

Temporal Cognitive Maps

Adrian Robert, David Genest and Stéphane Loiseau

LERIA, Université d'Angers, France

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Abstract: A cognitive map is an oriented graph whose nodes are labeled by concepts and edges represent influences. It provides a way to model strategies or influence systems. Cognitive maps do not take into account any temporal features. This article proposes a solution to this lack: a temporal cognitive map model defined on a temporal ontology. The temporal ontology is used to represent temporal domain knowledge and to temporally characterize the concepts of the cognitive map. An extension, named TCMQL, of the cognitive map query language CMQL, is proposed in order to access the concepts' temporality and compare them making inferences.

1 INTRODUCTION

The *cognitive map* (Axelrod, 1976) model is a semantic model coming from cognitive psychology. This model is used to represent strategies, or more generally, influence systems. Cognitive maps are close to Bayesian networks (Pearl, 2014). While Bayesian networks focus on the computation of influences based on conditional probabilities, cognitive maps focus mainly on the visualisation and are easier to understand for several types of user. A cognitive map is an oriented graph whose nodes are labeled by *concepts* and edges, called *influences*, are labeled by an *influence value*. Influence values belong to a predefined value set which can contain symbolic values such as $\{-, +\}$ (Tolman, 1948), $\{none, some, much, alot\}$ (Kosko, 1986), or numeric values such as $[-1, 1]$ (Kosko, 1986), $\{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ (Le Dorze, 2013). A sequence of influences from a node to another makes a *path*. The model can infer a *propagated influence value* from a node to another. A *taxonomic cognitive map* (Le Dorze et al., 2012) is a cognitive map defined on a *taxonomy*. The taxonomy organizes the concepts with 'kind of' type relations: the nodes of the taxonomic cognitive map are labeled by concepts of the taxonomy. The taxonomy can be used to infer the *taxonomic influence value* from a concept of the taxonomy to another one.

Cognitive maps are used in many fields such as social sciences (Axelrod, 1976), biology (Martin et al., 2000) and geography (Çelik et al., 2005). It is typ-

ically the case of the project in geography Kifanlo¹. This project aims to study the evolution of the fishing strategies in the Atlantic coast from 1970 to 2016. About fifty cognitive maps have been designed with fishermen to model their fishing strategies. Half of those maps represent fishing strategies in the seventies, the other half represent current fishing strategies, each map contains 25 to 50 nodes. A cognitive map edition software, VSPCC, has been used and improved².

In the Kifanlo project, there is a significant number of concepts that have a temporal semantics. These concepts usually repeat periodically over time like seasons, fishing seasons and so on... This periodicity of the concepts should be taken into account in cognitive maps. Notice that the only articles that study time in cognitive maps stem not on concepts but on influence : (Park and Kim, 1995; Zhong et al., 2008) consider the delay or duration process of an influence in fuzzy cognitive maps.

So, this paper introduces temporal cognitive maps, which is a new model that extends taxonomic cognitive maps with a temporal ontology for representation and reasoning.

Because of the periodicity of the concept's semantics, the *temporal ontology* aims to represent *periodic intervals* (Osmani, 1999). It uses *temporal assertions*

¹Kifanlo is a project financed by the Fondation de France. It has been led from 2013 to 2017.

²The edition software VSPCC (LeDorze and Robert, 2014) has been implemented after the thesis of Aymeric LeDorze (Le Dorze, 2013), for the project Kifanlo.

that are triples made of two periodic intervals related by a *comparison predicate*. The proposed temporal ontology could be added to other temporal ontologies of reference like owl-time (Hobbs and Pan, 2006a) which lacks of such periodic temporal entities.

A *temporal cognitive map* is defined on a temporal ontology; it contains a set of temporal assertions that link the nodes of the cognitive map to the temporal ontology. The nodes can thus be temporally characterized, meaning that a certain influence holds with respect to the temporal assertions of its nodes.

To reason with a temporal cognitive map, this article proposes the 'Temporal Cognitive Map Query Language' *TCMQL*, which is an extension of the query language for cognitive maps *CMQL* (Robert et al., 2018). *TCMQL* is made with two temporal primitives: *TimeInfo* and *Compare*. *TimeInfo* lets the user access the periodic interval associated with a node. *Compare* infers new information using temporal assertions of nodes and the temporal ontology. This extension provides a way to use the temporal information of the model for a further analysis of cognitive maps.

TCMQL, as well as *VSPCC* extended to temporal cognitive maps, have been delivered to the researchers in geography that work in the Kifanlo project for further analysis³.

This article is composed of three parts. The first recalls the taxonomic cognitive map model. The second introduces the temporal cognitive map model. The third one presents the *TCMQL* language.

2 TAXONOMIC COGNITIVE MAP

A taxonomic cognitive map is a graph whose nodes and edges are respectively labeled by a concept of a taxonomy and by an influence value; the taxonomy aims to organize the concepts. The taxonomy is even more useful when using a set of cognitive maps, to make sure that different cognitive maps use the same concepts (Chauvin et al., 2009).

2.1 Taxonomic Cognitive Map Model

The taxonomy organizes the concepts by specifying a specialization relation between them.

³This work is being led in the project *Analyse Cognitive de Savoirs* granted by the french region Pays de la Loire from 2017 to 2020.

Definition 1 (Taxonomy). *Let C be a concept set. A taxonomy $\mathcal{T} = (C, \leq)$ is a set of rooted trees of concepts that represents a partial order relation \leq whose meaning is 'kind of'.*

Example 1. \mathcal{T}_1 is the taxonomy of the figure 1. Some concepts are ordered by a relation of specialization. For instance, the relation $MultiPurposeShip \leq Ship$, meaning that *MultiPurposeShip* is a kind of *Ship*, is represented by an arrow in the figure.

The most specialized concepts of the taxonomy are said elementary.

Definition 2 (Elementary concepts). *Let $\mathcal{T} = (C, \leq)$ be a taxonomy. The elementary concepts of \mathcal{T} are: $elem_{\mathcal{T}} = \{c \in C / \forall c' \in C, c' \leq c \implies c' = c\}$.*

Example 2. In \mathcal{T}_1 (figure 1), the elementary concepts are $elem_{\mathcal{T}_1} = \{MultiPurposeShip, RemoteArea...\}$; only the concepts *Ship*, *FishingActivity* and *Pleasure* are not elementary.

A taxonomic cognitive map is a graph whose nodes and edges are respectively labeled by an elementary concept of a taxonomy and an influence value. The influence value represents the strength of the influence and belongs to a defined value set which can be qualitative or quantitative, discrete or continuous.

Definition 3 (Taxonomic cognitive map). *A taxonomic cognitive map defined on a taxonomy $\mathcal{T} = (C, \leq)$ and a value set I is an oriented labeled graph $CM = (N, E, labelN, labelE)$ such that:*

- N : the nodes of the graph.
- $E \subset N \times N$: the edges are called influences.
- $labelN : N \rightarrow elem_{\mathcal{T}}$ is a label function on the nodes.
- $labelE : E \rightarrow I$ is a label function on the edges.

Example 3. *CC1* and *CC2* are the two taxonomic cognitive maps of the figure 2. They are defined on the taxonomy \mathcal{T}_1 of the figure 1 and the value set $I = [-1, 1]$. Note that, in the figure, each node has a unique identifier per map n_1, n_2, \dots that is displayed only for clarity in this paper. An influence labeled by 1 (resp. -0.25) means that the source node influences strongly (resp. weakly) and positively (resp. negatively) the destination node. In

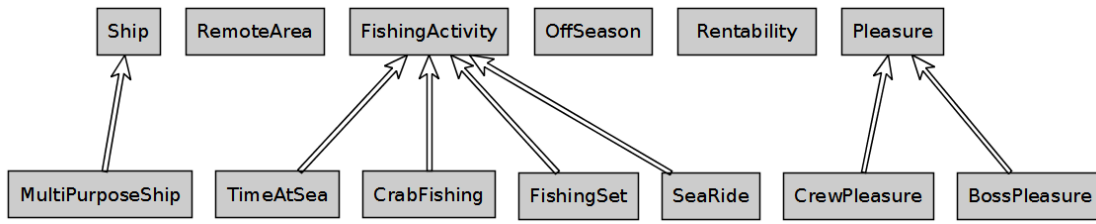


Figure 1: A taxonomy \mathcal{T}_1 .

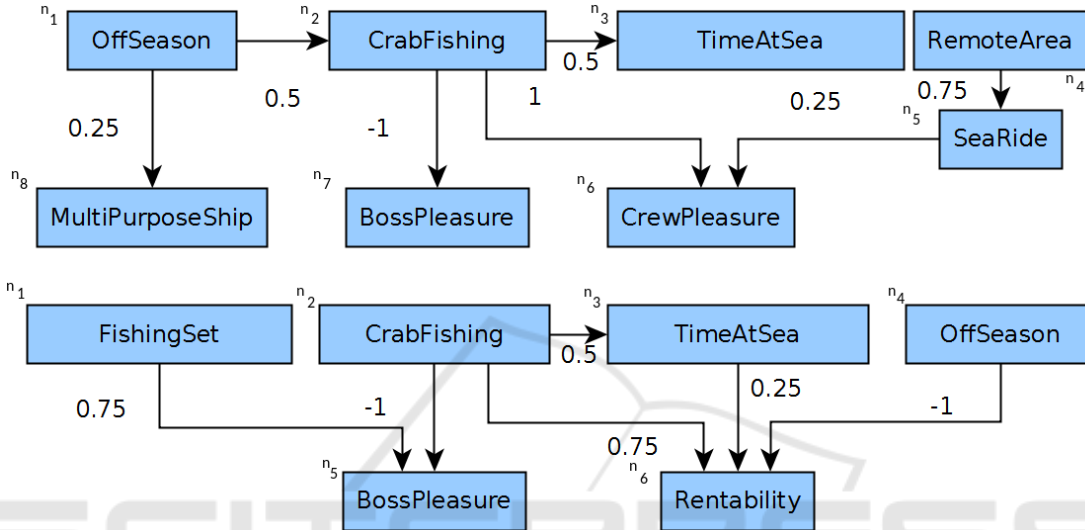


Figure 2: Two taxonomic cognitive maps, CC1 (top) and CC2 (bottom).

our application, each fisherman designs a cognitive map: CC1 has been designed by fisherman1, and CC2 by fisherman2. In CC2, the node n_2 (CrabFishing) influences strongly and negatively (-1) the node n_5 (BossPleasure); which means that the boss does not like fishing crab.

2.2 Taxonomic Cognitive Map Inference

A path is a sequence of influence which represents a way a node of the map influences another. A path is said minimal if it does not contain any cycle. Notice that between two nodes, there can be more than one minimal path.

Definition 4 (Path). Let $CM = (N, E, labelN, labelE)$ be a taxonomic cognitive map defined on $\mathcal{T} = (C, \leq)$ and I . Let $a, b \in N$ be two nodes of CM.

- A path P from a to b is a sequence of length $length_P \geq 1$ of influences $(u_i, u_{i+1}) \in E$ (with $i \in [0; length_P - 1]$) such that $a = u_0$ is the source of P and $b = u_{length_P}$ is the destination of P . This path is denoted by $a \rightarrow u_1 \rightarrow \dots \rightarrow b$.

- A path P is said minimal if $\forall i, j \in [0; length_P], i \neq j \Rightarrow u_i \neq u_j$.
- The set of all minimal paths on CM is denoted by $Paths_{CM}$.

Example 4. This example is based on CC2 (figure 2). $p_1 = n_2(CrabFishing) \rightarrow n_3(TimeAtSea) \rightarrow n_6(Rentability)$ is a minimal path of length=2, from the source node n_2 to the destination node n_6 . $p_2 = n_2(CrabFishing) \rightarrow n_6(Rentability)$ is a minimal path of length=1.

One of the main features of cognitive maps is their ability to infer the propagated influence from any node to any other one, which denotes a value of influence. To do that, every influence path from the node to the other is involved.

The propagated influence from a node to another can be calculated differently depending on the map's semantics and on the value set on which it is defined. In all cases, the computation of the propagated influence first assigns a path value for each path with a function, then secondly aggregates those values with an other function.

Definition 5 (Propagated Influence). *Let $CM=(N,E,labelN,labelE)$ be a taxonomic cognitive map defined on $\mathcal{T} = (C, \leq)$ and I .*

- *The path value is a function $PV_{path}: Paths_{CM} \rightarrow I$ which infers the propagated influence of a path.*
- *The propagated influence value is a function $PV: N \times N \rightarrow I$ which infers the propagated influence from a node to another one, aggregating the path values of each path between the two nodes.*

In this paper, we will use the value set $I = [-1, 1]$. A product function will be used as path value and a mean function for the propagated influence value as it is often done in cognitive maps (Genest and Loiseau, 2007).

Example 5. *This example is based on CC2 (figure 2). The paths p_1 and p_2 come from the example 4. Let's infer the propagated influence value between n_2 and n_6 , respectively labeled by *CrabFishing* and *Rentability*. The set of all minimal paths between those two nodes is $Paths_{n_2,n_6} = \{p_1, p_2\}$, it contains two paths. To infer the propagated influence value between n_2 and n_6 we need $PV_{Path}(p_1)$ and $PV_{Path}(p_2)$. From the chosen product function, we have $PV_{Path}(p_1) = 0.5 * 0.25 = 0.125$ and $PV_{Path}(p_2) = 0.75$. Then, aggregating the path values, $PV(n_2, n_6) = \frac{(0.125+0.75)}{2} = 0.44$. So the propagated influence value from n_2 to n_6 is 0.44.*

The taxonomic cognitive map model can also infer a taxonomic influence value which is used to infer the influence value between any pair of concepts of the taxonomy. Note that the propagated influence value is a particular case of the taxonomic influence value where the concepts are elementary. The taxonomic influence value is not presented in this article, but is described in (Chauvin et al., 2009).

3 TIME REPRESENTATION

This section introduces the periodic intervals, then proposes a temporal ontology defined on those periodic intervals and temporal assertions that compare pairs of them. So, the temporal cognitive map can be introduced, it is a taxonomic cognitive map defined on a temporal ontology.

3.1 Periodic Intervals

A periodic interval (Ermolayev et al., 2014; Ermolayev et al., 2008; Poveda-Villalón et al., 2014; Osmani, 1999) is a type of non-convex interval (Ladkin, 1986), which is an interval composed of several unconnected convex subintervals. Periodic intervals have the particularity to be composed of subintervals that have the same length and are equally spaced. For instance 'winter' is a periodic interval.

The periodic intervals of Osmani and Balbiani (Osmani, 1999; Balbiani and Osmani, 2000) that also considers qualitative relations between them are chosen. This approach is relevant to the Kifanlo project and, in general, seems suited for cognitive maps as it offers more flexibility and handles the lack of precise information.

Definition 6 (Periodic Interval). *A periodic interval is a non-convex interval whose subintervals are equally spaced and have equal length.*

Example 6. *January is a periodic interval since all its subintervals last one month and occur every year. Summer is also a periodic interval, with subintervals lasting three months and occurring every year.*

This paper proposes to specify those periodic intervals with qualitative relations between two intervals using a comparison predicate. Those predicates are the 16 relations of Osmani (Osmani, 1999) plus 5 relations. The relations of Osmani are very similar to the 13 relations of the Allen's intervals, except that the precedence and its inverse are replaced by 5 relations which consider the periodicity. This paper also considers two relations (Inside/Disjoint) that combine some of Osmani's relations and three relations ($<, >, =$) that compare duration of intervals, which can not be done with Osmani's intervals.

Definition 7 (Comparison predicate). *A comparison predicate is a binary relation whose domain and range are periodic intervals. \mathcal{P} is the set of the 21 comparison predicates: $\{m, mi, s, si, d, di, f, fi, o, oi, eq, ppi, mmi, moi, omi, ooi, in, dis, <, =, >\}$*

The table below shows the 16 relations of Osmani & Balbiani, the column *meaning* explains the relations through an ordering of the boundaries (A1,A2,B1,B2) of the periodic intervals A and B. This ordering comes from the CYCORD theory (Röhrig, 1994). Two added relations are: 'in'(Inside) which

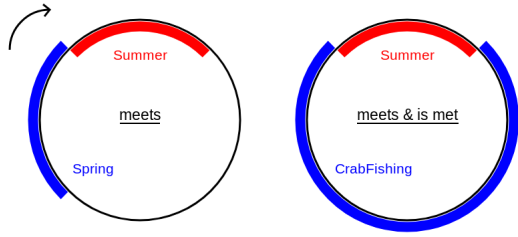


Figure 3: Two cyclic representations of relations between periodic intervals.

is the disjunction of 's','d','f','eq' and 'dis'(Disjoint) which is the disjunction of 'm','mi','mmi','ppi'.

To these relations are also added three relations to compare the duration of periodic intervals : '<','>' and '='.

name	meaning	inverse
eq (equals)	$A1 = B1, A2 = B2$	(eq)
m (meets)	$A1, A2 = B1, B2$	mi
s (starts)	$A1 = B1, A2, B2$	si
d (during)	$A1, A2, B2, B1$	di
f (finishes)	$A1, A2 = B2, B1$	fi
o (overlaps)	$A1, B1, A2, B2$	oi
ppi (precedes & is preceded)	$A1, A2, B1, B2$	(ppi)
mmi (meets & is met)	$A1 = B2, A2 = B1$	(mmi)
moi (meets & is overlapped)	$A1, B2, A2 = B1$	omi
ooi (overlaps & is overlapped)	$A1, B2, B1, A2$	(ooi)
in (inside)	$s \vee d \vee f \vee eq$	
dis (disjoint)	$m \vee mi \vee mmi \vee ppi$	
< (is shorter)	$A1A2 < B1B2$	>
= (has same length)	$A1A2 = B1B2$	(=)

Periodic intervals and comparison predicates defined above are used to represent temporal knowledge through temporal assertions. A temporal assertion is an assertion which represents a relation between two periodic intervals. It is a triple (interval,predicate,interval).

Definition 8 (Temporal assertion). \mathcal{P} is the set of the 21 comparison predicates. A temporal assertion is an assertion which constitute a triple (e_1, p, e_2) such that $p \in \mathcal{P}$ and e_1 and e_2 are periodic intervals.

Example 7. The relations between periodic intervals are often represented on a circle (figure 3) which is to be read clockwise. The first circle represents the temporal assertion (Spring, meets, Summer) and it matches the ordering ('SpringBegins', 'Spring

gEnds'='SummerStarts', 'SummerEnds') of the second line of the table. Its inverse relation is *mi* (is met by), so we have (Summer, is met by, Spring). The second circle illustrates the temporal assertion (CrabSeason, meets&ismet, Summer). CrabSeason is related to Summer by the relation 'meets&ismet' which means that the crab season starts when summer ends and ends when summer starts. Some comparison predicates are used to compare duration, for instance in the temporal assertion (Day, <, Month) the comparison predicate '<' is used to compare the duration of Day and Month.

3.2 Time Ontology

Many temporal ontologies exist, amongst those owl-time ontology (Hobbs and Pan, 2006b) is a W3C reference and one of the most used. It turns out that time ontologies do not take into account periodic intervals and certainly not the qualitative relations to compare them. That is why this paper introduces a new temporal ontology that considers periodic intervals and could be added to existing heavier temporal ontologies like owl-time. Our light-weight temporal ontology is composed of the class PeriodicInterval, the 21 comparison predicates as object properties, a set of instances of PeriodicInterval and a set of temporal assertions on these individuals.

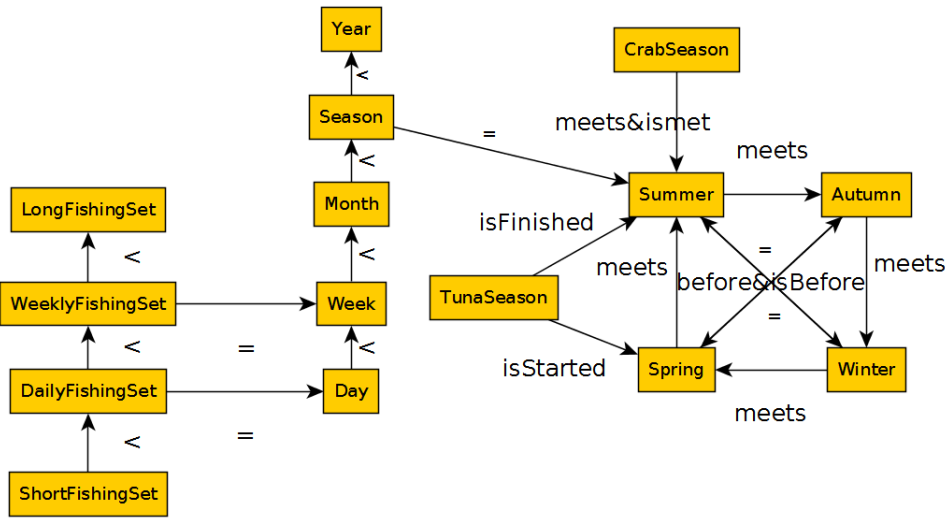
Definition 9 (Temporal Ontology). A temporal ontology $\mathcal{O} = (\mathcal{P}, \mathcal{E}, \mathcal{A})$ is an ontology such that :

- \mathcal{P} is the set of the comparison predicates.
- \mathcal{E} is a set of periodic intervals.
- \mathcal{A} is a set of temporal assertions of the ontology.

Example 8. The figure 4 represents the temporal ontology \mathcal{O}_1 . The periodic intervals of this ontology are $\mathcal{E} = \{\text{Spring, CrabSeason, Year ...}\}$ and the temporal assertions are $\mathcal{A} = \{(\text{Season}, <, \text{Year}), (\text{CrabSeason}, \text{meets\&ismet}, \text{Summer})\dots\}$.

3.3 Temporal Cognitive Map

A temporal cognitive map is a taxonomic cognitive map defined on a temporal ontology. Each node of the map is labeled by a periodic interval and a set of temporal assertions links those periodic intervals to the ontology. This way, nodes may be temporally characterized.

Figure 4: A partial representation of the temporal ontology \mathcal{O}_1 .

Definition 10 (Temporal cognitive map). Let $\mathcal{O} = (\mathcal{P}, \mathcal{E}, \mathcal{A})$ be a temporal ontology. Let $CM = (N, E, labelN, labelE)$ be a taxonomic cognitive map defined on $\mathcal{T}(C, \leq)$ and I . A temporal cognitive map TCM defined on \mathcal{O} is a triple $(CM, labelT, \mathcal{A}_{TCM})$ such that :

- $labelT$: is a label function on the nodes of the map which attaches a unique periodic interval e_n to a node n .
- \mathcal{A}_{TCM} is a set of temporal assertions (e_1, p, e_2) where $labelT^{-1}(e_1) \in N$ and $e_2 \in \mathcal{E}$.

In the next part, a set of temporal cognitive maps based on the same taxonomy, value set and temporal ontology is considered. So to specify an associated periodic interval, the following notation is used.

Notation 1 (Associated Periodic interval). The periodic interval associated with a node labeled by a concept 'c' of a map 'm' is noted 'm_c'.

Example 9. This example describes the two temporal cognitive maps of the figure 5: $TCM1$ and $TCM2$. A temporal assertion (in yellow) of a temporal cognitive map is visually represented below the node (in blue) that it characterizes. The periodic interval attached to the node is visually omitted, that is why temporal assertions are written as couples and not triples. For instance in $TCM1$, the node labeled by $OffSeason$ is characterized by the temporal assertion $(TCM1_OffSeason, si, Summer)$ where $TCM1_OffSeason$ is the omitted periodic interval attached to this node and 'si' is the comparison predicate 'isStartedBy'. Notice that several temporal assertions can be attached to the same node, as it is

the case for the node labeled by $CrabFishing$ in $TCM1$. This node is characterized by a periodic interval that lasts one month ($=, Month$) at the end of the $CrabSeason$ ($f, CrabSeason$). The fisherman I fishes crab for one month at the end of the crab season.

4 TCMQL

CMQL is a query language whose syntax is close to the one of SQL and whose semantics is similar to the one of the relational domain calculus (Louis and Pirotte, 1982; Robert et al., 2019). CMQL's particularity resides in the use of many *primitives* that allow to access the various features of a taxonomic cognitive map set. TCMQL is the extension of CMQL that integrates two temporal primitives, *TimeInfo* and *Compare*, which allow to access the concepts' temporal assertions and compare them. TCMQL is designed to query a set of temporal cognitive maps defined on the same temporal ontology.

4.1 Primitive 'TimeInfo'

The extraction primitive *TimeInfo* links a cognitive map, a concept of this map, and the periodic interval associated with the node labeled by this concept in this map.

Definition 11 (Primitive: *TimeInfo*). Let S be a set of temporal cognitive maps defined on the same temporal ontology $\mathcal{O} = (\mathcal{P}, \mathcal{E}, \mathcal{A})$, taxonomy $T = (C, \leq)$

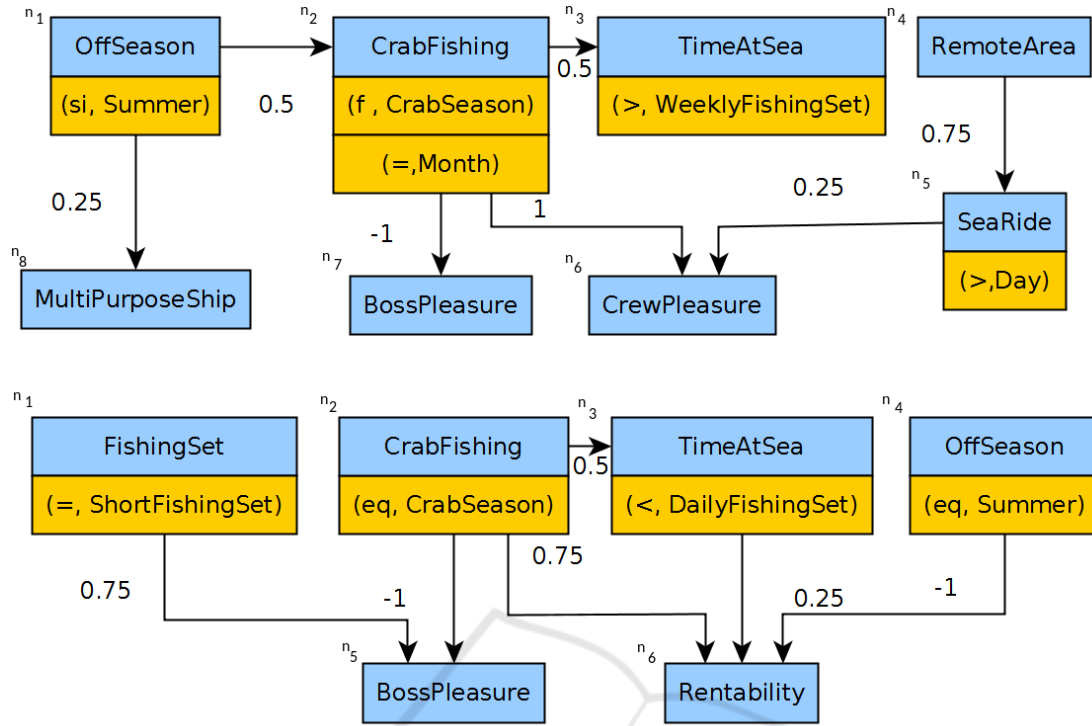


Figure 5: Two temporal cognitive maps, TCM1 (top) and TCM2 (bottom).

and I . Let \mathcal{E}_S be the set of all periodic intervals associated with the nodes of the maps in S . The primitive $\text{TimeInfo}(\text{map}:S, \text{concept}:C, \text{interval}:\mathcal{E}_S)$ is a relation made of the set of the triples (map,concept,interval) such that $\exists n \in N_{\text{map}}, \text{Label}_{T_{\text{map}}}(n) = \text{interval}$ and $\text{label}_{N_{\text{map}}}(n) = \text{concept}$.

Example 10. In TCMQL, a variable is a syntactic expression prefaced with "?" like in SPARQL (Harris et al., 2013). The following examples uses the maps TCM1 and TCM2 (figure 5).

$\text{TimeInfo}(\text{?map}, \text{TimeAtSea}, \text{?interval})$ is a primitive formula, which is the syntactic expression of a primitive. Its meaning is a binary relation whose value is the set of tuples ($\text{?map}, \text{?interval}$) in which ?interval is associated with the node labeled by the concept TimeAtSea in ?map :

?map	?interval
TCM1	TCM1_TimeAtSea
TCM2	TCM2_TimeAtSea

The primitive TimeInfo is used here to link concepts and maps to associated periodic intervals, TimeAtSea is then used in TCM1 and TCM2.

Used alone the usefulness of this primitive is limited, it is often used in conjunction with the primitive Compare defined below.

4.2 Primitive 'Compare'

When the designer of a temporal cognitive map adds his domain knowledge, he adds the least amount of temporal assertions and expects the implicit ones to be taken into account: an inference is thus necessary. 151 inference rules are used for these inferences, they are OWL2(Hitzler et al., 2009) rules of two types described in the next example. The comprehensive list of rules is not given in the paper as it is too long but available online (Robert, 2019). The rules about the 16 Balbiani's relations can be found also in the references (Balbiani and Osmani, 2000), a few other rules about the new predicates are added.

Example 11. Here are some inference rules:

- $\text{SubObjectPropertyOf}(\text{during inside})$ which means $(e_1, \text{during}, e_2) \rightarrow (e_1, \text{inside}, e_2)$
- $\text{SubObjectPropertyOf}(\text{starts } <)$ which means $(e_1, \text{starts}, e_2) \rightarrow (e_1, <, e_2)$
- $\text{SubObjectPropertyOf}(\text{ObjectPropertyChain}(\text{meets startedBy}) \text{ meets})$ which means $(e_1, \text{meets}, e_2) \wedge (e_2, \text{startedby}, e_3) \rightarrow (e_1, \text{meets}, e_3)$

Using the ontology of the figure 4 and the cognitive maps figure 5, the inference produces assertions like :

- (*CrabSeason*, *disjoint*, *Summer*) which means that the crab season is outside the summer. This assertion comes from the assertion (*CrabSeason*, *mmi*, *Summer*) of \mathcal{O}_1 and the rule $(e_1, mmi, e_2) \rightarrow (e_1, disjoint, e_2)$.
- (*TunaSeason*, *>*, *Season*) which means that the season of tuna is longer than a calendar season. This assertion comes from the assertions (*TunaSeason*, *fi*, *Summer*), (*Season*, *=*, *Summer*) of \mathcal{O}_1 and the rules $(e_1, fi, e_2) \rightarrow (e_1, <, e_2)$ and $(e_1, >, e_2) \wedge (e_2, =, e_3) \rightarrow (e_1, >, e_3)$.
- (*TCM1_CrabFishing*, *meets*, *Summer*) which means that Summer starts when the crab season ends. This assertion comes from the assertions (*TCM1_CrabFishing*, *f*, *CrabSeason*) of *TCM1*, (*CrabSeason*, *mmi*, *Summer*) of \mathcal{O}_1 and the rule $(e_1, f, e_2) \wedge (e_2, mmi, e_3) \rightarrow (e_1, meets, e_3)$.

Inferences can be carried out on a set that contains the temporal assertions of the ontology and the temporal assertions of each temporal cognitive map. The saturated set is the set of temporal assertions that can be deduced from all these temporal assertions and the inference rules.

Definition 12 (Saturated set). Let \mathcal{R} a set of rules and $S = \{(CM_1, labelT_1, \mathcal{A}_1), \dots, (CM_k, labelT_k, \mathcal{A}_k)\}$ be a set of k temporal cognitive maps defined on $\mathcal{O} = (\mathcal{P}, \mathcal{E}, \mathcal{A})$, \mathcal{I}_S is the saturated set of temporal assertions resulting from the inference of the rules of \mathcal{R} on the set $\mathcal{A} \cup \bigcup_{i=1}^k \mathcal{A}_i$.

The primitive *Compare* uses the saturated set of temporal assertions, it is a relation between two periodic intervals and a comparison predicate which is a valid comparison between these intervals.

Definition 13 (Primitive: Compare). Let S be a set of temporal cognitive maps defined on the same ontology $\mathcal{O} = (\mathcal{P}, \mathcal{E}, \mathcal{A})$ where \mathcal{P} is the set of comparison predicates. Let \mathcal{E}_S be the set all periodic intervals associated with the nodes of the maps in S . Let \mathcal{I}_S the saturated set of all temporal assertions. The primitive *Compare*($e1: \mathcal{E}_S \cup \mathcal{E}, p: \mathcal{P}, e2: \mathcal{E}_S \cup \mathcal{E}$) is a relation made of the triples $(e1, p, e2) \in \mathcal{I}_S$.

Example 12. The following examples use the ontology \mathcal{O}_1 (figure 4) and the temporal cognitive maps *TCM1* and *TCM2* (figure 5).

- *Compare*(*TunaSeason*, *?pred*, *?interval*) is a primitive formula which

aims to compare the periodic interval *TunaSeason* (which is from Spring to Summer according to \mathcal{O}_1) to any other periodic interval. There are many result tuples like $(>, Summer)$ since the summer finishes the *TunaSeason*:

?pred	?interval
isStarted	Spring
isFinished	Summer
>	Summer
>	Week
isFinished	TCM2_OffSeason
...	...

- *Compare*(*TCM1_TimeAtSea*, *?pred*, *Month*) is a primitive formula. There is no answer since we can not evaluate the comparison of two durations both greater than a week (*TCM1_TimeAtSea*, *>*, *Week*) and (*Month*, *>*, *Week*):

?pred

- *Compare*(*Winter*, *?pred*, *CrabSeason*) is a primitive formula. This primitive formula asks the relations between the Winter and the CrabSeason. Since the CrabSeason starts at the end of the summer and ends at its beginning, the winter is during the CrabSeason and thus shorter. We obtain the three following tuples:

?pred
during
Inside
<

Although the complexity of the inferences is high (at least EXPTIME), it has not been a problem in our system for two reasons. Firstly, a cognitive map is hand designed and it is a visual model so it is usually quite a small graph, for instance in the Kifanlo project a thirty nodes map is a big one. Secondly, the saturated set is precomputed and queries give an answer in an acceptable time in our application. Nevertheless, to go one step further, a study should be done to evaluate the theoretical complexity and how to face it depending on maps structure.

4.3 Query Examples

Some primitives of CMQL are recalled here for the following examples. The primitive *IsInMap* is one of them: it is a binary relation whose first attribute is a map and the second a concept. This primitive is verified if the concept appears in this map. *Path* is another primitive : it is a relation of four attributes,

a map, two concepts of this map and a path whose source and destination nodes are labeled by the two concepts. `KindOf` is a binary primitive with two concepts of the taxonomy where the first is a kind of the second. `Value` is a relation of four attributes which links a map, two concepts and the taxonomic influence value of the first second to the second in the map. TCMQL's syntax is close to SQL's syntax : `SELECT` selects variables `?x...`, `FROM` indicates the maps to query and `WHERE` describes the conditions. Four examples are given here along with their results and comments, they are based on \mathcal{T}_1 (fig. 1), \mathcal{O}_1 (fig. 4) and TCM1,TCM2 (fig. 5).

Example 13. *The primitives `IsInMap` and `KindOf` are used in this example. In plain English this query means : 'In which maps are used the concepts types of Pleasure?'*

```
SELECT ?map, ?concept FROM TCM1, TCM2
WHERE{
  KindOf(?concept, Pleasure)
  AND IsInMap(?map, ?concept)
}
```

The first condition allows to obtain the concepts that are types of Pleasure in the taxonomy. The second condition gets the couples (map, concept) such that the concept belongs to the map. The result of this query is the list of the following tuples (`?map,?concept`):

?map	?concept
TCM1	BossPleasure
TCM1	CrewPleasure
TCM2	BossPleasure

The result shows what are the types of the concept pleasure and in which maps they appear.

Example 14. *In plain English this query means : 'When does fisherman1(TCM1) fish crabs in comparison to fisherman2(TCM2)?'*

```
SELECT ?pred FROM TCM1, TCM2 WHERE{
  TimeInfo(TCM1, CrabFishing, ?e1)
  AND TimeInfo(TCM2, CrabFishing,
  ?e2) AND Compare(?e1, ?pred, ?e2)
}
```

The first two conditions allow to get the temporal entities of the concept `CrabFishing` in TCM1 and TCM2. The third condition allows to get all comparison predicates between those two temporal entities that are characterized by "finishes CrabSeason" and " = Month" for the one in TCM1 and by "equals CrabSeason" for the other. The result is made of the tuples (`?pred`):

?pred
finishes
<

The result shows that the fisherman1 fishes at the end of the fisherman2's fishing period, for a shorter period.

Example 15. *In plain English this query means : 'Which duration of FishingSets influences BossPleasure?'*

```
SELECT ?p, ?e2 FROM TCM1, TCM2
WHERE{
  Path(?map, FishingSet, BossPleasure, ?path)
  AND TimeInfo(?map, FishingSet, ?e1)
  AND Compare(?e1, ?p, ?e2))}
```

The first condition allows to get the maps in which FishingSet influences BossPleasure (TCM2). The two following conditions allow to get the temporal information about FishingSet in the right map.

?p	?e2	?m
=	ShortFishingSet	TCM2
<	Day	TCM2
...

The result shows that according to the fisherman2 (TCM2), the BossPleasure is influenced by a short period of fishingset.

Example 16. *This query asks the concepts in summer which influence a concept kind of Pleasure.*

```
SELECT ?map, ?c1, ?i, ?c2
FROM TCM1, TCM2
WHERE{KindOf(?c2, Pleasure) AND
Value(?map, ?c1, ?c2, ?i) AND ?i != 0
AND
TimeInfo(?map, ?c1, ?e1) AND
Compare(?e1, in, Summer) }
```

The first condition allows to get all concepts `?c2` kind of Pleasure. The second and third ones allow to get the concepts `?c1` that influences `?c2` with their influence value. The two last conditions filter only the concepts `?c1` in summer.

?map	?c1	?i	?c2
TCM1	OffSeason	-0.5	BossPleasure
TCM1	OffSeason	0.5	CrewPleasure

The results show that, according to the fisherman1, the OffSeason which is in Summer influences negatively the pleasure of the boss and positively the pleasure of the crew.

5 CONCLUSION

This paper introduces an extension of the cognitive map model called temporal cognitive map which allows to temporally characterize concepts of the map. To do this the temporal cognitive map is defined on a temporal ontology which uses periodic intervals. This paper proposes also an extension of CMQL, named TCMQL, which allows to query a set of temporal cognitive map and its new temporal features.

The temporal cognitive map model has been implemented and tested into the VSPCC software which provides tools to edit and use cognitive maps. This software can also execute TCMQL queries, it is available online (LeDorze and Robert, 2014). The implementation uses the temporal ontology owl-time to which is added a class `PeriodicInterval` as a subclass of the main class `http://www.w3.org/2006/time#TemporalEntity` and comparison predicates as properties. owl-time contains other temporal entities, such as instants or Allen's intervals. They could also be used once adequate inference rules are added.

The temporal features introduced in this paper come from real application needs for a better modelling of the fishermen's strategies and for a more in-depth analysis in the Kifanlo project. The ACS project that succeeds the Kifanlo project is currently in progress, using these new tools.

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