Solitons in a Dual-core System with a Uniform Bragg Grating and a Bragg Grating with Dispersive Reflectivity

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Abstract: The existence and stability of gap solitons in a dual-core optical fiber made of a uniform and a nonuniform Bragg grating with Kerr nonlinearity are considered. The nonuniformity in the one of the cores originates from the presence of dispersive reflectivity. It is found that quiescent soliton solutions exist throughout the bandgap. Stability analysis shows that there exist vast areas within the bandgap where stable solitons exist.

1 INTRODUCTION

Solitons are nonlinear waves that maintain their profile for a long distance (or time). They have been observed in a variety of physical systems such as water, plasma and optical materials. In optical materials, solitons are formed when the nonlinearity of the medium is balanced by the dispersion (Chiao et al., 1964).

In periodic optical media such as Fiber Bragg gratings (FBGs), due to the coupling between the forward and reflected waves gives rise to an induced dispersion which can be up to 6 orders magnitude greater than that of silica. Gap solitons (GSs) are formed when the induced dispersion in the FBG is balanced by the nonlinearity of medium (De Sterke and Sipe, 1994). In the last few decades, GSs have attracted much attention and have been studied in numerous theoretical (Aceves and Wabnitz, 1989; Malomed and Tasgal, 1994; Barashenkov et al., 1998) and experimental works (De Sterke et al., 1997; Eggleton et al., 1999).

GSs have a number of interesting features one of which is that their velocity can range from zero to speed of light in the optical medium. Zero velocity or slow solitons have potential applications in optical buffers and memory elements (Krauss, 2008). The existencxe and dynamics of GSs have been studied in different structures and nonlinearities such as dual-core systems (Mak et al., 1998a; Atai and Malomed, 2000), nonuniform Bragg gratings (Atai and Malomed, 2005; Baratali and Atai, 2012), photonic crystal waveguides (Neill and Atai, 2007; Monat et al., 2010), cubic-quintic nonlinearity (Atai and Malomed, 2001; Dasanayaka and Atai, 2010) and quadratic nonlinearity (Conti et al., 1997; Mak et al., 1998b).

Dual-core and dual-mode systems possess rich dynamical features (Atai and Chen, 1992; Mak et al., 2004; Chen and Atai, 1998; Chen and Atai, 1995). In particular, a dual-core system with non-identical cores can provide superior switching performance than the dual-core systems with identical cores (Atai and Chen, 1993; Bertolotti et al., 1995). In this paper, we consider the existence and stability of gap solitons in a dual-core system with Kerr nonlinearity where one core has a uniform Bragg grating and the other is equipped with a Bragg grating with dispersive reflectivity.

2 THE MODEL

Propagation of light in a dual-core nonlinear coupled system with one core having a uniform Bragg grating and the other being equipped with a Bragg grating and dispersive reflectivity can be represented mathematically by the following equations:

$$
\begin{aligned}
&iu_{1t}+iu_{1x}+u_1\left(\tfrac{1}{2}|u_1|^2+|v_1|^2\right)\\
&\quad +v_1+\lambda u_2+mv_{1xx}=0,\\
&iv_{1t}-iv_{1x}+v_1\left(\tfrac{1}{2}|v_1|^2+|u_1|^2\right)\\
&\quad +u_1+\lambda v_2+mu_{1xx}=0,\\
&iu_{2t}+iu_{2x}+u_2\left(\tfrac{1}{2}|u_2|^2+|v_2|^2\right)\\
&\quad +v_2+\lambda u_1=0,\\
&iv_{2t}-iv_{2x}+v_2\left(\tfrac{1}{2}|v_2|^2+|u_2|^2\right)\\
&\quad +u_2+\lambda v_1=0.
\end{aligned}
\tag{1}
$$

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In Eqs. (1), $u_{1,2}$ and $v_{1,2}$ stand for forward- and backward-propagating waves of the cores 1 and 2, respectively. $m > 0$ denotes dispersive reflectivity and λ is the coupling coefficient between the two cores. It is worth noting that $m > 0.5$ may not be physically realizable (Atai and Malomed, 2005). Therefore, we have limited our analysis to $0 \leq m < 0.5$.

To determine the bandgap within which GSs may exist, the linear spectrum of the system needs to be analyzed. To this end, the dispersion relation for the model by is derived by substituting $u_{1,2}, v_{1,2} \sim \exp(ikx - i\omega t)$ into linearized form of Eqs. (1) which leads to the following equation:

$$\omega = \pm\left(k^2 - mk^2 + \lambda^2 \mp \tfrac{1}{2}\left(m^4k^8 - 4m^3k^6 + 4m^2\lambda^2k^4 + 4m^2k^4 - 16m\lambda^2k^2 + 16\lambda^2k^2 + 16\lambda^2\right)^{\frac{1}{2}} + 1 + \tfrac{1}{2}m^2k^4\right)^{\frac{1}{2}}. \tag{2}$$

Figure 1 represents the dispersion diagram or bandgap specturm in (k, ω) plane. It is evident that the bandgap shrinks as λ increases.

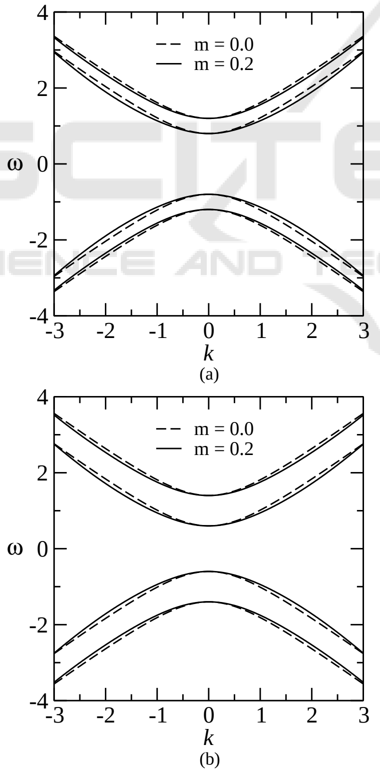

Figure 1: Examples of bandgap spectrum for (a) $\lambda = 0.2$, $m = 0.0$, 0.2 and (b) $\lambda = 0.4$, $m = 0.0$, 0.2.

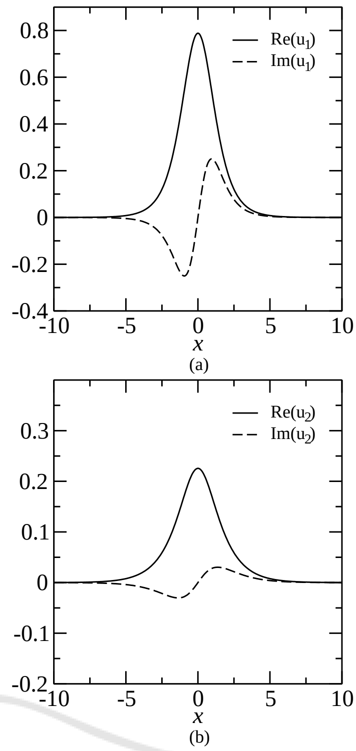

Figure 2: Examples of for (a) u_1 and (b) u_2 at $\lambda = 0.2$, $m = 0.2$, $\omega = 0.4$.

3 QUIESCENT GAP SOLITON SOLUTIONS

The stationary GS solutions must be obtained numerically using the relaxation algoritm since there are no analytical solutions for Eqs. (3). The soliton solutions are sought as $u(x,t) = U(x)\exp(-i\omega t)$ and $v(x,t) = V(x)\exp(-i\omega t)$. Substituting these ansatz into Eqs. (1) results in the following equations:

$$\begin{aligned}
&\omega U_1 + iU_{1x} + U_1\left(\tfrac{1}{2}|U_1|^2 + |V_1|^2\right) + V_1 + \lambda U_2 + mV_{1xx} = 0,\\
&\omega V_1 - iV_{1x} + V_1\left(\tfrac{1}{2}|V_1|^2 + |U_1|^2\right) + U_1 + \lambda V_2 + mU_{1xx} = 0,\\
&\omega U_2 + iU_{2x} + U_2\left(\tfrac{1}{2}|U_2|^2 + |V_2|^2\right) + V_2 + \lambda U_1 = 0,\\
&\omega V_2 - iV_{2x} + V_2\left(\tfrac{1}{2}|V_2|^2 + |U_2|^2\right) + U_2 + \lambda V_1 = 0.
\end{aligned} \tag{3}$$

Figure 2 shows the real and imaginary parts of u_1 and u_2 (note that $v_1 = -u_1^*$ and $v_2 = -u_2^*$) and Figure 3 displays the amplitudes of u_1 and u_2. Our analysis shows that quiescent soliton solutions exist throughout the bandgap.

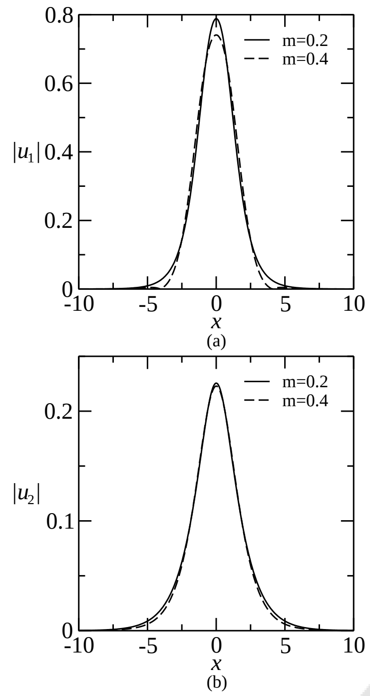

Figure 3: Examples of amplitude variation of (a) u_1 and (b) u_2 at $\lambda = 0.2$, $\omega = 0.4$, $m = 0.2, 0.4$.

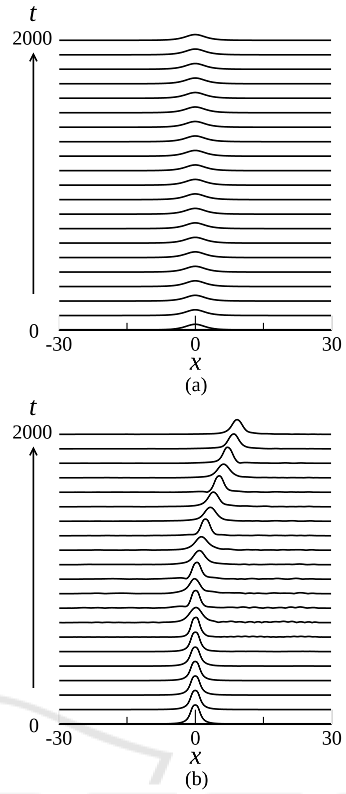

Figure 4: Examples of propagation of (a) stable soliton at $\lambda = 0.2$, $m = 0.2$, $\omega = 0.7$ and (b) unstable soliton at $\lambda = 0.2$, $m = 0.2$, $\omega = -0.7$. Here only u_1 component is shown.

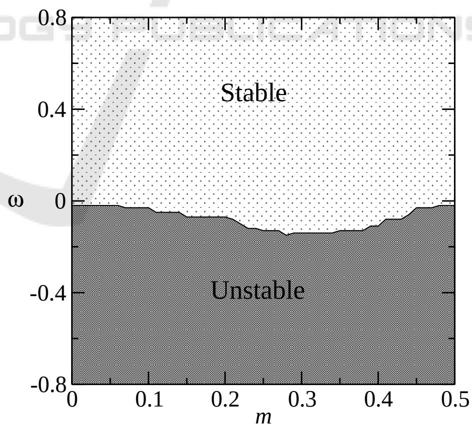

Figure 5: The stability diagram of quiescent gap solitons at $\lambda = 0.2$.

4 STABILITY ANALYSIS OF QUIESCENT GAP SOLITONS

To analyze the stability of the solitons in the model, we have solved Eqs. (1) numerically using the symmetrized split-step Fourier method. Figure 4 shows the examples of propagation of stable and unstable solitons. In the case of unstable solitons, it is found that they generally shed some energy in the form of radiation and they either evolve to a moving soliton (see Figure 4(b)) or if they are highly unstable they are completely destroyed. Figure 5 shows the stability diagram for $\lambda = 0.2$ in the plane of (m, ω). An important feature of this stability diagram is that there is a vast region within the bandgap where stable quiescent solitons exist. Moreover, the stabilization effect of dispersive reflectivity is more pronounced for moderate values m (i.e. when m is in the range $0.2 < m < 0.4$).

5 CONCLUSIONS

The existence and stability of gap solitons are considered in a coupled system with Kerr nonlinearity where one core has a uniform Bragg grating and the other has a Brag grating with dispersive reflectivity. The analysis of the linear spectrum of the model shows that there exists a genuine bandgap in the model where solitons can exist. The size of the bandgap shrinks as the coupling coefficient between the cores is increased. Quiescent soliton solutions are found throughout the bandgap.

Stability analysis of quiescent solitons shows that stable and unstable solitons exist in the system. Unstable solitons may either evolve to a moving soliton or are completely destroyed. Nontrivial stability borders have been identified in the plane of (m, ω).

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