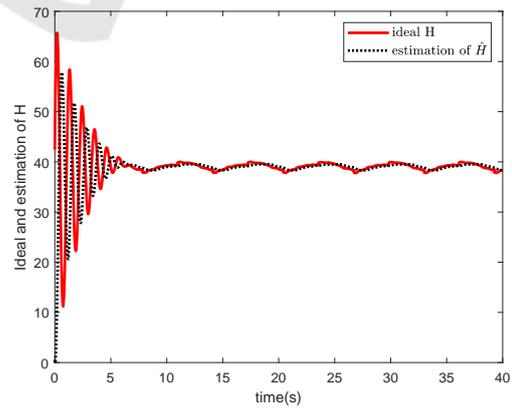


Figure 8: The estimation of the impedance parameter of the tele-rehabilitation process.

presented in Figure. 7. The figure 8 shows the estimation of  $\hat{H}$  by the adaptive impedance control of

the slave manipulator. The contact force between the patient and the slave manipulator is displayed in fig-

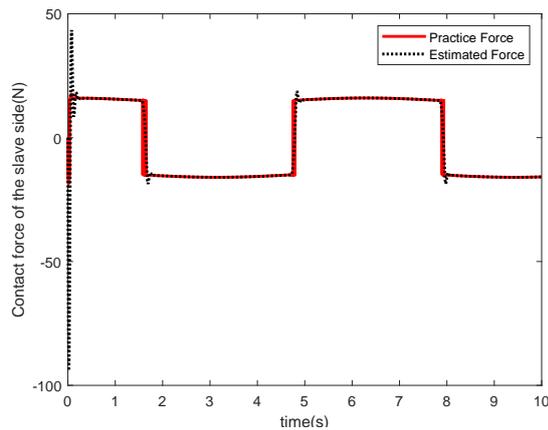


Figure 9: The contact force of between the slave manipulator and the patient.

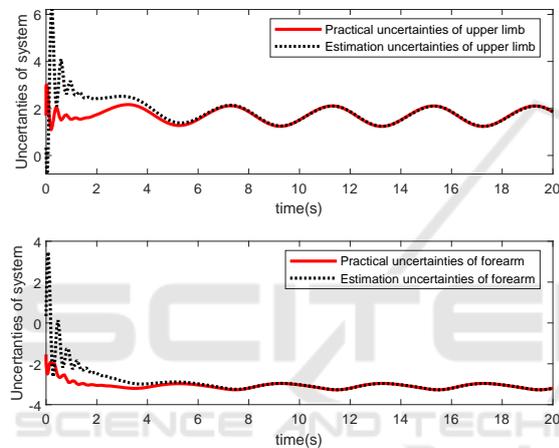


Figure 10: The model uncertainties of the tele-rehabilitation process.

ure 9. Model uncertainties of the upper limb and the forearm are explained in Figure. 10. Figure. 11 introduces the estimation of the external disturbances.

From results of numerical simulations, it may be concluded that the patient may follow the training exercises trajectory according to the therapist through the master trajectory tracking control. In the meantime, the therapist may sense the contact force of the patient through the slave adaptive impedance control, so that the therapist can momentarily adjust the training trajectory according to the severity of the patient.

## 5 CONCLUSIONS

The tele-rehabilitation process is studied in this paper. In order to guide the training exercises of remote patients, the contact force is an important issue to let the therapist know how to strengthen or attenuate the training exercises. At the same time, the trajectory

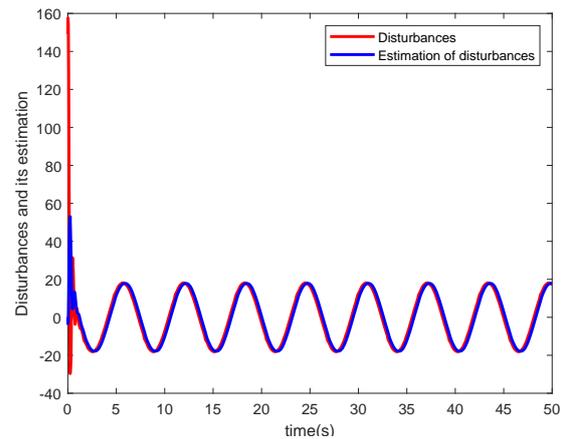


Figure 11: The external disturbances of the tele-rehabilitation process.

tracking is also important so that different strengths must be realized by different kinds of training exercises. Aiming to solve the problems, we propose a trajectory control based on the HJI theorem and the RBF neural network for the master manipulator. Simultaneously, an adaptive impedance control is designed so as to get the appropriate contact force of the patient, for the purpose of protecting the patient not to be impaired twice. Both of the trajectory control and the adaptive impedance control are analyzed by the Lyapunov theorem. Although the impedance control is simple, it is easily to achieve in the practice. The proposed control methods are implemented by numerical simulations. Results show the efficiency and high performances.

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