# Digital Technologies in Specialized Mathematics Education: Application of GeoGebra in Stereometry Teaching

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Abstract: The purpose of the paper is to improve methodology of teaching Mathematics via the use of digital technologies. The task of the paper is to identify the issues that require a theoretical and experimental solution. The objective of the paper is the educational process in the higher education institution, the subject of the paper is modern ICT. The result of the study is the learning tools of pedagogically considered and adequate bending of conventional and modern learning environment implemented into the educational process. The possibilities of using cloud technologies and Dynamic Mathematics system GeoGebra in the educational process through Stereometry specialized training have been revealed. The use of GeoGebra Dynamic Mathematics in Stereometry teaching will favourably influence the formation of students' STEM competencies. In order to encourage Mathematics and Computer Science teachers to implement effectively the elements of STEM education, it is suggested that cloud-based learning tools such as GeoGebra be used in the teaching process.

1 INTRODUCTION

Velikova and Petkova (Velikova and Petkova, 2019) point out that every society needs STEM professionals, talented people who design new technologies, materials, constructions and processes. One of STEM training related fields is an integrated problem-based Mathematics training.

The results of the research study regarding the development of teachers' STEM competencies have shown that most of them are not aware of the peculiarities of students' STEM competencies formation. And some contradictions between the necessity to form the students' STEM competencies and insufficient attention to the training of future teachers of Mathematics occur.

One of the most effective tools for STEM-based Mathematics teaching is the system of dynamic mathematics, e.g. GRAN 2D tool and cloud-based GeoGebra. Lavicza et al. (Lavicza et al., 2020) claim that GeoGebra was designed to integrate arithmetic,

algebra, geometry, calculus, statistics. In recent times it is supposed to support STEM subjects as a single, integrated system available on most technology platforms. Moreover, teachers and students are offered free access to it all over the world. GeoGebra was originally created to integrate Algebra and Geometry into a single environment. GeoGebra enables to use such new technologies as Augmented (Striuk et al., 2018) and Virtual Reality (Lavrentieva et al., 2020), 3D Printing (Hevko et al., 2020) and MID (Modlo et al., 2019) in the learning Mathematics.

The use of GeoGebra Dynamic Mathematics in Stereometry teaching will enhance the formation of students' STEM competencies. It can also be helpful in the formation of key mathematical and digital competences (Astafieva et al., 2020; Moiseienko et al., 2020), self-study competences, as well as in the development of spatial thinking.

In order to encourage Mathematics and Computer Science teachers to implement the elements of STEM education, it is suggested the cloud-based learning tools, such as GeoGebra, be used in the learning process.

The 3D Graphics application (3D feature graphs, surface, and 3D geometry) can be used while develop-

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ing the visuals with AR. The application is now available not only for users of gadgets in the iOS operating system, but also for Android version 4.0 and higher. This program includes some examples of 3D mathematical objects that can be placed, such as on a desktop, floor, or any other flat surface. The fixed models will be managed. They can be resized. Such visualization will allow you to see more mathematics in the surrounding world.

More detailed information about the application 3D Graphics can be found in our article (Kramarenko et al., 2019b), teaching aids visualization during lectures and practical classes, in particular in Mathematics, allows students to understand the learning material better, to increase the applied orientation of learning and the communication competence both learners and teachers.

Gartner attributes Artificial Intelligence Education Applications, Conversational User Interfaces, Blockchain in Education, Immersive Technology Applications in Education, Design Thinking, Competency-Based Education Platforms and Adaptive Learning Platforms to the main tendencies of using ICT in education (Panetta, 2019).

Since augmented reality technology already has an important place in innovative development, it can also have significant potential for implementation in Mathematics learning. That is why this technology needs more detailed study. Because augmented reality is intrinsically linked to 3D construction, its usage in conjunction with Dynamic Mathematics systems like GeoGebra, can significantly increase the level of visualization in Mathematics and enhance students learning. In addition, Augmented Reality can become a tool for enhancing STEM-based learning for students majoring in Mathematics and Computer Science.

2 LITERATURE REVIEW

According to the recommendation of the European Commission (EC, 2018), competence is defined as a combination of knowledge, skills and attitudes. In the updated list of key competences of a personality eight major ones are emphasized (EC, 2018). We are keen to foster the development of students' STEM competencies: mathematical competence and competence in science, technology and engineering.

Rakov et al. (Rakov et al., 2009) explores the possibilities of mastering mathematical competencies in the learning by future teachers through the research approach using the system of dynamic mathematics DG. The scientist distinguishes procedural,

logical, technological, research and methodological competencies of Mathematics teacher. In particular, methodological competence implies the ability to assess the appropriateness of using mathematical methods to solve individual and social issues.

Consider in detail the opportunities which GeoGebra gives to a teacher. Hohenwarter et al. (Hohenwarter et al., 2008) emphasize a significantly important role of free open-source software for teaching mathematics world-wide. Open-source software do not only offer opportunities for teachers and students to use them both at home and in the classroom without any restrictions, but they also provide the tools for developing support and user communities reaching across borders. Such collaboration as well contributes to the equal access to technological resources and democratization of learning and teaching mathematics.

GeoGebra is available in multiple languages. Using GeoGebra a teacher can create interactive materials to solve the main tasks of teaching Geometry – the development of spatial imagination, practical understanding and logical thinking.

GeoGebra can be used both in standalone and online modes on mobile Internet devices (MID) which offer new opportunities to improve learning and teaching either inside or outside of classrooms. Further advantage of GeoGebra is the Resources tab containing a number of teaching materials: textbooks, articles, instructions, and illustrations. Also, a registered users can share their own educational materials on the official website via GeoGebraTube (this is a kind of file-sharing site). Its essential feature is that the created dynamic constructions can be accessed online via the use of View Sheet tool. Any user is allowed to comment on GeoGebraTube content.

A user is provided with a possibility to create their own GeoGebraBook, a folder where selected materials can be added and subfolders can be created. A user can publish a game, a collection of visuals, lesson workbook that may include text messages, GeoGebra models, web pages, questions, videos, images and manuals public to anyone. GeoGebraTube content can be shared via social networks Facebook, Twitter, emailed and uploaded to Google Drive. Cloud-based tools affect favourably teacher-student interactions in an informal learning environment.

Diković (Diković, 2009) statistically confirms the fact that the use of GeoGebra applications in learning calculus had a positive effect on the development of students' reasoning and visualization skills.

The issues of using GeoGebra are highlighted in (Drushlyak et al., 2020; Semenikhina, 2017; Semenikhina and Drushliak, 2014). Application of Ge-

oGebra 5.0 to the solution of solid geometry problems has been analyzed in (Semenikhina and Drushliak, 2014). Examples of solid geometry problems with detailed solution and learning clues have been presented. Among them there are problems on auxiliary section, polyhedron net, locus problems, space transformations etc. Semenikhina (Semenikhina, 2017) discusses the relevance of the study of the dynamic Mathematics software for future teachers of Mathematics in Ukraine.

The purpose of study by Zetriuslita et al. (Zetriuslita et al., 2021) was to identify improvements in Self Efficacy and selfregulated through GeoGebra Based Teaching seen from the level of students' initial mathematical abilities. The research method used was a mixed-method with a sequential explanatory strategy, while the research design was an untreated control group design with pretest and posttest. the experimental group obtained direct learning using GeoGebra software while the control group obtained conventional learning. The population in this study were university students of mathematics education department at the Islamic University of Riau who took the field and space analytic geometry course. The research instruments used were self-efficacy questionnaire and self-regulated questionnaire for getting quantitative data and interview guidelines for getting qualitative data. It was concluded that GeoGebra based teaching was effective in improving students' self-efficacy and self-regulated. Judging from the level of students' initial mathematical abilities, there was an increase in self-efficacy and self-regulated through GeoGebra based teaching from both high, medium and low levels.

Modelling implies simplifying thus providing the model with its own characteristics, independent from the original reality. Consequently, geometry, in its interpretation and processing, acts like a new reality both abstract in its logic and concrete in its representation modes.

Richard and Blossier (Richard and Blossier, 2012) begin with classical theories of modelling representation processes in order to apply them to 3D dynamic geometry software, GeoGebra3D, computing development seeking to bring geometric models closer to the reality of a school setting. The authors introduce three key notions, developed from Mathematics teaching, in order to support the exploration of two interaction situations between mathematics and their teaching. Finally, they finish on a few general considerations for the teaching of mathematics.

A number of works of scientists and software developers are devoted to the research of integration issues of the augmented reality technology into the ed-

ucational process. In particular, (Brzezinski, 2019; Striuk et al., 2018), focus on the general trends and special issues of the augmented reality application in education.

In teaching Stereometry, the assignment worksheets on the construction of sections through the trace method, internal projection and combination method by Sidoruk (Sidoruk, 2018) should be taken into consideration. Each of the assignments is provided with the possibility of step-by-step procedure of the construction building. Thus, students are given an opportunity to develop both mathematical competence and the ability to learn independently. Some assignments include constructions created on a 3D canvas.

Fariha and Lestari (Fariha and Lestari, 2019) describe in detail how to realize the Dandelin Sphere more realistically, using GeoGebra Classic 5, which supports the creation of 3D images.

At a profound methodological level, a number of assignments on combinations of geometrical solid shapes was created by Rykovskyi (Rykovskyi, 2018). These models are designed as constructions of plane geometric shapes through the use of parallel projection properties. They are intended to be used to visualize the properties of geometrical shapes. However, students might face a challenge with reproducing models as the algorithm of the construction and details of the construction are not available for them.

The models are designed as constructions of plane geometric shapes with the use of the properties of parallel projection. In particular, a book / collection of visuals covering the topic "Prism" is recommended. There is a number of visuals that represents combinations of stereometric shapes: sphere and pyramid, sphere and prism, sphere and cylinder etc. Consider the visual "There is a cone inscribed in the sphere. And there is the right triangular pyramid inscribed into the cone. A pyramid is inscribed in the sphere". Even the name itself suggests that integrity is inherent to visuals. It can be used in the process of teaching Stereometry in school or teaching Methodology of Mathematics on various topics. While using visual aids, users are advised to study the properties of shapes inscribed into the sphere, to look into the relationship between the sphere radius, radius of cone base, the height of the cone and element of the cone.

These visuals are equipped with the Checkbox tool which allows a user to do the revision and answer to the questions: Which cone is called inscribed into the sphere? What element of the cone is the center of the sphere? Which pyramid is called inscribed in the cone? Which pyramid is called inscribed into the sphere? How to find the axial section of the cone?

Which circle of the sphere is called a great circle? Which properties of the diameter of the sphere perpendicular to its great circle?

Mobile applications such as Geometry, Graphing Calculator, 3D Graphing Calculator have been relatively recently launched and now they are available on the official GeoGebra website. These applications are still undergoing improvements. However, the methodology of their use, and especially in teaching Stereometry, is not yet well developed. Therefore, this paper aims at elucidating the features of creating visuals with GeoGebra and their use in teaching how to solve Stereometric problems.

3 RESULTS

3.1 GeoGebraBook "Models for Stereometric Problems" as Training Manual on the Methodology of Teaching Mathematics

The methodical materials of the paper authors related to the use of the GRAN software are presented in the manual "Innovative information and communication technologies of teaching Mathematics". The issue of GeoGebra use has been considered in an updated version of the manual (Kramarenko et al., 2019a). On the GeoGebra site, users can find the materials which contain relevant visuals at GeoGebraBook "Models for stereometric tasks" (Kramarenko, 2019). One can find here problem situations to be solved by students independently, self-assessment tests.

The study of the effectiveness of the use of GeoGebra mobile applications in the process of teaching Stereometry was conducted by us within the preparation of future teachers of Mathematics during Mathematics teaching methodology classes, advanced training courses at Kryvyi Rih State Pedagogical University, in teaching students of Kryvyi Rih State College of Economics and Technologies, and students of classes with mathematical specialization of Kryvyi Rih schools.

Particular interest in using GeoGebra was shown during the advanced training courses by Kryvyi Rih teachers of Mathematics who have been working at school for a considerable time. The study was also conducted on the effectiveness of the use of GeoGebra applications by students in order to enhance students' motivation during extra-curricular vocational activities at the university. And it was studied as well during the course of Sereometry lessons conducted by masters majoring in Mathematics education complet-

ing professional practice in secondary education institutions.

When using applications, it is useful for a student to be able to reproduce the construction in the browser window, open other temporarily hidden drawing elements in case of necessity, and receive text notifications. However, a considerable number of constructions should be created later or simultaneously with the viewing via the mobile application.

Consider the peculiarities of the implementation of specific constructions and provide recommendations concerning their use.

One of the main issues that school students and students of higher education establishments face is the recognition of a 2D drawing representing the image of a 3D shapes. In order to develop the spatial imagination of students at schools and universities it is necessary to provide them with the assignments on constructing sections of polyhedron by planes.

3.2 Construction of Sections of Polyhedron With Plane via the Applications GeoGebra Geometry and 3D Graphics

Consider examples of problems for constructing a section of a polyhedron with a plane passing through three given points that are not on the same line; through a straight line and a point that does not belong to it etc. It is necessary to construct a linear angle of a dihedral angle and measure them; the angle between the straight line and the plane.

Look into what four approaches to constructing a section of a polyhedron in a plane which should be distinguished by a teacher in order to be able to provide their students with the most appropriate one for their educational trajectory (Kramarenko et al., 2020).

If a student has not fully mastered GeoGebra tools, it is advisable to provide them with a sheet of paper guiding the algorithm to follow in order to build a construction. Moreover, a student should be supplied with information regarding the icons that correspond to the tools in question. It is appropriate to place QR codes on printable worksheets for demonstrations.

1. The construction on a 2D canvas is rather cumbersome and requires logically justified steps for the construction. At first, it is necessary to follow step-by-step construction procedures to build several drawings, created via "the trace method" on a 2D canvas. To intensify the material dissemination, students may be at once provided with a constructed polyhedron. However, in the course

of the study, there were cases when the students constructed the polyhedron wrongly. That is, after its construction on the plane, they were not able to change it dynamically. For example, to build a prism image on a 2D canvas made of an *n*-sided polygonal base ($n > 3$), it is necessary to follow the following steps. Firstly one should build an *n*-sided polygon via the use of the appropriate tool. Then it is preferable to build a vector along one of the edges of the prism. Further, one should apply a parallel translation of the *n*-sided polygon to the constructed vector. The given vertexes are connected. Such a construction is represented taking into account the properties of the parallel projection.

2. Next it is necessary to compare the previous drawing with the one presented according to the same problem situation but on the 3D canvas. In this case, all straight lines for the construction of the section can be presented step by step. The advantages of such a construction include the possibility to change the constructed polyhedron dynamically and the location of points which the secant plane passes through. Via the GeoGebra 3D Geometry (GeoGebra Team German, 2018) app, one can simulate external actions aimed at geometrical solid shapes which are necessary for a student to be able to apply internal thinking regarding them and as a result develop spatial thinking. In our opinion, this approach is the most appropriate for the development of students' spatial imagination, logical thinking. In figure 1 the construction of the section of the prisms with a secant plane which passes through the point on the lateral edge and a straight line drawn in the plane of the base is presented.

3. The third option for constructing a section on the 3D canvas. Firstly, a polyhedron, and a secant plane should be built, and then the tool "Intersection of two surfaces" should be applied. A student, by "turning" the polyhedron, will have an opportunity for the better perception and understanding of the construction. The approach in question is appropriate when augmented reality tools are used enabling to align the drawings with the image of a real object (Kramarenko et al., 2019b). It is advisable to encourage students to find examples of such sections among the surrounding objects, in architecture etc.

4. The use of both 2D and 3D canvases simultaneously. Such a combination is advisable if there is some kind of difficulty in constructing a polygon that is for a base of the pyramid or specifying the position of the vertex of the pyramid, etc. For instance, if a trapezoid, in particular, a rectangular or equilateral one, is for a base of the pyramid or prism (figure 2).

It is worth pointing out that such constructions can only be built in the classic version of GeoGebra, and the construction preview may also be available via MID after uploading the file to the GeoGebra cloud repository.

3.3 The Peculiarities of the Definition and Angle Construction between Planes and Linear Angle of the Dihedral Angle

Via the assistant of GeoGebra (3D canvas) it is possible to build the base of the pyramid (OXY) on the canvas, and in the process the constructions can be synchronously displayed on the 3D canvas (figure 3).

Attention should be drawn to the convenience and ease of the construction with the help of the linear angle of the dihedral angle at the base. According to the definition, to determine the linear angle of the dihedral angle at the base a plane perpendicular to the edge of the dihedral angle should be drawn. Then the angle between the rays formed as a result of the intersection of this plane with the edges of the dihedral angle will determine its linear angle (Bevz et al., 2011).

It should be stated that the measure of the angle between the planes varies from 0° to 90°. While the measure of the dihedral angle can vary from 0° to 360° (Rykovskyi, 2018). Here is an example of the problem from the textbook with major in Mathematics (No. 836, (Bevz et al., 2011)).

At the base of the pyramid there is an isosceles triangle with angle β at the vertex and radius *R* of the circle described. The plane of each face of the pyramid forms an angle α with the plane of the base. Find the area of the side surface.

It is recommended to use 2D and 3D canvases simultaneously to represent the drawing corresponding to the set problem (Kramarenko, 2019).

The reason why mistakes are often made is that only one case is under consideration. Whereas the concept of "angle between the plane of the lateral face and the plane of the base" is substituted by "dihedral angle at the base". If the problem situation said that "dihedral angles at the base were congruent", then we would consider the pyramid the vertex of which is projected into the center of the circle inscribed into the triangle of the base (Pyramid of Type 1).

Since it concerns only congruent angles between planes, the problem will have two solutions. It is also
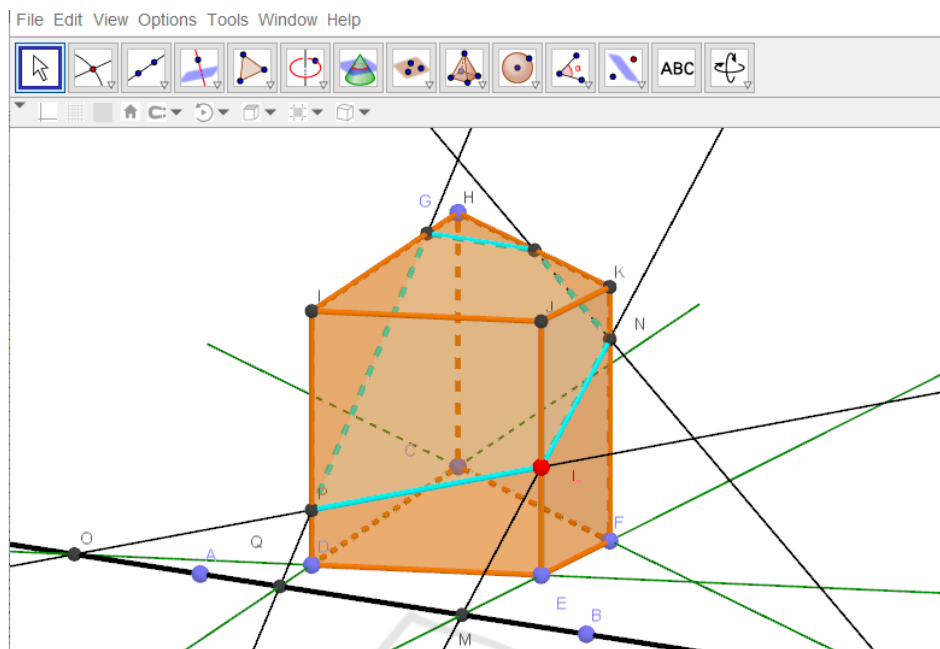
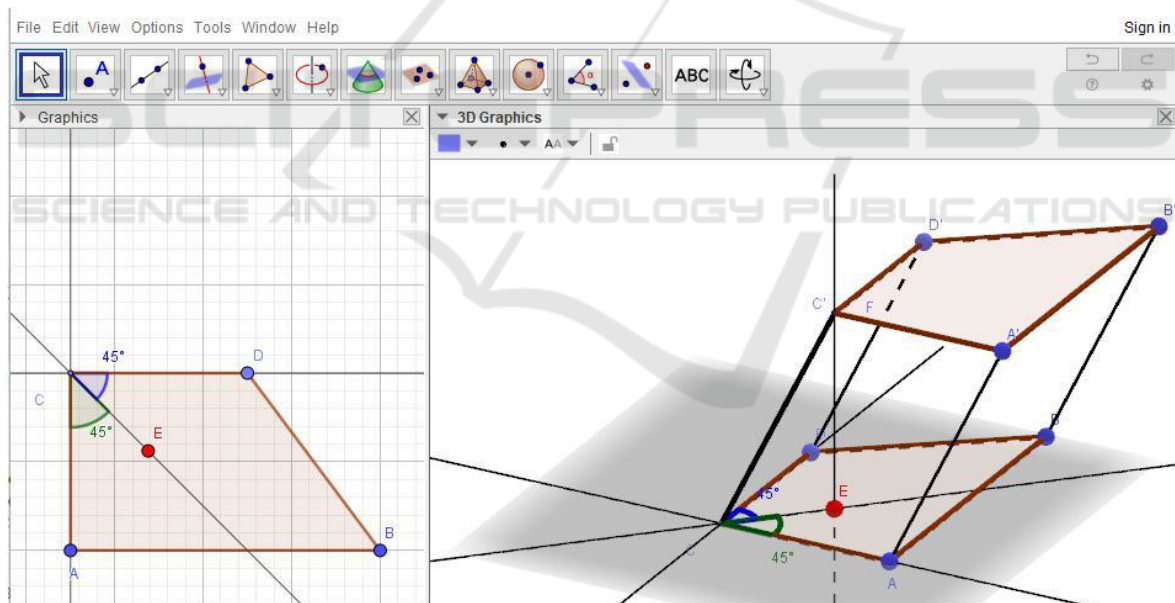Figure 1: Construction of a section of the prisms with a plane.

Figure 2: Construction of a prism, using both 2D and 3D canvas simultaneously.

necessary to take under consideration the case when one of the dihedral angles at the edge of the base is obtuse (Pyramid of Type 2). In this case, the vertex of the pyramid is projected into the center of the inscribed circle lying outside (figure 3).

Using the problem mentioned above and similar ones we have conducted a research on the basis of the two 11th grade classes specialized in Mathematics, and in particular with the teachers of Mathematics Svitlana Shakhmatova and Ruslan Kaluhin (Kramarenko et al., 2019a). 16 students of the experimental group (EG students) and 18 students of the control group (CG) have participated in the research. The similar research has also been conducted at the advanced training courses among the teachers of Mathematics in Kryvyi Rih State Pedagogical University
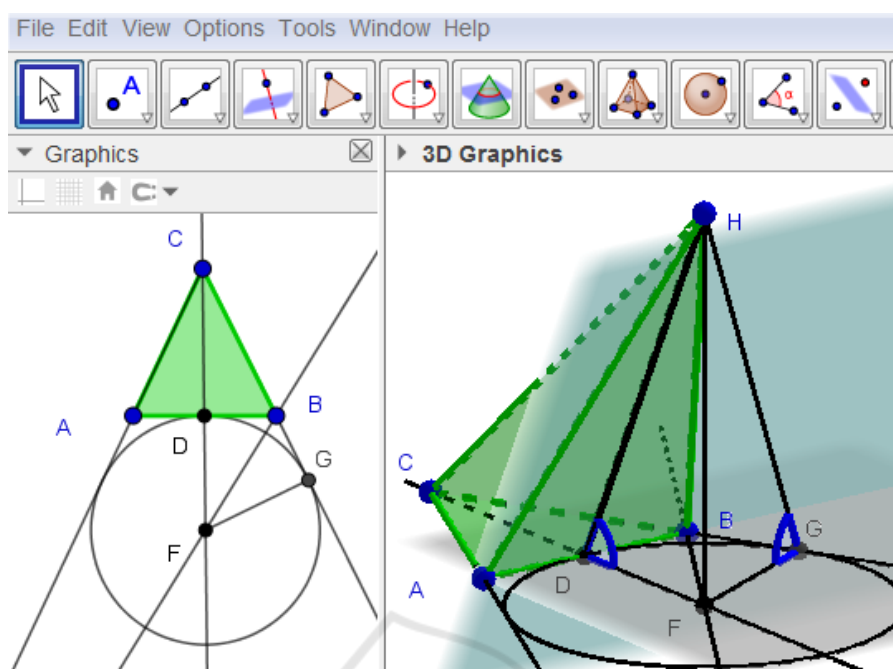
Figure 3: Image of the pyramid and linear angles.

(group 1 – 22 teachers, group 2 – 26 teachers). Some teachers among the participants of the courses did not teach students in high school that is why they do not deal with solving these very problems in their teaching.

The suggested questionnaire consisted of several questions and corresponded mainly to the Pyramid of Type 2. The participants were supposed to deal with the questions during the problem solving. Alongside the questionnaire we present the results of the survey.

1. On reading the problem situation, have you distinguished two types of the Pyramid?

A positive answer: EG – 1 student (6%), CG – no students (0%), group 1 – 5 teachers (23%), group 2 – 6 teachers (23%).

2. The participants received a clue about the way it is possible to construct the Pyramid with the obtuse dihedral angle at the base (Pyramid of type 2). The question was the following: Is it possible to construct the vertex of the pyramid projection on the plane base and in fact the Pyramid itself?

A positive answer: EG – 6 students (28%), CG – 5 students (28%), group 1 – 12 teachers (55%), group 2 – 13 teachers (50%).

3. First, a dynamic model, constructed with GeoGebra, was demonstrated to the participants. The algorithm of the Pyramid construction via the use of the tool GeoGebra Construction Steps was displayed. The task was to construct the Pyramid

of Type 2 and create a problem solving algorithm individually. Next, a paper Pyramid model was demonstrated to the participants. The algorithm of the construction was discussed. The task was to construct the Pyramid of Type 2 and create a problem solving algorithm.

A positive answer: EG – 12 students (75%), CG – 8 students (44%), group 1 – 19 teachers (86%), group 2 – 17 teachers (65%). Fisher's angular transformation (Stevens, 1953) was used for the research results processing. For this purpose, the task performance proportions in the groups during the first and second stages were compared.

φ (EG, CG) = 0,61 < 1,64; φ (group1, group2) = 0,31 < 1,64: the received results do not differ considerably both in the groups of students and teachers.

After stage 3, the proportions of successful task performance in groups were compared one more time. φ (EG, CG) = 1,85 > 1,64; φ (group1, group2) = 1,73 > 1,64: the results differ significantly this time.

It has been defined that the use of dynamic models created with GeoGebra was more effective in comparison with the static paper ones.

On discussing the use of dynamic models created with GeoGebra, participants pointed out the following advantages: variability; dynamic visuals which enhance the development of spatial imagination; multiple reproducibility of the construction, which will contribute to the development of algorithmic thinking.

3.4 Tasks on Combinations of Polyhedron and Solids of Revolution

Consider the way it is possible to inscribe a sphere into the right rectangular pyramid via the use of 3D Geometry. In order to construct the base of the pyramid, it is necessary to use the Right Polygon tool, by pointing two points on the 3D canvas – adjacent vertexes of the base, and indicating that the right polygon has 4 vertexes. Then one should construct the diagonals of the square (the Segment tool) and define the center (Intersection point). Then through the center of the square, which is also the center of the circle inscribed in the square, one draws a straight line perpendicular to the plane of the square. On this straight line, one chooses an arbitrary point (Point on the object) and constructs a polyhedron (Pyramid). The perpendicular to the plane of the square straight line is the geometric location of points, equidistant from the sides of the base of the right pyramid.

To determine the position of the center inscribed sphere in the pyramid, one constructs a geometric location of points that are equidistant from the edges of the dihedral angle at the base of the pyramid. Since there is no construction of the bisector plane in the GeoGebra tools, it is necessary to construct a linear angle of the dihedral angle at the base and then bisector of the very angle. The plane passing through the vertex of the pyramid perpendicular to the edge of the base is built (Plane through the point perpendicular to the straight line; Intersection point). Instead of a plane, it is possible to draw a straight line from the vertex of the pyramid perpendicular to the edge of the base (straight, perpendicular to straight). Next, one should find the intersection point of the constructed plane / perpendicular with the edge of the base (Intersection point of the straight line and the plane / Intersection point of two straight lines). Then one builds the bisector of the obtained linear angle.

The point of its intersection with the perpendicular to the base of the pyramid, drawn from the top of the pyramid, will determine the center of the inscribed sphere (Point of intersection). Finally, one constructs the inscribed sphere (Sphere outside the center and radius), specifying in sequence the center of the sphere and the point of intersection of the diagonals of the square (Kramarenko et al., 2020).

For better understanding and mastering of the algorithm the construction of the sphere inscribed around the pyramid the students setting of the canvas are adjusted to be able to show the step-by-step procedure of the construction.

With AR, the students can understand the basic concepts of 3D geometrical shapes, their relationships and ways to construct the 3D shapes and the objects in 3D space. Importantly, AR can provide a dynamic visualization of 3D structures of geometrical shapes. This feature helps the students to understand a comprehensive background of 3D geometrical shapes and improve the abilities of geometrical structures. Moreover, the hand gesture based interactions furnish an intuitive and convenient way for the students to directly control and interact with geometrical shapes in 3D space.

GeoGebra Augmented Reality application allows you to transfer the constructed figure into the space of the room (figure 4). Having built a figure, we press the "AR" button. Next, you need to use the camera to select the environment in which we plan to move the object. For example, on the table. By tapping on the screen, the figure will be transferred to the real world (Kramarenko and Pylypenko, 2021b) where it can be explored. The phone camera will serve our eyes. Immersing the phone in a virtual figure we will see it from the inside, we can bypass it, also the application allows you to resize, color (Kramarenko and Pylypenko, 2021a).

With the experiences of interacting with the 3D shapes using their own hand gestures, the students can improve their own awareness of the relationships of the 3D shapes and easily remember or retain the knowledge about the 3D shapes.

3.5 Stereometric Problems of Applied Content

Geometry is an abstract science, often taught without proper implementation of its applied orientation. This leads to the fact that a significant part of students do not feel the need to study this subject, because they do not see the possibility of using the acquired geometric knowledge, in particular in stereometry, in the future. And so there is a need to connect stereometric problems with life. We propose to consider two problems of applied direction, for the solution of which we consider it expedient to involve the GeoGebra 3D application. We offered these tasks to students of the State University of Economics and Technologies.

Problem 1. What percentage of wood goes to waste when made of wooden logs, 5 m long and 20 cm and 15 cm in diameter, beams with a rectangular cross-section of the maximum cross-sectional area?

Problem 2. Calculate the volume of the largest beam with a base in the shape of a rectangle, which can be carved from a log of cylindrical shape. The length of the log is 5 m and the thickness is 20 cm. What percentage of wood will go to waste?

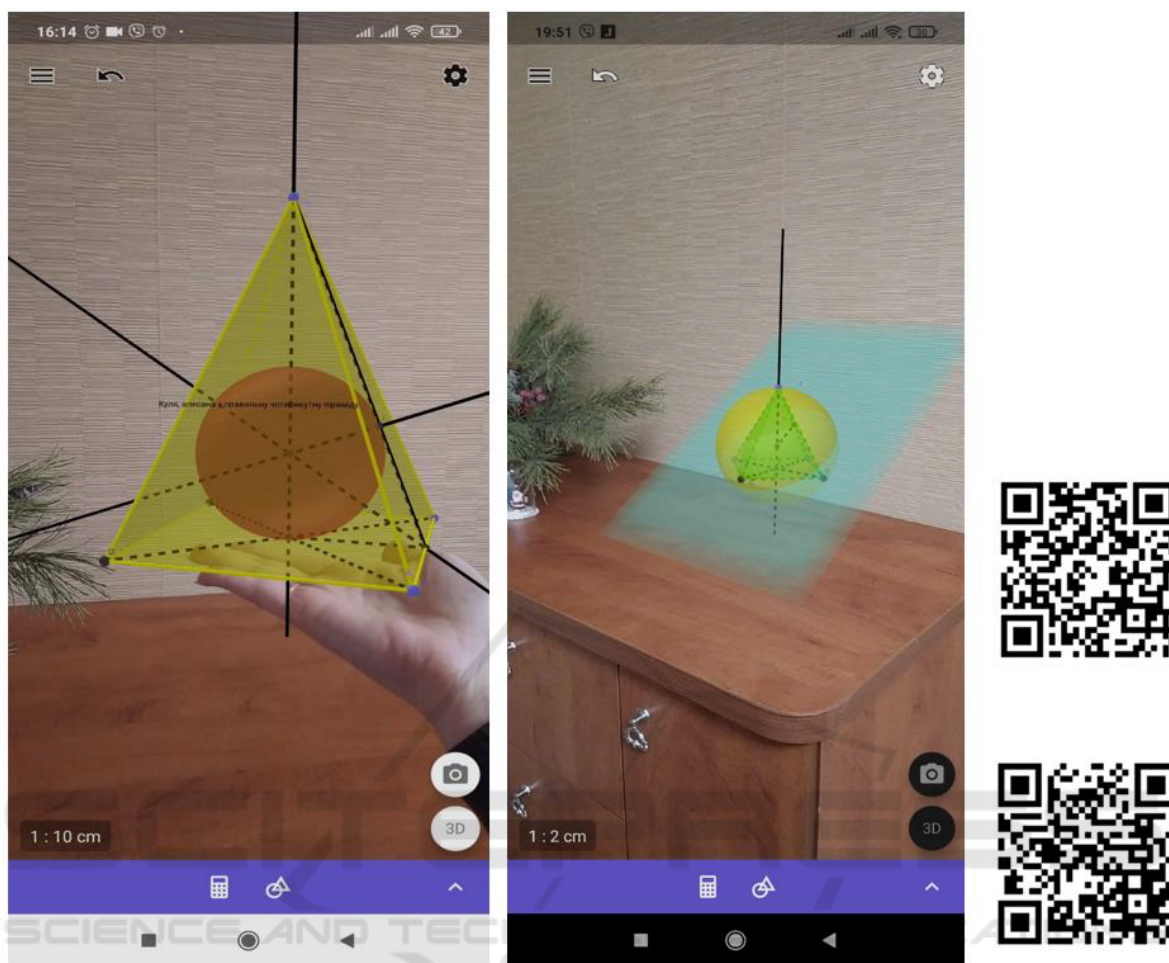Using these tasks, we conducted research on the

Figure 4: GeoGebra AR demos.

basis of two parallel groups majoring in "Finance and Credit". 18 students of the experimental group (EG) and 17 students of the control group (CG) took part in the study. In the experimental group, the task was to solve problems based on a dynamic figure, the control group solved the same problems, but with the help of static.

The proposed questionnaire consisted of several questions that students answered while solving problems.

1. What figures will we work with? Positive answer: CG – 6 students (35%), EG – 7 students (39%).

2. How are the figures relative to each other? Positive answer: CG – 7 students (41%), EG – 7 students (39%).

3. What shape should be the cross section of the beam to maximize its size? The volume of the beam will be the largest if the cross section of the beam is square. It is not necessary to compose a

function and study it to the extreme, it is enough to use the formula to calculate the area of a quadrilateral inscribed in a circle. Positive answer: CG – 4 students (24%), EG – 3 students (17%).

In the second stage, the CG group was shown a figure for the problem on paper, the EG group was shown a figure in GeoGebra (figure 5) and considered in dynamics.

4. After that, the groups were asked the last question about the cross-section again, the statistics of positive answers improved: CG – 6 students (35%), EG – 9 students (50%).

5. What is meant by waste from the manufacture of logs? The positive answer that this is the difference between the volume of the truncated cone and the volume of the parallelepiped was given by: CG – 9 students (53%), EG – 12 students 67%.

The dynamic image in GeoGebra helped the EG

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$$V_{frustum} = \frac{1}{3} * \pi * H * (R^2 + R * r + r^2)$$
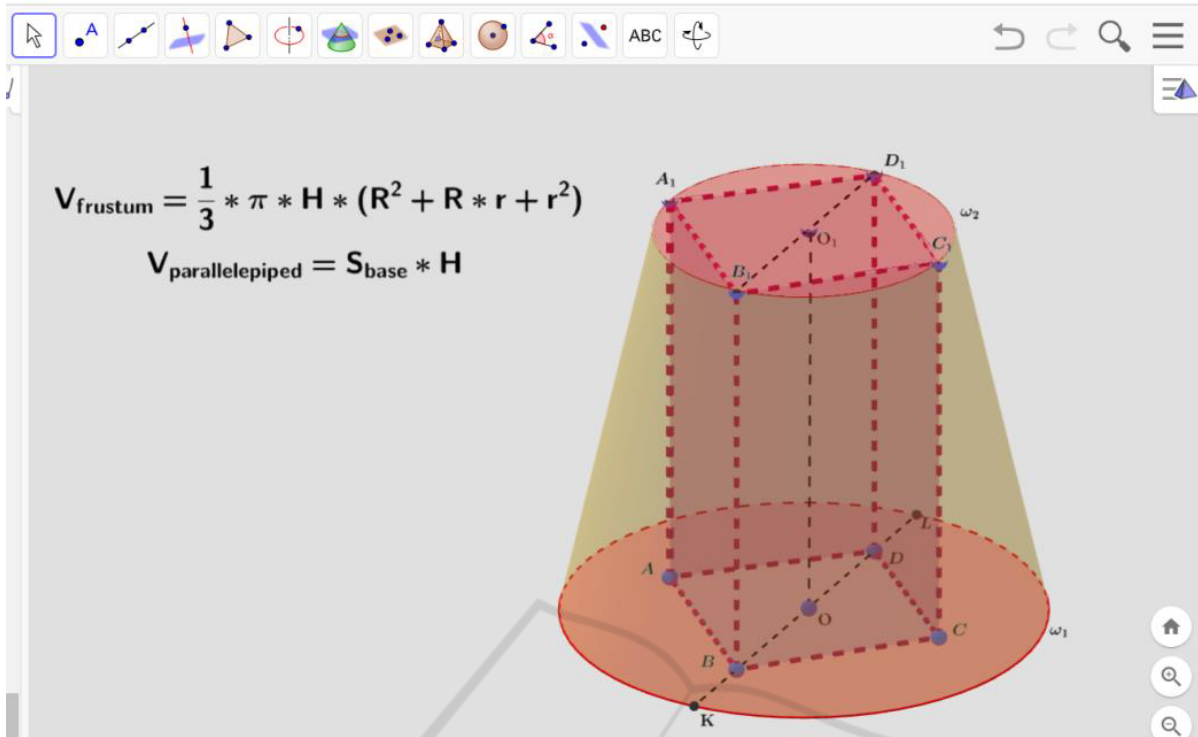
$$V_{parallelepiped} = S_{base} * H$$

Figure 5: A parallelepiped inscribed in a truncated cone.

group to improve the statistics of responses, after the demonstration of the figure on paper this effect could not be achieved. The results of the survey showed that the highest efficiency is achieved when demonstrating dynamic models.

Optimization tasks using Geogebra were proposed by us in the textbook (Kramarenko and Pylypenko, 2021b). We supplemented the sets of tasks using Geogebra with visual aids for specialized teaching of mathematics, realization of interdisciplinary connections of the beginnings of mathematical analysis and stereometry. In this case, you can use the expressions to calculate the volume of the body to track the change in this value and find the optimal size of the beam. It is also advisable to use the "Function Inspector" tool in GeoGebra to find the extreme values of the function and visualize the abstractions.

It is convenient to write the formulas on the canvas at once, and then open them step by step during the discussion. To make such a blank in the application GeoGebra 3D, you must first build a truncated cone (by crossing the cone plane), then through the center of a smaller circle and a point on it build a line. Draw a perpendicular line to the obtained line, choosing the center of a smaller circle as a point. Mark the points of intersection of the lines with the circle and through the obtained 4 points build a square (using the Polygon tool), connecting the points in series. From the

vertices of the square we lower the perpendiculars to the lower base of the cone (larger circle) and mark the points of intersection of the perpendiculars with the plane of the base of the cone, through the obtained 4 points we build a square, connecting the points in series. Using the Prism tool, build a prism by selecting a polygon of the base (square) and the vertex at one of the points of the smaller circle.

During the in-depth study of mathematics at the Kryvyi Rih Pokrovsky Lyceum, we offered students the problem of stereometry for optimization according to the textbook (Skanavi, 1990). After calculating the optimal dimensions of the prism / pyramid, the polygon scan was drawn and glued. Models in dynamics created by means of system of dynamic mathematics were offered for demonstrations. Here are examples of mathematical problems that students had to reformulate as problems of applied content.

1. (15.194) What are the dimensions of the base radius and the height of the open cylindrical tank, so that at a given volume V for its manufacture was spent the least amount of sheet metal?

2. (15.195) The side face of a regular quadrangular pyramid has a constant given area and is inclined to the plane of the base at an angle α. At what value of α is the volume of the pyramid the largest?

3. (15.196) In a regular quadrangular pyramid with the edge of the base a and the height H, a regular quadrangular prism is inscribed so that its lower base is located at the base of the pyramid, and the vertices of the upper base are placed on the side edges. Find the edge of the base and the height of the prism that has the largest side surface.

4. (15.197) The side edge of a right triangular pyramid has a constant given length and forms an angle α with the plane of the base. At what value of α will the volume of the pyramid be the largest?

5. (15.198) In a regular triangular pyramid, the side face has a constant given constant area and forms an angle α with the plane of the base. At what value of α is the distance from the center of the base of the pyramid to its side face the largest?

6. (15.199) A pyramid is inscribed in a cone with a given constant volume, which is based on an isosceles triangle with an angle at the vertex equal to α. At what value of α is the volume of the pyramid the largest?

7. (15.200) The generating cone has a constant length and forms an angle α with the height of the cone. A regular hexagonal prism with equal edges is inscribed in the cone (the base of the prism is located in the plane of the base of the cone). At what value of α is the side surface of the prism the largest?

Solving problems of applied content will provide an opportunity to motivate, intensify the educational and cognitive activities of students and promote the practical application of acquired knowledge.

3.6 Project Work in GeoGebra 3D

One of the effective means of developing students' cognitive activity is the project method. After all, the project method includes a set of research, search, problem, creative approaches, promotes the creative development of students, prepares them to solve problem situations in everyday life. Therefore, it is advisable to offer students to perform mini-projects while studying the section of stereometry.

The task of the project will be to build a playground in the GeoGebra 3D application, using the maximum number of studied geometric shapes: prisms, pyramids, spheres, cones, cylinders, etc. (figure 6). Performance appraisal is a mandatory element of the organization of project work. The effectiveness of the project lies in the ratio of planned expectations with the final results. Created designs can be designed in the yard with an augmented reality application.

There are three stages of self-regulated, namely the Planning Phase, at this stage students set steps for learning, namely (1) Analyzing learning tasks, (2) Determining learning objectives, and (3) Planning learning strategies. In the analyzing stage, students implement a plan that is constantly monitored to ensure it leads to learning goals. In the determining stage, students determine how well the learning strategy is chosen and how to achieve these learning goals (Tomaschko et al., 2018).

Students were also asked to develop a project "Artist's Room", in which students will model a room from improvised means, and before that it is advisable to offer to make a layout in GeoGebra. In this way, students will already know where to start, what sizes of objects to take, what colors will impress, what shapes are needed to create a room, they will learn to break an object into simple geometric bodies and shapes.

Project work interests students in the subject, increases mental activity and creative thinking, helps to mobilize knowledge in practice and quickly adapt to unusual situations. During the construction of a playground or an artist's room, students use innovative abilities, invention, STEM competencies are formed, such as critical thinking, creativity, organizational skills, teamwork, emotional intelligence, ability to interact effectively, cognitive flexibility.

4 CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

1. Investigating the possibilities of using GeoGebra in the learning calculus and geometry, found out that engaging students to research using GeoGebra helps to expand the range of educational tasks, including STEM problems. This allow to achieve the high level of learning motivation and individualize the learning process.

2. Implementation of applied aspect in teaching mathematics using GeoGebra 3D Calculator with AR will help to solve one of the main problems of STEM education –individualization. We can explore AR objects because this application brings 3D math to the real world. Systematic using of GeoGebra 3D Calculator with AR can help to develop students' research skills, enhance their socialization opportunities through the acquisition of ICT, which should lead to the systematic development of universal STEM competencies.
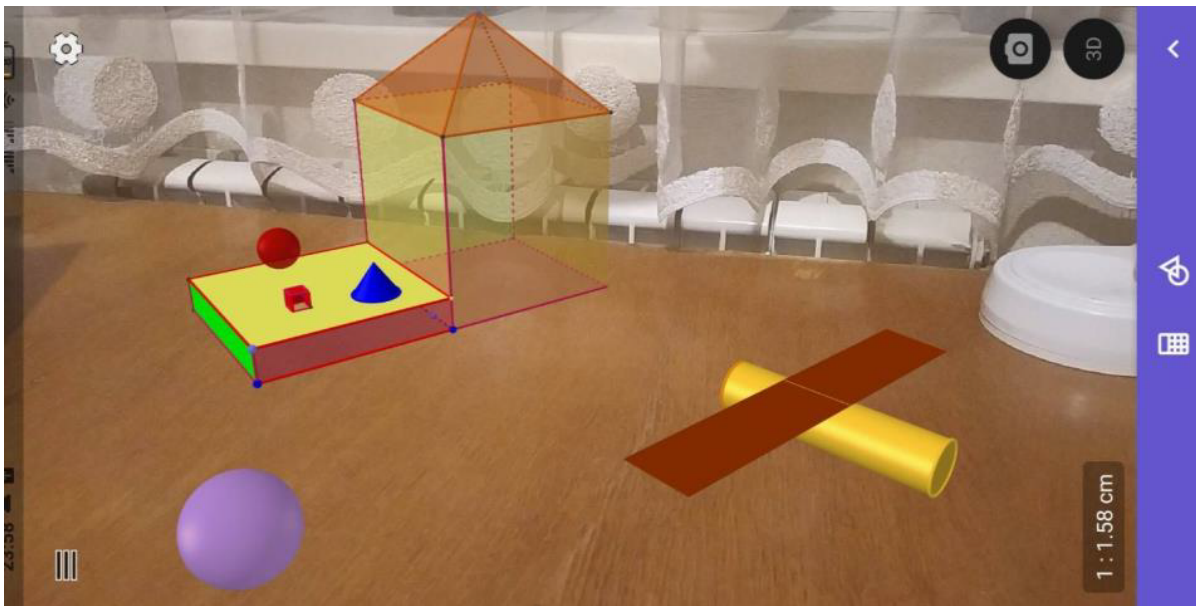
Figure 6: Sample implementation of the project "Playground".

3. The use of dynamic geometry software GeoGebra as a modeling tool makes it possible for scientific experimental logic to engage students in discovering new mathematical facts. Teachers who are concerned by the issues of instrumented learning help to create activities which imply that the main students' motivation is in the understanding of mathematical concepts.

4. In the process of Stereometry teaching, it necessary to divide the students into subgroups according to the type of spatial thinking. This will contribute to the implementation of the individual approach to the development of the spatial imagination, taking into account individual peculiarities, gradually complicating the task, supplementing the teaching material with visual aids, focusing on the practical application of knowledge. Actions with the models created with GeoGebra serve as an intermediate link between external actions with geometrical solid shapes and internal mental processes. Thinking must precede the external ones to engage and develop a person's imagination.

5. In order to increase the efficiency of the perception and assimilation of stereometric material, to overcome the difficulties in transcoding the symbols of a spatial body and to create an adequate spatial image, it is necessary to supplement the theoretical material with multimedia demonstration models created by the means of ICT, to encourage students and to engage them in the process of creating such models for classes independently. After all, the acquisition of mathematical competence depends not on memory, but on the activity in which the person is involved, on the system of mental operations that they perform in the process of knowledge acquisition.

6. In general, the result of the research study is the improvement of teaching methods of Mathematics via the use of cloud technologies, the implementation in the educational process of methodological materials of pedagogically balanced and appropriate blending of traditional and innovative learning tools, which contribute to the intensification of teachers' readiness for the use of STEM training in teaching.

In the future, it is necessary to explore the conditions for the effective use of other GeoGebra mobile applications in STEM learning.

REFERENCES

Astafieva, M., Zhyltsov, O., Proshkin, V., and Lytvyn, O. (2020). E-learning as a mean of forming students' mathematical competence in a research-oriented educational process. *CEUR Workshop Proceedings*, 2643:674–689.

Bevz, H., Bevz, V., Vladimirova, N., and Vladimirov, V. (2011). *Geometry 11 grade*. Geneza, Kyiv.

Brzezinski, T. (2019). GeoGebra 3D with AR (iOS): Explorations & Lesson Ideas. https://www.geogebra.org/m/rmfptnzu.

Diković, L. (2009). Applications GeoGebra into teaching some topics of mathematics at the college level. *Com-*

puter Science and Information Systems, 6(2):191–203.

Drushlyak, M. G., Semenikhina, O. V., Proshkin, V. V., Kharchenko, S. Y., and Lukashova, T. D. (2020). Methodology of formation of modeling skills based on a constructive approach (on the example of GeoGebra). *CEUR Workshop Proceedings*, 2879:458–472.

EC (2018). Council Recommendation of 22 May 2018 on key competences for lifelong learning. https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32018H0604(01)&from=EN.

Fariha, N. F. and Lestari, H. P. (2019). Construction of dandelin sphere on definition of conics using geogebra classic 5. *Journal of Physics: Conference Series*, 1320:012085.

GeoGebra Team German (2018). Learn geogebra 3d calculator. https://www.geogebra.org/m/aWhYSpvy.

Hevko, I., Potapchuk, O., Sitkar, T., Lutsyk, I., and Koliasa, P. (2020). Formation of practical skills modeling and printing of three-dimensional objects in the process of professional training of IT specialists. *E3S Web of Conferences*, 166:10016.

Hohenwarter, M., Hohenwarter, J., Kreis, Y., and Lavicza, Z. (2008). Teaching and Learning Calculus with Free Dynamic Mathematics Software GeoGebra. In *ICME 11 – 11th International Congress on Mathematical Education*. Nuevo Leon, Monterrey.

Kramarenko, T. and Pylypenko, O. (2021a). GeoGebra AR Demo: A sphere described around a right quadrangular pyramid. Video. https://youtu.be/JSkdu1lgWlg.

Kramarenko, T. and Pylypenko, O. (2021b). GeoGebra AR Demo: The sphere is inscribed in a pyramid. Video. https://youtu.be/oohbLc19Llo.

Kramarenko, T., Pylypenko, O., and Muzyka, I. (2020). Application of GeoGebra in Stereometry teaching. *CEUR Workshop Proceedings*, 2643:705–718.

Kramarenko, T. H. (2019). Selected questions of Elementary Mathematics from GeoGebra. https://www.geogebra.org/m/gqpk8yfu.

Kramarenko, T. H., Korolskyi, V. V., Semerikov, S. O., and Shokaliuk, S. V. (2019a). *Innovative information and communication technologies of Mathematics teaching*. Kryvyi Rih Pedagogical University, Kryvyi Rih, 2 edition.

Kramarenko, T. H., Pylypenko, O. S., and Zaselskiy, V. I. (2019b). Prospects of using the augmented reality application in STEM based mathematics teaching. *Educational Dimension*, 53(1):199–218.

Lavicza, Z., Prodromou, T., Fenyvesi, K., Hohenwarter, M., Juhos, I., Koren, B., and Diego-Mantecon, J. (2020). Integrating STEM related technologies into mathematics education at a large scale. *International Journal for Technology in Mathematics Education*, 27(1):3–12.

Lavrentieva, O., Arkhypov, I., Kuchma, O., and Uchitel, A. (2020). Use of simulators together with virtual and augmented reality in the system of welders' vocational training: Past, present, and future. *CEUR Workshop Proceedings*, 2547:201–216.

Modlo, Y., Semerikov, S., Nechypurenko, P., Bondarevskyi, S., Bondarevska, O., and Tolmachev, S. (2019). The use of mobile Internet devices in the formation of ICT component of bachelors in electromechanics competency in modeling of technical objects. *CEUR Workshop Proceedings*, 2433:413–428.

Moiseienko, M., Moiseienko, N., Kohut, I., and Kiv, A. (2020). Digital competence of pedagogical university student: Definition, structure and didactical conditions of formation. *CEUR Workshop Proceedings*, 2643:60–70.

Panetta, K. (2019). 5 Trends Appear on the Gartner Hype Cycle for Emerging Technologies. https://www.gartner.com/smarterwithgartner/5-trends-appear-on-the-gartner-hypecycle-for-emerging-technologies-2019/.

Rakov, S., Gorokh, V., and Osenkov, K. (2009). Mathematics, computer mathematical systems, creativity. In Braman, J., Vincenti, G., and Trajkovski, G., editors, *Handbook of Research on Computational Arts and Creative Informatics*, pages 253–279. IGI Global, Hershey.

Richard, P. and Blossier, M. (2012). Instrumented modelling and preliminary conceptions in threed-imensional dynamic geometry with GeoGebra-3D. In Bastiaens, T. and Marks, G., editors, *Proceedings of E-Learn: World Conference on E-Learning in Corporate, Government, Healthcare, and Higher Education 2012*, pages 322–330. Association for the Advancement of Computing in Education (AACE), Montréal, Quebec, Canada. https://www.learntechlib.org/p/41611.

Rykovskyi, M. Y. (2018). Mykhailo Yosypovych Rykovskyi. https://www.geogebra.org/u/mirinf.

Semenikhina, O. V. (2017). Methodological approaches to the formation of professional readiness of mathematics teacher for the use of mathematical knowledge computer visualization tools. *Pedahohichni nauky: teoriya, istoriya, innovatsiyni tekhnolohiyi*, 2(66):129–138.

Semenikhina, O. V. and Drushliak, M. H. (2014). GeoGebra 5.0 tools and their use in solving solid geometry problems. *Information Technologies and Learning Tools*, 44(6):124–133.

Sidoruk, V. A. (2018). Construction of polyhedra sections. https://www.geogebra.org/m/Jd4va4rs.

Skanavi, M. I., editor (1990). *Collection of problems in mathematics for applicants to universities*. Minsk.

Stevens, W. L. (1953). Tables of the angular transformation. *Biometrika*, 40(1/2):70–73.

Striuk, A., Rassovytska, M., and Shokaliuk, S. (2018). Using Blippar augmented reality browser in the practical training of mechanical engineers. *CEUR Workshop Proceedings*, 2104:412–419.

Tomaschko, M., Kocadere, S., and Hohenwarter, M. (2018). Opportunities for participation, productivity, and personalization through GeoGebra mathematics apps. In Khan, A. and Umair, S., editors, *Handbook of Research on Mobile Devices and Smart Gadgets in K-12 Education*, pages 45–56. IGI Global, Hershey.

Velikova, E. and Petkova, M. (2019). Analysing students' creativity in integrating GeoGebra applets in solving geometrical problems. *Baltic Journal of Modern Computing*, 7(3):419–429.

Zetriuslita, Z., Nofriyandi, N., and Istikomah, E. (2021). The Increasing Self-Efficacy and Self-Regulated through GeoGebra Based Teaching reviewed from Initial Mathematical Ability (IMA) Level. *International Journal of Instruction*, 14(1):587–598.