The Implementation of Inquiry-based Learning in the Organization of Students' Research Activities on Mathematics

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Abstract: The article looks into the issue of developing an interest of students' research activities on Mathematics. The study is dedicated to the feasibility of involving the inquiry-based learning to the organization of students' scientific research during the practice on the Approximation Theory and Fourier Series. The research considers the results of the survey among students who helped to evaluate their emotional state during the workshop. To collect the data we used the tool of express evaluation of positive and negative emotionality the Differential Emotion Scale by Izard. The article discusses the positive influence of the environment developed through the inquiry-based learning on students' emotional state and forming their interest in scientific research while organizing practic classes. We have grounds to conclude that there is the efficiency of implementing workshops based on the inquiry-based learning. The index reduction of students' negative emotions encouraged their activity during the practice and the improvement of interest in research activities.

1 INTRODUCTION

One of the main objectives of higher education is to form scientific competencies among would-be specialists that are necessary for further successful professional or academic development. Nechypurenko and Soloviev (Nechypurenko and Soloviev, 2018), Yarullin et al. (Yarullin et al., 2015) have called the organization of students' research activities one of the mechanisms to form their research competence. During such activities, skills that allow a graduate student to create new actual methods of professional activity in the future, develop new ideas and approaches that correspond to the changing modern requirements, are formed. In particular, this idea is supported in pedagogical literature dedicated to mathematical education where the organization of research activities on Mathematics is considered to have a positive influence on the further graduate student's activities in professional researches (Jahnke et al., 1983; Turner, 2010; Vintere and Zeidma, 2016; Proulx, 2015; Koichu and Pinto, 2018). Taking it into ac-

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count, the matter of organizing research activities on Mathematics is still actual in pedagogical researches.

Traditional learning methods focused on the teacher do not provide active students' involvement in research activities (Yore, 2001). According to the results of the researches conducted by the European Association for Quality Assurance in Higher Education, the European University Association, and the Higher School Teachers European Society (EURASHE, 2015), the success of forming students' research activities depends on the selection of learning strategy that is determined as a priority of its methods, where methods of student-focused education come first. It is connected with the variety and increasing expectations from higher education that in its turn requires fundamental changes in providing it and is focused on flexible learning ways of students' involvement in research activities. One of the methods of realizing such an approach is inquiry-based learning that has an official status in many countries of the world (National Research Council, 2000; Rocard et al., 2007; National Research Council, 2006). inquiry-based learning is included in the student-focused educational paradigm where students have to build their activities in the same way as scientists during the process of learning and knowledge grounding. In Mathematics this is emphasized in works (Sandoval and Reiser, 2004; Jahnke et al., 1983; Artigue and Blomhøj, 2013; Dorier and Maass, 2020), where it is stated that Inquiry is one of the most important contexts while learning mathematics. So, the matter of organizing research activities in Mathematics through the implementation of inquiry-based learning corresponds to the requirements of the most important issues of modern fundamental education.

Many authors have emphasized the necessity to support active students' research activities. Lithner (Lithner, 2000) pointed out that international tendency in Mathematics education is acquiring mathematical knowledge not only in terms of context but in terms of getting skills connected with carrying out mathematical research. Bonwell and Eison (Bonwell and Eison, 1991), quoted in (Fallon et al., 2013), stated that students have to do more than just listen. They have to read, discuss, and do research on the problems. Jones et al. (Jones et al., 2019) confirm that at every level of university students' training it is necessary to form their creative thinking and their investigative skills. Scientists emphasize that the organization of students' research activities during their training encourages the development of research competence, necessary both for solving practical problems and for being able to adapt fast to the changeable conditions of the modern time and master their skills constantly. We also took into account the ideas of Dreyfus et al. (Dreyfus et al., 2018), who considers research activities during Mathematics learning as a natural part of the educational process, which is directed at forming research competence among students.

According to Yore (Yore, 2001), the formation of interest in research activities is the first stage during the development of research competencies while learning Mathematics. This idea is agreed with the conclusions by Hernandez-Martinez and Vos (Hernandez-Martinez and Vos, 2018), who have described the critical state of the matter to form students' interest in research activities. Scientists emphasized the importance of organizing students' activities, the formation of their positive attitude to research projects. While learning the literature, the authors of this research were especially interested in the work (Mathiassen, 2000) that describes a research project that included a group of researchers and practitioners who have worked for three years to understand, support, and improve the methods of Systems Development. The work proved a positive influence of practice on theoretically strict research processes and suggested the means of developing research projects that are based on combining traditional theoretical research with experiments and practice. However, not every practical class can be considered a research stimulus. In the organization of research activities, the key aspects of inquiry-based learning are the ability of students to develop new ideas based on previous knowledge and scientific facts; restructure their previous ideas about the scientific concept by adding new studied information; take into account each other, monitor and evaluate their own learning. Only due to this, it is possible to transfer new knowledge into a real context.

In order to organize practice-focused research activities scientists offer to use special courses dedicated to special scientific researches in the priority areas of modern Mathematics. This fact is evidenced by the opinion of Yarullin et al. (Yarullin et al., 2015), Biza et al. (Biza et al., 2016), Telegina et al. (Telegina et al., 2019) about the significant potential in the researches on forming a positive attitude to students' research activities using the materials of different mathematical branches. In scientists' opinion, the use of interesting mathematical theories encourages students to get a more meaningful education of theoretical materials, facts, and methods of solving mathematical problems and it allows getting particular experience. We can also meet the confirmation of this opinion in the works by Matejko and Ansari (Matejko and Ansari, 2018), Sevinc and Lesh (Sevinc and Lesh, 2018), who investigated the organization of research activities related to particular branches of Mathematics.

The idea caught on, that is why guided by the conclusions made by the above-mentioned scientific researches we decided to research the formation of students' interest in research activities on Mathematics through the implementation of practice on approximation theory following inquiry-based learning. The choice of this branch results from its extensive use in practice. This is explained by the fact that the modern stage of science and technology development is characterized by the use of a considerable amount of information. As experience shows this tendency will only enhance in the future – the development of computer science, telecommunication, and registration equipment lead to steady growth of the data amount. Therefore, the tools and methods of their processing and analysis are growing. The creation of a single methodical approach based on general mathematical principles is actual for several tasks such as to get, model, register, and process data. The series finds a mass use as a tool to represent a considerable class of functions, carrying out analytical transformations, approximate calculations in many applied tasks. Algorithmic and computer software that is created on their basis is characterized by high universality and is included in computer and hardware-computer complexes of different purposes, which is confirmed by the numerous researches conducted by (Malvar, 1992; Pankratov et al., 2009), etc.

The research is aimed at forming students' interest in research activities on mathematics through the implementation of practice on approximation theory following inquiry-based learning.

2 METHOD

At the first stage of the research, we used a survey method to assess students' interest in Mathematics research activities. We used the Differential Emotions Scale by Izard (Izard, 1977) to survey students. The relevance of involving this methodology to assess students' interest in research activities is proven by the researches where the direct dependency between the subject's interest in cognitive activities and their emotional state during its implementation is emphasized. Since the feeling is a dynamic component of the emotion (Panksepp, 2003) and two psychobiological processes are connected with it – fascination and individuation (Langer, 1967), motivating, managing, and informative functions of feelings allow them to capture or simplify and organize the thing that can become (especially in difficult situations) a great number of impulses in concentrated cognitive processes. During 2015–2019 we surveyed master's degree students of Physics-Mathematics departments of Kryvyi Rih State Pedagogical University and Berdyansk State Pedagogical University. 49 master's students took part in the survey (17 male students and 32 female students aged from 20 to 28). The use of the online survey, first through Google form, posted on the Internet, and then, moved to the forum of the platform "Higher School Mathematics Teacher" (Vlasenko and Sitak, 2019) had an advantage in comparison to the survey on paper as it encouraged the respondents' frankness and prevented missed questions.

According to the chosen methodology, we selected the Likert scale to assess each of the basic emotions where 1 – "feeling is completely absent"; 2 – "feeling is slightly expressed"; 3 – "feeling is moderately expressed"; 4 – "feeling is strongly expressed"; 5 – "feeling is fully expressed". At the beginning of the research, the most significant ($>$ 9 points) positive emotion related to the experience of Mathematics research activities was "interest", negative – "shame" and "fear". Students usually face the last two emotions while learning Mathematics.

Students believe that the key problem of learning mathematical theory is the absence of the connection between theory and practice and the abstract character of the subject.

At the second stage of the research, we determined the structure of practice regarding Approximation theory and the main aspects of the content that ensure its correspondence to inquiry-based learning. While selecting resources for the analysis of possibilities to use inquiry-based learning we were focused on those that represent the efficiency of its use during the education. Among them, we can name TeachThought (Lesley University Online, 2017), Lesley University (Lesley University Online, 2019), The National Academies Board on Science Education (Bybee, 2009), Alberta Education (Alberta Learning, 2004) (table 1).

We also found out what the purpose of using inquiry-based learning by other scientists was. Cheng et al. (Cheng et al., 2016) noted the efficiency of using the approach to increase the motivation of students' learning. Duran and Duran (Duran and Duran, 2004) describe the use of inquiry-based learning in programs of professional development in education. Supasorn and Promarak (Supasorn and Promarak, 2015) see the use of inquiry-based learning as an efficient method of improving students' understanding of natural processes.

In conclusions of scientific researches (Bybee

Table 1: The analysis of the resources that represent the efficiency of using inquiry-based learning.

Resources	**Used while learning a subject**	**Features**	**What are the efficiency grounds**
Teach Though	Biochemistry and Molecular Biology Education, Mathematics	Joint activities	The solid knowledge foundation through an active part
Lesley University	Mathematics, Life sciences	Constructing knowledge based on experience	Possibility for the full cycle of education
The National Academies Board on Science Education	Biological sciences	Structure and sequence of education are directed at creating a challenging situation	Integration of learning activity with laboratory experience
Alberta Education	Librarianship, work with information	Student's involvement in metacognition; encouragement of critical and creative thinking	Focus on achieving defined learning outcomes in different subjects

et al., 2006; Abdi, 2014; Ong et al., 2018) we also find the confirmation of the efficiency to use the above-mentioned approach to improve students' achievements in science. Considering it, we believe that inquiry-based learning will encourage the alignment of teaching processes with the formation of better students' understanding of scientific knowledge and skills during practice.

The practice program consists of six classes.

1. The history of the development of approximation theory and Fourier series.
2. The ways of periodic function classification.
3. Approximation methods that are based on matrix series summing.
4. Main tasks of approximation theory: approximation of individual function, class approximation, precise, and asymptomatically precise ratio.
5. Examples of researches by subject.
6. Examples of using approximate aggregates in computer complexes of broad purpose.

The practice was aimed at the formation of students' interest in research activities through their implementation in the real process of using series in applied tasks.

The practice was held for a group of 7–8 students twice a month for three months. Every class included two hours of classwork and three hours of extracurricular work. The classes were held by the prominent teachers of Mathematics departments who took part in the development of the practice and looked for the method, the implementation of which would encourage the formation of students' interest in research activities during the practice.

During the organization of practice classes, we developed recommendations for every practice stage that have to encourage the increase in students' interest in mathematics research activities.

At the first stage, the teacher has to determine what students already know regarding the concept that is considered and what kind of knowledge they still need. In order to master new educational material, it is necessary to help students to revise Mathematics sections such as Algebra, Mathematical Analysis, Functional Analysis, and Function Theory. Moreover, at this stage, the teacher is only a consultant who helps students to prepare short reports encouraging students' interest and motivation. For this purpose, the teacher presents the actuality of the researches dedicated to learning approximate features of approximation methods that are generated by certain transformations of partial sums of Fourier series and allow building the sequence of trigonometric polynomials that would equally coincide for any function (table 2).

Table 2: Recommendation for the teacher on the organization of the first stage.

Appropriate	**Inappropriate**
encourage students to raise their questions offer to compare their ideas with others	read the lecture give definitions to terms explain or give tasks

The second stage is aimed at strengthening students' activities regarding knowledge and skills. At this stage, students can revise the tasks that use the methods of Approximation theory on special subjects that they learn. As a rule, students cite examples of tasks on periodic signal approximation in the theory of control engineering, pattern recognition, nondestructive testing, etc. Students can discuss and write down approximation methods in every particular case. The teacher is only a consultant who offers students such research methods as observation, hypothesis generation, forecasting. Students' communication and work in groups without the direct teacher's involvement are encouraged to equally coincide for any function (table 3).

Table 3: Recommendation for the teacher on the organization of the second stage.

Appropriate	Inappropriate
encouragement of search for several ways to solve the problems comparison of ideas self and mutual survey	use of traditional explanation implementation and involvement of a great amount of terminology

At the next stage, students can describe their point of view regarding the search for solving extreme problems of approximation theory. After this, the teacher has to introduce common terminology and acquaint the students with the general scheme of researching integral images of trigonometric polynomial variations that are generated by linear methods of summing Fourier series, from periodic functions. Generating students' new ideas on methods of approximation improvement, their comparison with the ideas of the previous stage is possible. At this stage, the teacher also has to prevent possible mistakes while explaining misconceptions that could arise at the stage of engagement and exploration. During the classes of this stage, the teacher involves interactive methods and presentations for mathematical modeling of periodic processes (table 4).

After getting an explanation about the research main scheme regarding integrated images of trigonometric polynomial variations during the classes of periodic functions it is important to involve students in further research activities. Further work includes significant analytical calculations connected with exact and approximate methods. Starting from the integral image students can learn asymptotic behavior of exact upper bounds of deviations of trigonometric polynomials from periodic functions to infinity. The stage is aimed at helping students to develop a deeper understanding of general methods of mathematical analysis and the use of approximation processes in practical

Table 4: Recommendation for the teacher on the organization of the third stage.

Appropriate	Inappropriate
teacher's explanation expression of the ideas using generally accepted terms idea review and formation of new ones	forming a great amount of terminology focus on independent work

tasks. Students can carry out additional researches, develop new approximation methods, exchange ideas, and use acquired research experience to integrate Approximation theory in practice (table 5).

Table 5: Recommendation for the teacher on the organization of the fourth stage.

Appropriate	Inappropriate
enhancement of understanding through strengthening the ideas acquired by experience use of algorithms that are close to new situations grounds for conclusions support of forming student's proper ideas	development of the ideas that are not connected with previous experience generating a great number of ideas without deepening in the essence of the theory

The practice of working in small groups is important at this stage. The lessons include planning and preparation of students' proper development on using the considered approximation methods from every group of students. It is possible to create an algorithmic and program-algorithmic product based on the created methods. As the simplest and at the same time the most natural example of a linear process of approximation of continuous periodic functions of the real variable can be the approximation of these functions using the sequence elements of partial sums of Fourier series, the greater majority of students have a basic idea about the techniques of using these methods while creating an algorithmic product. But, as it is well known, the sequences of partial sums of Fourier series $S_n(f;x)$ are not equally similar for the entire class of continuous periodic functions. Thus, a considerable number of students' developments in this area are directly dedicated to the learning of approximate features of other approximation methods that are generated by particular transformations of partial sums of its Fourier series for this function and allow building the sequence of trigonometric polynomials that would be completely similar for every function (Rovenska, 2019). Fejer sums $\sigma_n(f;x)$ are arithmetic averages for the first n of partial Fourier sums for this function and, as it is known, the sequence of polynomials $\sigma_n(f;x)$ equally coincides with its function. Sums of de la Vallee Poussin $V_{n,p}(f;x)$ are a synthesis of sums $\sigma_n(f;x)$ and have approximate features that depend a lot on the parameter p. Trigonometric polynomials $V_{n,p_1,p_2}(f;x)$ that are generated by the repeated use of de la Vallee Poussin summation method are the further synthesis of classical Fourier methods, de la Vallee Poussin and Fejer (Novikov and Rovenska, 2017). Choosing particular parameters p_1 and

p_2 these polynomials coincide with the sums $S_n(f;x)$, $V_{n,p}(f;x)$, $\sigma_n(f;x)$. The works of practice participants should be dedicated to the learning of approximate features of such approximation methods showing graphically the advantages of its use (figure 1, 2). For the visualization, students can be recommended a system of computer mathematics Maple that includes developed graphic means.

The demonstration of the efficiency of the selected approximation methods can be done by comparing the results of numerical experiments held simultaneously for the operators $S_n(f;x)$, $V_{n,p}(f;x)$ and $V_{n,p_1,p_2}(f;x)$. Meanwhile, it is necessary to pay students' attention to the fact that the aggregate of all the harmonicas that are used to build the operators $S_n(f;x)$; $V_{n,p}(f;x)$ coincides with a similar aggregate for the operator $V_{n,p_1,p_2}(f;x)$. At the same time, the program for the numeric implementation of the values $S_n(f;x)$, $V_{n,p}(f;x)$ and $V_{n,p_1,p_2}(f;x)$ can be developed using Python. This tool is easy to use for students–non-programmers and is suitable for easy calculations.

The final stage of practice is dedicated to evaluation. Evaluation is considered to be a permanent process during which the teacher only observes the students and supports them during report presentations, idea introduction, and question tasks. The use of peer assessment is relevant. Such a form of evaluation can be complemented by students' self-assessment of their level. During the classes of this stage, the teacher involves interactive methods and presentations for mathematical modeling of periodic processes (table 6).

Table 6: Recommendation for the organization of the final stage.

Appropriate	Inappropriate
evaluate the progress in general in comparison to the initial level evaluate the ability to use approximate methods to solve complex problems give students feedback regarding the feasibility of their ideas encourage questions that enhance a deeper understanding of the influence of individual function features on the approximation order	evaluate single facts and separate elements of approximation theory offer a survey in a test form

The use of inquiry-based learning does not oblige the teacher to strictly follow the indicated stages. If necessary, it is possible to repeat them several times (Bybee and Landes, 1990). This fact proves the flexibility of using this approach for the implementation of scientific practice.

3 RESULTS

During the preparation stage, we selected the target type as a selection strategy, because the selection had to include the students who have a high achievement level in mathematical branches. By high level, we understand the absence of the final mark "satisfactory" and lower following the national 4–level scale "unsatisfactory", "satisfactory", "good", "excellent" for each of the subjects "Algebra", "Mathematical analysis", "Functional analysis" and "Function theory". The target selected analysis provided us with a sample size n=49 of students that represents 23% of the general number of master's degree students of the first year during 2015–2019. At the stage of organizing data collection, we used the tool of express-evaluation of positive and negative emotional states called the Differential Emotion Scale (Izard, 1977), which ensures diagnostics of a wide range of emotional states. Each of the ten basic emotions (x_i, $i = 1, 2, ..., 10$) is represented by three independent changeable 5–character scales for factors that describe emotional states. The points on every scale correspond to the level of emotional feedback and can be in total from 3 to 15 points. The stage of data analysis of every profile implies the selection of significant ($>$ 9 points) emotions, creation of "emotion profile", determination of the dominant emotional state.

At the beginning of the research, the most significant positive emotions regarding the experience of research activities are "interest", negative – "shame" and "fear" (table 7).

While processing every profile we defined the indexes of emotional states that characterize the level of subjective students' emotional attitude to the present experience of research activities. The Index of positive emotions and Index of critically negative emotions could range from 9 to 45 points, the Index of anxious–depressive emotions ranged from 12 to 60 points. We defined that the positive emotional state turned out to be dominant among 69.4% of students; a strong level ($>$ 36 points) of expressing a positive emotional state was marked only among 6.1% of respondents. Also, a distinct (from 29 to 36 points) level of positive emotional state was fixed among 10.2% of students. Other students (53.1%) showed moderate (from 20 to 28 points) and weak ($<$ 20 points) level. So, most students' attitude to the research process can be mainly characterized as positive. However, this positive attitude is weakly expressed, unstable, and cannot ensure the proper motivation in overcoming difficulties that inevitably arise during research activities. This fact plays an important (if not the most important) role in the failure of attempts to involve an

$$s[f]=\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^{n} n^{3}\sin nx$$

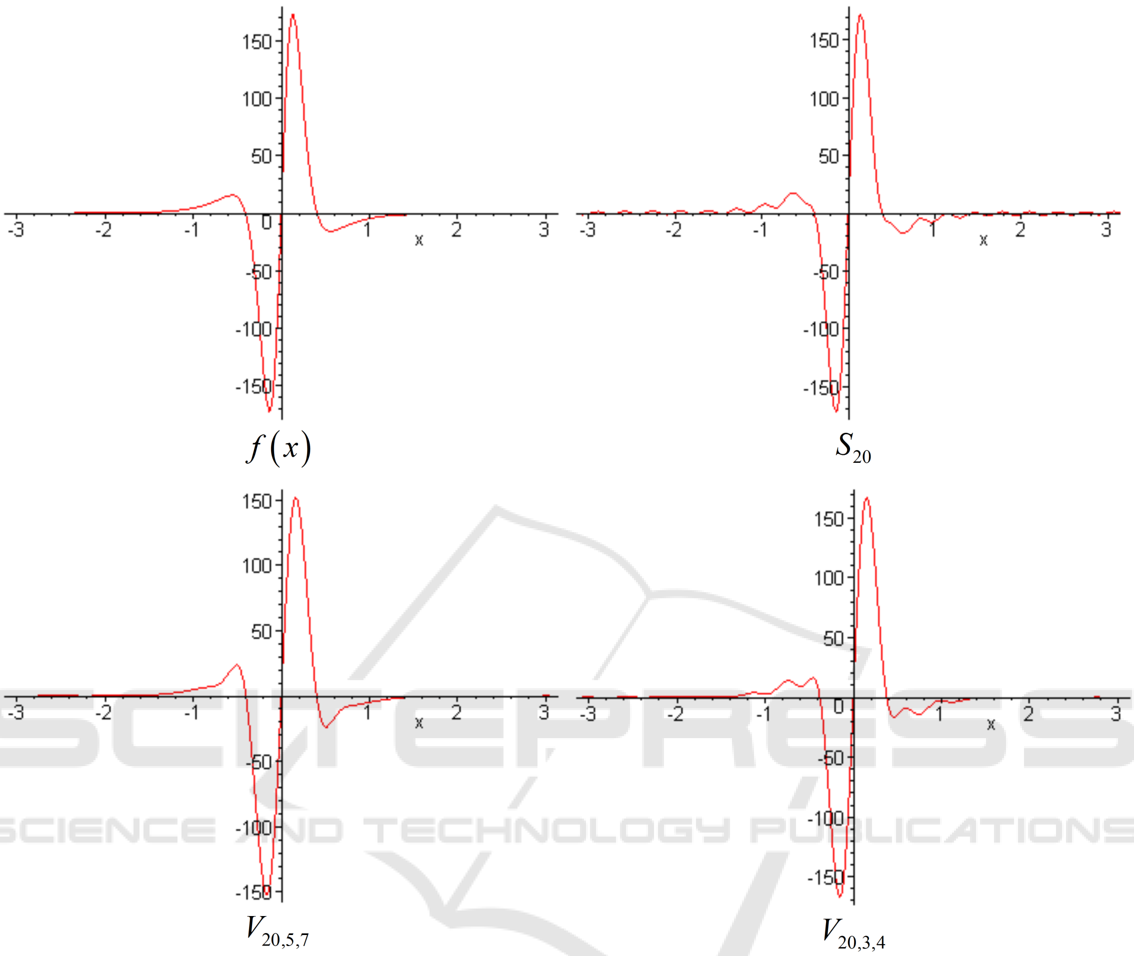


Figure 1: Visualization of functions and trigonometric sums that are generated in different methods of summarizing Fourier series in the system of computer mathematics Maple.

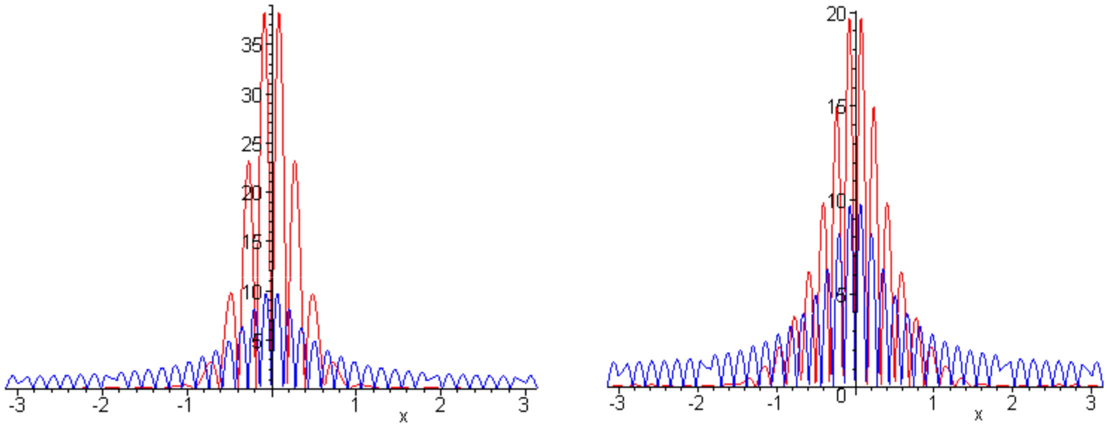
Table 7: Distribution of significant emotions at the beginning of the research.

Emotion	**Number of students who have this emotion as dominant (>9 points)**	**Comparison with the general number of students**
Interest	32	65.3%
Fear	45	91.8%
Shame	27	55.1%

unprepared student in research activities in any area, including Mathematics.

The dominant critically negative emotional state regarding the present experience of research activities was fixed among 12.2% of respondents, half of whom had a strong (> 32 points) or distinct (from 25 to 32 points) level. It is important that among all the students who had the critically negative state as dominant, the factor "Dull" took no less than 4 points, and, accordingly, made the greatest contribution to the calculation. It testifies a stereotype regarding the complexity and absence of interest in research activities among young people. We considered this aspect while searching for methods of practice implementation.

As mentioned above, the emotions "fear" and "shame" were detected as significant among 91.8% and 55.1% of respondents. These emotions are in-



$|f - S_{20}|, \quad |f - V_{20,5,7}|$ $\qquad$ $|f - S_{20}|, \quad |f - V_{20,3,4}|$

Figure 2: Visualization of deviation of Fourier series and repeated de la Vallee Poussin repeated series from the function $f(x)$ in the system of computer mathematics Maple.

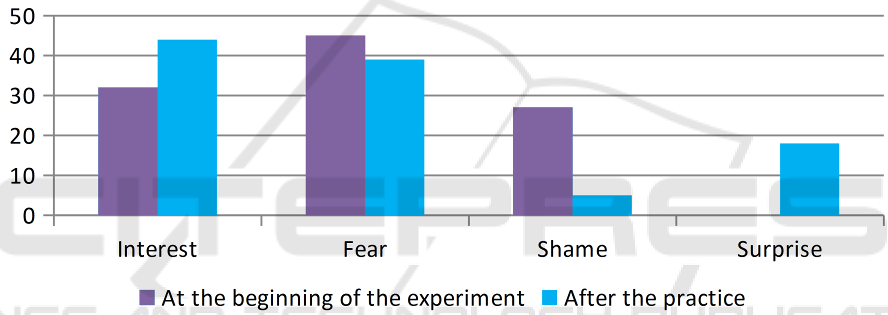


Figure 3: Distribution of a significant emotion.

Table 8: Distribution of significant emotions after taking practice.

Emotion	**Number of students who have this emotion as dominant (> 9 points)**	**Comparison with the general number of students**
Interest	44	89.7%
Surprise	18	36.7%
Fear	39	79.5%
Shame	5	10.2%

cluded in the third group of emotions that determine the anxious–negative emotional state of the subject regarding the experience of research activities. Despite this fact, the given state is dominant only among 18.4% of students. It demonstrates that these two emotions influence the formation. 4.1% of respondents have strong (> 30 points) level of emotional state, distinct (from 21 to 30 points) – 10.2%, moderate (from 12 to 20 points) and 4.1% of respondents – weak (< 12 points). Such a noticeable selection of two emotions in the general image of the emotional state confirms the idea that fear and shame prevent students from implementing their interest in the research process and take an active position while conducting research.

The repetitive survey was carried out after finishing the practice. The distribution of significant emotions after taking practice is represented (table 8).

Interest turned out to be a significant positive emotion among 44 students. We can note that the number decrease in students who had shame as a significant negative emotion is well seen – 17 respondents. At the same time, the number decrease of students who had fear as a significant emotion is minor – 6 students (figure 3).

Despite this fact it is impossible to claim that this emotion in the context of the given research is badly adapted. The profile analysis of respondents' emotions shows the decrease of fear expression to varying degrees among 77.5% of students. The presence of surprise among the significant emotions, as well as interest, which is included in the positive group, is predictable.

More detailed analysis of the feasibility of implementing practice that was carried out using the index calculations of students' emotional states. We detected the increase of students with the dominant positive emotional state up to 81.7%, where 63.2% of respondents had a strong and distinct level. At the beginning of the practice, the same indicator was 16.3%. Thus, we managed to form a stable positive attitude to research activities among more than half of the practice participants.

The number of students who have a critically negative emotional state as dominant remained at the level of 12.2%, though the qualitative structure of this subgroup changed. In our opinion, it is connected with a greater amount of working practice in small groups during classes in comparison to individual work. As teachers pointed out certain students perceived such a format negatively.

The dominant anxious–negative subject's attitude to experience of research activities after taking a practice was fixed among 6.1% of students. Among them 4% of respondents have moderate and 2.1% – weakly expressed level of emotional state. The comparative analysis of the students' number regarding dominant emotional states is displayed (figure 4).

The analysis of the results proved that creating the environment based on inquiry-based learning during the scientific practice where students did not feel negative emotions to research activities encouraged the increase of their interest in research activities.

4 DISCUSSION

Searching for ways of forming students' interest in research activities on mathematics we faced the researches (Sandoval and Reiser, 2004; Rocard et al., 2007). The scientists point out that in order to form students' impression of the real world it is necessary to show them how to organize their activities as real scientists do during the process of learning and knowledge grounding. Fallon et al. (Fallon et al., 2013) offered to seek the possibilities to organize students' research activities through the method selection and forms of a learning organization that influences active students' involvement.

Traditional educational methods, which are focused on the teacher, don't provide an active students' involvement in research activities (Yore, 2001; Lin et al., 2014; Vlasenko et al., 2019). The scientists emphasize the importance of searching for educational models that encourage the strengthening of students' learning activities. The Deductive Content Analysis Method helped us to choose inquiry-based learning as the foundation of developing a scientific environment for students' education.

The efficiency of inquiry-based learning to encourage students' research activities is proved in (Duran and Duran, 2004; Bybee and Landes, 1990; Suparsorn and Promarak, 2015; Cheng et al., 2016). Also, we support the opinion by Vlasenko et al. (Vlasenko et al., 2019), who believe that learning has to be built so that students can research, explain, extend and estimate their progress, and the introduction of ideas assumes students' awareness of the reason or necessity of their use. The indicated aims are fully agreed with the content of inquiry-based learning.

Alshehri (Alshehri, 2016) believes that while organizing research activities it is necessary to direct students to the main models of subject matters. One of the key subject matters of Mathematics is Approximation theory, its broad influence on the modern state of innovation and technology development is widely known. The research is aimed at searching for ways of implementing a practice on Approximation theory to form students' interest in Mathematics research activities. The main research result testifies that the use of the approach inquiry-based learning influenced efficiently the formation of students' positive attitude towards research activities. Within this approach, the involvement of the practice on Approximation theory encouraged the increase of the level of expressing students' positive emotional state (particularly interest, surprise increase) and decrease of anxiety level. These results are agreed with the conclusions by Chin and Lin (Chin and Lin, 2013), Abdi (Abdi, 2014), Jung et al. (Jung et al., 2014), Ong et al. (Ong et al., 2018), who studied the connection between interest growth and a person's emotional state. This justifies the use of methodology Differential Emotions Scale by Izard (Izard, 1977) during the experiment.

5 CONCLUSION

The actuality of involving students in research activities in education arises from the fact that research competence is considered as one of the components of professional competence. Enhancing students' interest in research content and research activities during the studies also requires the use of an approach

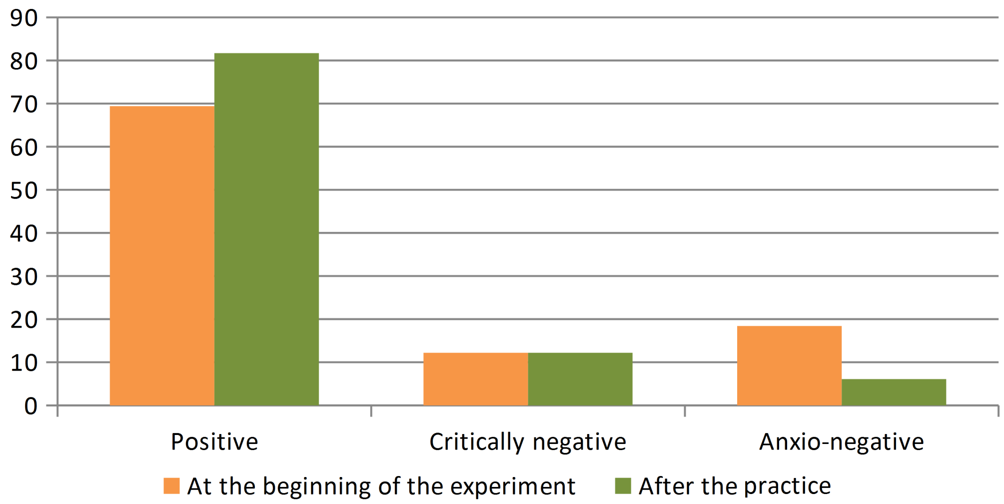


Figure 4: Distribution of dominant states.

that implies complete students' awareness of the importance of the research problem. The Deductive Approach to Content Analysis helped us determine the possibility to involve inquiry-based learning to the organization of practical classes on Approximation theory, determine its characteristics and efficiency parameters, predict that the approach can ensure the formation of a better understanding of scientific knowledge and students' skills. According to inquiry-based learning, we developed the content of the practice on Approximation theory. Based on the analysis of the current recommendations on using inquiry-based learning while studying different subjects we offered recommendations on the organization of practice. It should be noted that the course should be provided following the indicated recommendations that encourage students' activity, their interest in the research activity.

Forming a positive attitude to research activities is the first step to the development of the research competence of pre-service specialists. The analysis of the works on the connection between the person's interest and emotional state allowed formulating the most important positive and negative emotions that are connected with the experience of the research activities. The results of calculating the indexes of students' emotional states proved that the creation of the environment according to inquiry-based learning where students do not feel negative emotions to research activities encourages emotional state and interest in research activities.

The perspectives of future research involve the creation of Math courses that use inquiry-based approaches with the purpose of further research on forming research competence among students.

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