Using Dynamic Vector Diagrams to Study Mechanical Motion Models at Agrarian University with GeoGebra

Leonid O. Flehantov[1][a], Yuliia I. Ovsiienko[1][b], Anatolii V. Antonets[1][c]
and Vladimir N. Soloviev[2][d]

[1]*Poltava State Agrarian Academy, 1/3 Skovorody Str., Poltava, 36000, Ukraine*
[2]*Kryvyi Rih State Pedagogical University, 54 Gagarin Ave., Kryvyi Rih, 50086, Ukraine*

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Abstract: This article is devoted to study of effectiveness of one of the visualization options we use when teaching the students of an agricultural university the basics of math modeling. The main goal of this research is to test the hypothesis that visibility and visualization improve the educational achievements of students if for visualization we use the dynamic vector diagrams of characteristics of mechanical movement (in particular, these are velocity and acceleration or the acting force). There are discussion of the methodology and examples of the use of dynamic vector diagrams when teaching the basics of mathematical modeling. We described how our students are computing and plotting graphs and dynamic vector diagrams when they doing their practical learning exercises in math modeling and how they using them to analyze the mechanical motion of a body. Primarily we research and discuss the effectiveness of using dynamic vector diagrams. Other visualization options such as using plots of the dynamic characteristics of mechanical movement by Excel and GeoGebra was discussed earlier in (Flehantov and Ovsiienko, 2019). In this article we described our experience of using of dynamic vector diagrams in teaching the basics of math modeling at agrarian university and compared the research results were obtained with previous ones. Also we compare the educational achievements of students in the basics of math modeling were obtained by them using Excel and by GeoGebra software which allow to plot and study the dynamic vector diagrams much easy.

1 INTRODUCTION

Recently there are a lot of changes in education technologies: in fact we are living at a new time. Now it is clear that some of researches in the field of education that were made earlier are obsolete. The one of the main things today is the transition from verbal channel of communication in learning to visual. Although the scientists have not yet had time fully to study and investigate this process but the young people already have mastered all these.

The approach to teaching the basics of mathematical modeling (BMM) that this study focuses on is based on visualization (Ivanova et al., 2020) and it can be described by the saying: "I hear and forget. I see

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[a] https://orcid.org/0000-0002-4689-1457
[b] https://orcid.org/0000-0002-4873-9061
[c] https://orcid.org/0000-0002-2332-6711
[d] https://orcid.org/0000-0002-4945-202X

and remember. I do and understand" (Xunzi, 2020).

This article devoted to results of our research on the impact of visualization to teaching BMM with the use of popular mathematical computer program – the GeoGebra dynamic geometry system (GeoGebra, 2021; Kramarenko et al., 2020a). At first, we points out some researches that were the starting point for the hypothesis and objectives of this work.

The methodological principles of using ICT in the teaching of mathematical disciplines at universities we used were elaborated in (Bobyliev and Vihrova, 2021; Klochko and Bondarenko, 2013; Kramarenko et al., 2020b; Rakov, 2005; Tarasenkova et al., 2019; Tryus, 2005). Some important issues of the use of ICT in the educational process at the universities were shown in (Abdula et al., 2020; Bakum and Morozova, 2015; Polhun et al., 2021; Spivakovsky et al., 2019; Hrybiuk et al., 2014).

Note in passing that to using of Excel spreadsheets in the teaching of BMM which is also mentioned in

this article were devoted to a number of scientific publications about an array of the some aspects of this direction. For review of the early studies may be of utility the works by Illia O. Teplytskyi (Teplytskyi and Semerikov, 2002; Teplytskyi, 2010; Semerikov et al., 2018).

We can't be said the same about the GeoGebra dynamic geometry system. There are many articles devoted to this software but most of them describe of only the capabilities and show some examples how to use it in teaching. As examples we can point out for publication by Osypova and Tatochenko (Osypova and Tatochenko, 2021) on improving the learning environment for future mathematics teachers with the use application of the dynamic mathematics system GeoGebra AR. In the article by Drushlyak et al. (Drushlyak et al., 2020) the methodology of formation of modeling skills based on a constructive approach was shown the on the example of GeoGebra. It shows the basic techniques for using GeoGebra in teaching and studying mathematics, with an emphasis on examples from geometry. Noteworthy alone is the article by Kyslova et al. (Kyslova et al., 2014) devoted to the problems of teaching higher mathematics for students of engineering specialties and to the creation of dynamic models in GeoGebra.

Caligaris et al. (Caligaris et al., 2015a,b) were concluded that using GeoGebra applets is an effective teaching methodology for teaching Calculus. Arbain and Shukor (Arbain and Shukor, 2015) noted that the students have more positive perception towards learning and have better learning achievement using GeoGebra. The article by Valdés y Medina and Medina Valdés (Valdés y Medina and Medina Valdés, 2015) is show in practice how the GeoGebra software can be used to exemplify the different mathematical concepts.

In addition let's we say that a lot of examples the using of GeoGebra software for teaching mathematics and science daily replenished on the GeoGebra website (GeoGebra, 2021; Hall and Lingefjard, 2016). It is also known that MS Excel has built-in functions for visualizing changes in calculated values just like GeoGebra has the built-in SpreadSheet component that allows you to partially perform the tasks the same of Excel. However, the using of these tools needs the results of the previous teaching of information technology which are not provided for curriculum of agrarian universities.

The problems of teaching the basics of mathematical modeling are closely related to the issues of applied orientation of mathematical (fundamental, natural sciences) disciplines. In this regard we note the researches (Blomhøj and Kjeldsen, 2006; Blomhøj and Jensen, 2007; Burghes, 1980; Finlay and King, 1986; Flores et al., 2016; Geiger et al., 2010; Kaiser et al., 2011; Kapur, 1982; Klymchuk et al., 2008; Lofgren, 2016; Oke, 1980; Schukajlow et al., 2018; Soloviev et al., 2019; Temur, 2012; Verschaffel and De Corte, 1997; Vos, 2011). There are some applied issues of mathematical modeling in agroengineering are considered in the textbook (Flehantov, 2006).

In general, despite a significant number of studies and publications, the problems of teaching the basics of mathematical modeling using ICTs still remain outside the attention of the integrated approach of researchers.

Our interest for the main topic of this research based on its practical importance for the training of modern agricultural production engineers, since "teaching the basics of mathematical modeling (BMM) is an important component in the training of modern agricultural production engineers. Its practical value is due to the fact that training on the basis of mathematical modeling can be an effective strategy in modern realities" (Flehantov and Ovsiienko, 2019). This research is a logical extension of article (Flehantov and Ovsiienko, 2019) where we also were tested the hypothesis that visualization of the dynamic characteristics of mechanical processes using GeoGebra improves the student's learning outcomes for BMM (in this article for visualize the dynamic characteristics of mechanical motion were used graphs of trajectories of moved bodies in Excel and GeoGebra).

After the experiment with the simultaneous use of Excel and GeoGebra we were conducted in the fall of 2018 we observed a reversal of negative dynamics in the results of training of BMM the students of engineering and technical specialties of the Poltava State Agrarian Academy, Poltava, Ukraine (PSAA) (2014 – 75.8; 2015 – 75.6; 2016 – 72.6; 2017 – 70.1). According to the results of the fall semester of 2018 the average score in this discipline was 74.8 (Flehantov and Ovsiienko, 2019). At the same time in the experimental groups E (Excel users) and G (GeoGebra users) we obtained almost the same average values of student performance, equal to 73.4 and 73.7, in fact at the level of 2016 (taking into account the permissible statistical error). Whereas in the EG group (simultaneous use of Excel and GeoGebra) the mean value was 77.4. You can try to explain this result by the fact that already in 2017 we recognized the negative trend of lowering student performance as a problem and showed some enthusiasm in solving it. The results of the EG group obtained on the basis of the assumption of the leading role of visualization in teaching the basics of mathematical modeling with the simultaneous use of Excel and GeoGebra showed a statistically

significant difference from the results of the E and G. The average score in the EG group exceeded even quite high learning outcomes of 2014 and 2015. Thus it can be already considered now as a fact that the visualization of modeling results creates additional conditions for improving students' knowledge taking into account the specifics of their professional training.

The main goal of this article is to test with new idea the hypothesis that visibility and visualization improve the educational achievements of students of an agricultural university when studying the basics of mathematical modeling. This research is devoted to the study of the effectiveness of one of the visualization options, which we use when teaching the basics of mathematical modeling to students of an agricultural university. As a way of visualization in this research were used the dynamic vector diagrams of the characteristics of mechanical movement. Our students build these diagrams when they perform practical educational tasks in mathematical modeling, and use them to analyze the mechanical motion of a body thrown at an angle to the horizon, as shown in this article. Further in this article we describe the experience of using dynamic vector diagrams in teaching the basics of mathematical modeling, compare the obtained research results with previous ones and also compare the educational achievements of students in the basics of mathematical modeling which were obtained by them using Excel and GeoGebra.

2 EXPERIMENT DESCRIPTION

2.1 General Design

This article based on the results of the pedagogical experiment that was performed by authors in September-November 2019 and in September-November 2020. The experiment was enrolled by 167 students of the Faculty of Engineering and Technology. The provided sample size makes it possible with ANOVA method to establish significant differences between group means at the level of 1 point: at a significance level of 0.05, number of groups 3 and a power of 80% the required sample size for groups is at least 50 units.

At the beginning of the experiment the students was attended were categorized into three groups called E, G and EG; for this we used the technique described in (Flehantov and Volchkova, 2010) which allow us to initially form groups homogeneous by the criterion of academic performance (Flehantov and Volchkova, 2012). The group E used Excel spreadsheets during the training of BMM. The group G used

the GeoGebra. The group EG used simultaneously Excel and GeoGebra. Throughout the entire study period, these groups were not permanent: students remaining in their academic group could arbitrarily migrate from one experimental group to another. The final composition of the groups was fixed at the end of the experiment. All groups underwent BMM training in a single program using the methodology of a differentiated approach to training presented in (Flehantov and Ovsiienko, 2019). The training time in all groups was the same. Each of group was offered complete the same learning task has a direct connection with the topics important in the training of engineers of agrarian production. Learning outcomes of the students of groups E, G and EG were evaluated on the results of solving a set of typical tasks for individual independent work.

The main learning task for all students was: modeling the movement of spherical body was thrown at an angle to the horizon, with taking into account air resistance, gravitational interaction, electrostatic interaction, magnetic interaction, etc.

We have applied a differentiated approach in learning by doing considered the gradual construction of end-to-end mathematical models (MM) of several levels of complexity their computer implementation and study through numerical calculus. The purpose of these works is to familiarize students with the practice of creating MM, the construction of computational circuits and their computer implementation, forming the skills of conducting a computer experiment and interpreting its results.

The typical task for the students' practice was: create MM of free motion of the body in the gravity field and find its solution. Further we build a calculation algorithm and in the environment of the selected software implement the calculation and computing scheme for MM, conduct a computing experiment and draw conclusions from it. The basic of such MM is the classical model of the motion of a body thrown at an angle to the horizon well known from the school physics course. It's generalized in the first year of engineering faculties of agricultural universities in the study of the discipline "Physics", also used in "Higher Mathematics" to illustrate the physical meaning of the solutions of differential equations and in the second year in "Theoretical Mechanics". Several previously published articles have considered some options for solving this training exercise using Excel (Teplytskyi, 2010; Horda and Flehantov, 2015) and MathCAD (Flehantov and Antonets, 2017). A similar technique was also used by authors for implementation a differentiated approach in teaching of BMM (Flehantov and Ovsiienko, 2016). The algo-

rithm for solving this learning problem has several stages are described in detail in the article (Flehantov and Ovsiienko, 2019).

The students' final learning outcomes were evaluated after completing the full course of discipline. We also retained the methodology for assessing student academic achievements: the learning outcomes of students were assessed on a 100-point scale (Flehantov and Ovsiienko, 2019) on the basis of the performance results of an individual independent learning problems.

2.2 Mathematical Model Creation

It's assumed that the students should use the competencies they acquired were learning "Higher Mathematics", "Physics", "Theoretical Mechanics", "Applied Mathematics", etc when constructing and analyzing math models.

The algorithm of actions by students when they performing the learning exercises is detailed in (Flehantov and Ovsiienko, 2019). The students creates three mathematical models (MM) in different levels of complexity named MM I, MM II and MM III using the mechanical meaning of the derivative and analyzing the vector equations of the resultant forces acting on the body in three different cases. The MM I model describes the motion of a body thrown at an angle to the horizon without air resistance. The MM II model describes the motion of a spherical body thrown at an angle to the horizon with air resistance but no rotation. The MM III model additionally takes into account the Magnus effect – the Magnus force arising from the rotation of the body during the translational-rotational motion of the body in dense medium. An approximate line of reasoning follows next.

Based on the mechanical meaning of the derivative all models will include these two ordinary differential equations of the first order:

$$\frac{dx}{dt} = v_x, \quad \frac{dy}{dt} = v_y. \quad (1)$$

If a body thrown at an angle to the horizon is affected only by gravity force $\overline{F}_T = \left(F_1^x, F_1^y\right)$ then $F_1^x = 0$ and $F_1^y = -mg$. Therefore, two more equations will be added to MM I:

$$\frac{dv_x}{dt} = 0, \quad \frac{dv_y}{dt} = -g. \quad (2)$$

The MM I is described by four differential equations (1) and (2). This elementary model describes the motion of a body thrown at an angle to the horizon in the field of gravity force without air resistance. It corresponds to the first (lowest) level of difficulty in BMM training.

The MM II is the second level of complexity. In this case the equivalent force acting on the body is $\overline{F}_2 = \overline{F}_T + \overline{F}_r$, where $\overline{F}_r = -k_2 v^2 \frac{\overline{v}}{v}$ – force of air resistance, $v = \sqrt{v_x^2 + v_y^2}$ – velocity of the body, k_2 – factor of medium resistance. From here: $F_2^x = -k_2 v_x v$, $F_2^y = -g - k_2 v_y v$. Thus in addition to (1) MM II will also include the following two equations:

$$\frac{dv_x}{dt} = -\frac{k_2}{m} v_x \sqrt{v_x^2 + v_y^2},$$
$$\frac{dv_y}{dt} = -g - \frac{k_2}{m} v_y \sqrt{v_x^2 + v_y^2}. \quad (3)$$

MM III corresponds to the third (highest) level of complexity in our training program. In this case there will be an equivalent force $\overline{F}_3 = \overline{F}_T + \overline{F}_r + \overline{F}_M$, where $\overline{F}_M = -k_3 v^2 (\overline{\omega} \times \overline{v})$, k_3 – coefficient related to the Magnus effect. From here: $F_3^x = -k_2 v_x v \pm k_3 v_y v$, $F_3^y = -g - k_2 v_y v \mp k_3 v_x v$ (the upper sign "+" is on if body rotate clockwise, the lower sign "–" is on if body rotate counterclockwise). Therefore in addition to (1) MM III will include next two equations:

$$\frac{dv_x}{dt} = \left(-\frac{k_2}{m} v_x \pm \frac{k_3}{m} v_y \right) \sqrt{v_x^2 + v_y^2},$$
$$\frac{dv_y}{dt} = -g + \left(-\frac{k_2}{m} v_y \mp \frac{k_3}{m} v_x \right) \sqrt{v_x^2 + v_y^2}. \quad (4)$$

In mathematical models we built the following notation used:

$x = x(t)$, $y = y(t)$ – coordinates of center of the body at the point of time t, s;

$v_x = v_x(t)$, $v_y = v_y(t)$ – the body's velocity projections on axis of reference at time t, m/s;

g – gravity acceleration, m/s^2; m – mass of the body ($m \neq 0$), kg;

$k_2 = \frac{1}{2} C_D \rho S$ – factor of medium resistance, kg/m;

$k_3 = \frac{1}{2} C_L \rho S$ – factor of Magnus force, kg/m;

C_D – drag coefficient, depends on body shape and characteristics of medium (for sphere in air $C_D = 0.47$), no the unit of measurement;

C_L – coefficient characterizing the Magnus effect depends on the shape of the body, the quality of its surface and the properties of medium (for sphere $0.1 \leq C_L \leq 0.6$), no the unit of measurement;

ρ – ambient density (for air $= 1,213$), kg/m^3;

S – normal cross-sectional area of the body relative to direction of motion, m^2; in general $S = S(t)$, for a spherical body $S = \pi r^2$, where r – radius of the sphere, m.

2.3 Performing the Learning Exercises by Excel (Group E)

This part of our research we presented here in a concise statement. A detailed description of the relevant methodology and other explanations are presented in the article (Flehantov and Ovsiienko, 2016).

The calculation scheme of the computer implementation of MM I (1), (2) in the MS Excel environment is obtained directly from its analytical solution (Klochko and Bondarenko, 2013; Kalitkin and Koryakin, 2013):

$$v_{xi} = v_{0x}, v_{yi} = v_{0y} - gt_i, x_i = x_0 + v_{0x}t_i,$$
$$y_i = y_0 + v_{0y}t_i - \frac{gt_i^2}{2}, i = \overline{0, n} \quad (5)$$

The following notation used here:

x_0, y_0 – initial coordinates the center of the body, m;

v_0 – initial velocity of the body, m/s;

α_0 – initial angle to the horizon which the body is thrown, radians;

$v_{0x} = v_0 \cos \alpha_0$, $v_{0y} = v_0 \sin \alpha_0$ – projections the initial velocity of the body v_0;

$t_i = i \cdot \Delta t, t_i \in [0, t_M], i = \overline{0, n}$ – the time points of observation; i – numbers of body observation positions (fixation points); n – numbers of fixation points; $\Delta t = t_M/n$ – time between the time points of observation, s; t_M – simulation time, s;

$x_i = x(t_i)$, $y_i = y(t_i)$ – coordinates the center of the body in the time point t_i;

$v_{xi} = v_x(t_i)$, $v_{yi} = v_y(t_i)$ – projections velocity of the body in the time point t_i; $v_i = \sqrt{v_{xi}^2 + v_{yi}^2}$ – velocity of the body in the time point t_i.

Models MM II (1), (3) and MM III (1), (4), unlike MM I, do not allow an analytical solution. Therefore the computational schemes of the models are based on numerical methods.

The computational scheme for MM II according to the Euler method for the fist order ODE systems is (Kalitkin and Koryakin, 2013):

$$v_{x\,i+1} = v_{xi} - \frac{k_2}{m} v_{xi} v_i \Delta t,$$
$$v_{y\,i+1} = v_{yi} - \left(g + \frac{k_2}{m} v_{yi} v_i \right) \Delta t, \quad (6)$$
$$x_{i+1} = x_i + v_{xi}\Delta t, y_{i+1} = y_i + v_{yi}\Delta t.$$

For MM III the calculation scheme is the same but the first two equations in (6) need to replace by following ones:

$$v_{x\,i+1} = v_{xi} - \left(\frac{k_2}{m} v_{xi} v_i - rot \frac{k_3}{m} v_{yi} v_i \right) \Delta t,$$
$$v_{y\,i+1} = v_{yi} - \left(g + \frac{k_2}{m} v_{yi} v_i + rot \frac{k_3}{m} v_{xi} v_i \right) \Delta t. \quad (7)$$

The initial data for calculations by formulas (5), (6), (7) are the values of parameters g, x_0, y_0, v_0, α_0, r, m, ρ , C_D, C_L, t_0, t_M, n and rot. The rot parameter can take three fixed values: (–1) – means counterclockwise rotation, (+1) – clockwise rotation, 0 – no rotation. Model MM III with $rot = -1$ further we denote as MM III–, and with $rot = +1$ – as MM III+. With $rot = 0$ the model MM III is equally matched to model MM II.

Numerical calculations by formulas (5), (6), (7) are performed by standard MS Excel tools. As a result the next tables of values are obtained: i, t_i, x_i, y_i, v_{xi}, v_{yi}, v_i, α_i, E_{ki}, E_{pi}, where $\alpha_i = arctg \frac{v_{yi}}{v_{xi}}$ – the angle of the trajectory of the body to the horizon; $E_{ki} = \frac{mv_i^2}{2}$ – kinetic energy of translational motion of the body; $E_{pi} = mgy_i$ – the potential energy of the body.

Figure 1 demonstrates of the data input interface of the models MM I, MM II and MM III in MS Excel. Figure 2 displays a part of table of calculations according to MM III for values of input parameters presented in figure 1.

	A	B	C	D	E	F
1	**Model of dynamics of translational-rotational motion**					
2	**of a body in a dense medium**					
3						
4	g=	9,81	m/s2	d=	0,24	m
5	x0=	0	m	m=	0,68	kg
6	y0=	0	m	CD=	0,47	
7	alpha0=	45	degrees	ro=	0	kg/m3
8	v0=	9,3	m/s	S=	0,045239	m2
9	v0x=	6,576093	m/s	k2=	0	kg/m
10	v0y=	6,576093	m/s	k2/m=	0	1/m
11	Time of modeling:			CL=	0,35	
12	Start t0=	0,0000	s	Direction of rotation:		
13	End t=	1,3914	s		0	
14	TM=	1,3914	s	k3=	0	kg/m
15	deltaT=	0,013914	s	k3/m=	0	1/m

Figure 1: Interface for input of initial data for math models MM I – MM III in MS Excel.

Figure 3 and figure 4 depicts the plots of trajectories and velocities of the bodies calculated for all the models under consideration. Visual comparison and analysis of such graphs allows students to formulate meaningful conclusions about the characteristics of those movements in various conditions.

In this way, the students using Excel analyze the obtained numerical results and graphs by changing the input parameters of the models. Thus, they can to study the motion of body thrown at an angle to the

	A	B	C	D	E	F	G
17	i	ti, s	vxi, m/s	vyi, m/s	vi, m/s	xi, m	MM I
18	0	0	6,576093	6,576093	9,3000	0,00000	0
19	1	0,013914	6,576093	6,439597	9,2040	0,09150	0,0915
20	2	0,027828	6,576093	6,3031	9,1090	0,18300	0,1811
21	3	0,041742	6,576093	6,166604	9,0151	0,27450	0,268802
22	4	0,055656	6,576093	6,030108	8,9223	0,36600	0,354604
23	5	0,06957	6,576093	5,893611	8,8306	0,45750	0,438507
111	93	1,294002	6,576093	-6,11807	8,9820	8,50948	0,384657
112	94	1,307916	6,576093	-6,25456	9,0755	8,60098	0,29953
113	95	1,32183	6,576093	-6,39106	9,1701	8,69248	0,212504
114	96	1,335744	6,576093	-6,52756	9,2657	8,78398	0,123579
115	97	1,349658	6,576093	-6,66405	9,3624	8,87548	0,032755
116	98	1,363572	6,576093	-6,80055	9,4600	8,96698	-0,05997
117	99	1,377486	6,576093	-6,93704	9,5586	9,05848	-0,15459
118	100	1,3914	6,576093	-7,07354	9,6582	9,14998	-0,25111

Figure 2: Fragment of the table of calculations for MM I in MS Excel.

horizon in various conditions to answer by itself to control questions and to prepare to exam.

2.4 Performing the Learning Exercises by GeoGebra (Group G)

The description and implementation of the mathematical model of the mechanical motion of bodies in GeoGebra has its own characteristics, which we were mentioned in the work (Flehantov and Ovsiienko, 2019).

The main feature of GeoGebra is an algebraic-geometric approach to the description of mathematical objects. To solve MM I, MM II and MM III by GeoGebra is much easier than by Excel. In GeoGebra is not necessary to build and describe in detail (step by step) complex calculation schemes for solving systems of differential equations of models. All you need is write down the models following the GeoGebra syntax and use the built-in NSolveODE command for numerical solutions of first order differential equations (wiki.geogebra.org, 2020).

The MM I represents in GeoGebra by formulas (1) and (2) as follow (all commands are entered through the command line of the program). From now on the GeoGebra's notation used:

```
x1'(t,x1,y1,vx1,vy1) = vx1
y1'(t,x1,y1,vx1,vy1) = vy1
vx1'(t,x1,y1,vx1,vy1) = 0
vy1'(t,x1,y1,vx1,vy1) = -g
```

Next input command used to solve the system of differential equations above by GeoGebra is:

```
NSolveODE({x1',y1',vx1',vy1'},
          0,{x0,y0,vx0,vy0},TM)
```

The NSolveODE command implements the 4th order numerical Runge-Kutta method in GeoGebra command line (GeoGebra, 2021; Hall and Lingefjard, 2016; wiki.geogebra.org, 2020). It generates the numerical solution the Cauchy problem per segment $t \in [0, t_M]$ with initial conditions

`{x0, y0, vx0, vy0}` and other input parameters of model – tabulates four functions $x_1 = x_1(t)$, $y_1 = y_1(t)$, $v_{x1} = v_{x1}(t)$, $v_{y1} = v_{y1}(t)$ (according to the number of unknown functions in the model). The resulting solutions are assigned with identifiers called `numericalIntegral` with sequential numbering (according to the order of unknown functions in the system of differential equations), namely:

```
numericalIntegral1 = x1(t)
numericalIntegral2 = y1(t)
numericalIntegral3 = vx1(t)
numericalIntegral4 = vy1(t)
```

The same way, the MM II by formulas (1) and (3) is represented as:

```
x2'(t,x2,y2,vx2,vy2)=vx2
y2'(t,x2,y2,vx2,vy2)=vy2
vx2'(t,x2,y2,vx2,vy2)=-k_2*vx2
                 *sqrt(vx2^2 + vy2^2)/m
vy2'(t,x2,y2,vx2,vy2)=-g - k_2*vy2
                 *sqrt(vx2^2 + vy2^2)/m
```

We find the solution of the MM II by the command:

```
NSolveODE({x2',y2',vx2',vy2'},0,
          {x0,y0,vx0,vy0},TM)}
```

This command also gives us four functions – MM II solution:

```
numericalIntegral5 = x2(t)
numericalIntegral6 = y2(t)
numericalIntegral7 = vx2(t)
numericalIntegral8 = vy2(t)
```

Same as previous the MM III by formulas (1) and (4) is represented next way:

```
x3'(t,x3,y3,vx3,vy3)=vx3
y3'(t,x3,y3,vx3,vy3)=vy3
vx3'(t,x3,y3,vx3,vy3)=-k_2*vx3*
sqrt(vx3^2+vy3^2)/m+rot*k_3*vy3*
sqrt(vx3^2+vy3^2)/m
vy3'(t,x3,y3,vx3,vy3)=-g-k_2*vy3*
sqrt(vx3^2+vy3^2)/m-rot*k_3*vx3*
sqrt(vx3^2+vy3^2)/m
```

And the MM III solution we will find by the command:

```
NSolveODE({x3',y3',vx3',vy3'},0,
          {x0,y0,vx0,vy0},TM)}
```

This way we have got the MM III solution:

```
numericalIntegral9 = x3(t)
numericalIntegral10 = y3(t)
numericalIntegral11 = vx3(t)
numericalIntegral12 = vy3(t)
```

Now then we have saw that due to the uniformity of actions the solutions of all three models are obtained by GeoGebra much faster than with Excel.
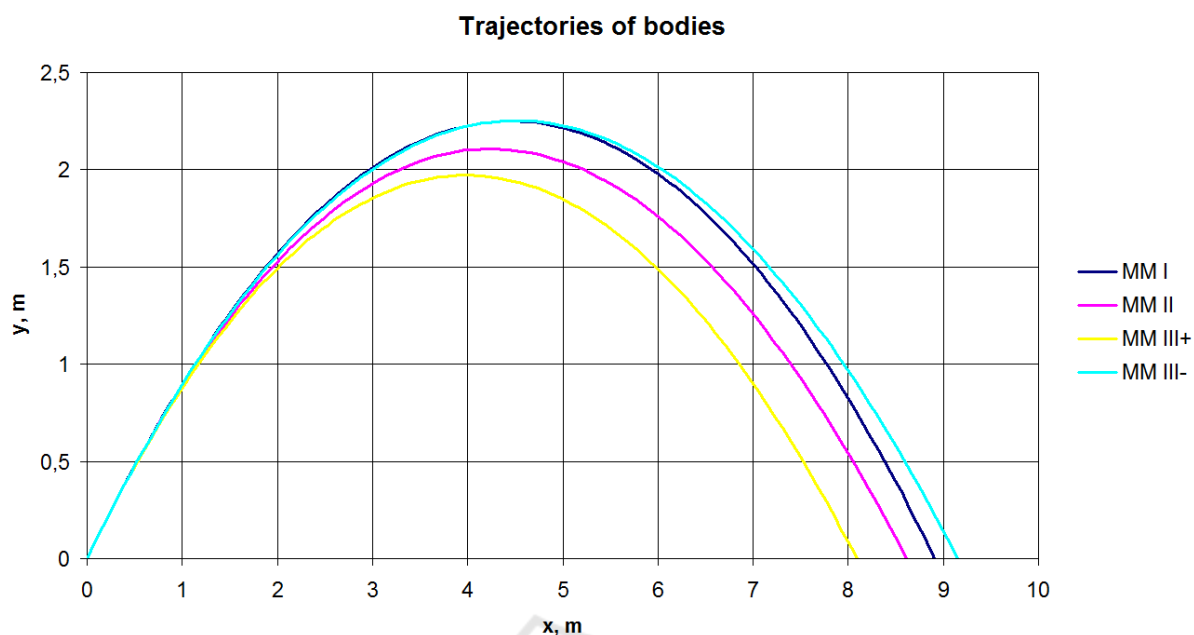
Figure 3: Plots of trajectories of bodies for MM I – MM III in MS Excel.
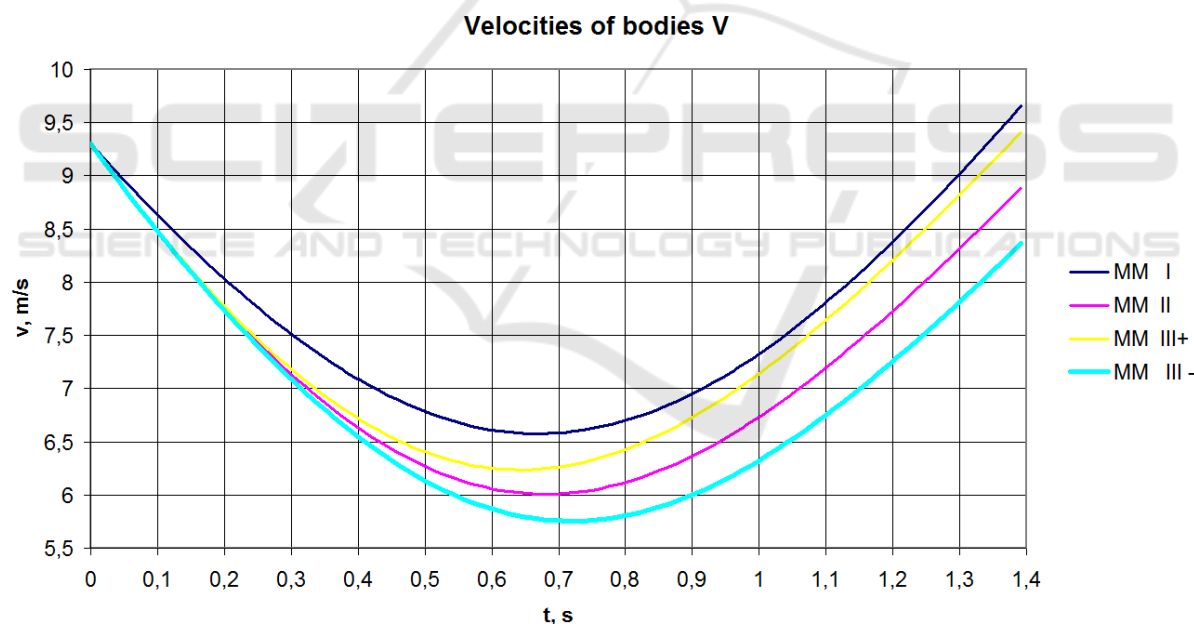

Figure 4: Graphs of velocities of bodies for MM I – MM III in MS Excel.

Also, if the students have this solutions, than they can fast and easy visualizing dynamic characteristics of the math models by GeoGebra as it shown in figure 5.

The trajectories of bodies visualized in GeoGebra according to the models MM I, MM II and MM III using the values of functions `x1(t)`, `y1(t)`, `x2(t)`, `y2(t)`, `x3(t)`, `y3(t)` as coordinates for three moving points A=(`x1`, `y1`), B=(`x2`, `y2`) and C=(`x3`, `y3`)

respectively. Figure 5 shows the trajectories of spherical bodies thrown at an angle to the horizon (for three different values of initial angles) for models: MM I – red line; MM II – green line; MM III – blue line. These colors we will use for convenience in all figures next. The current values of the basic parameters of the models are shown in the figure on the interactive controls. The arrows shown in figure 5 are vectors
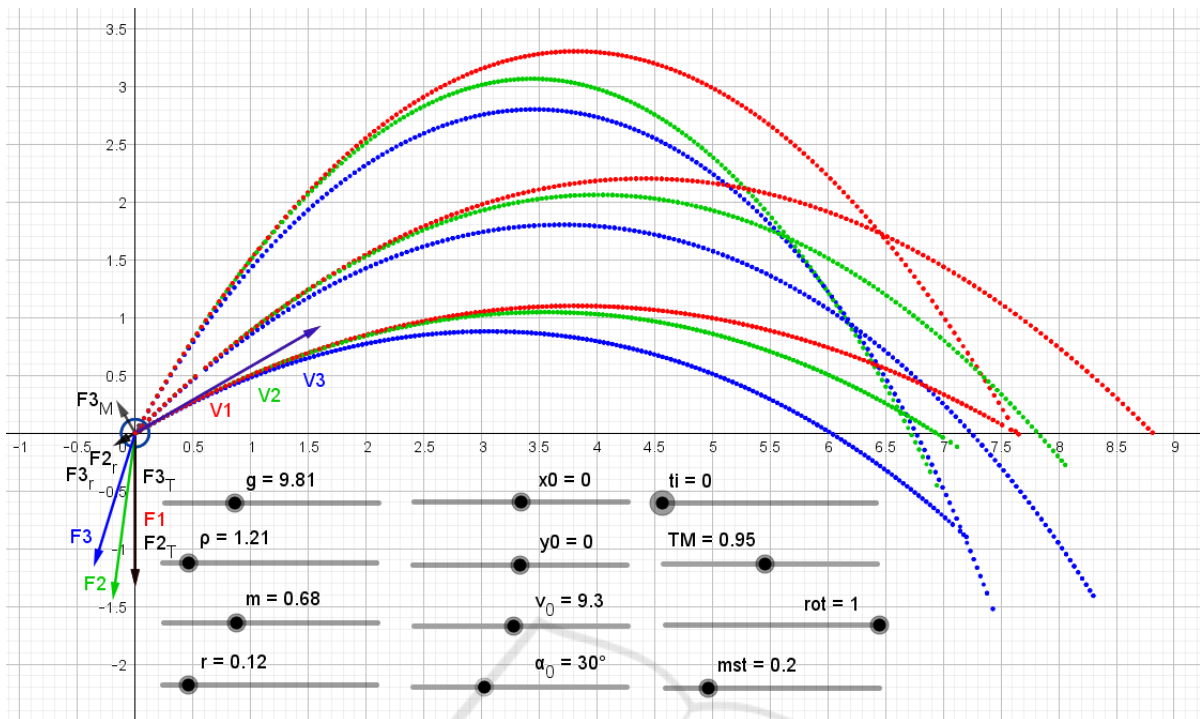
Figure 5: Graphs of trajectories of bodies for MM I – MM III in GeoGebra for different values of initial angles α_0: 30°, 45° and 60°.

of velocities and vectors of forces acting on the bodies as they moves.

As we see, the GeoGebra software has better graphics than Excel and provides additional visualization capabilities. In particular, using the animation feature, the students can observe with GeoGebra dynamic graphing of functions – a computer simulation the process of mechanical movement of the body. By using dynamic zooming in the GeoGebra window the students can to view graph details on a larger scale. Figure 6 and figure 7 shows some fragments trajectories of bodies for opposite directions of body rotation which controlled in MM III by parameter *rot*. The plots in these figures are presented for compare one to others as an example.

Vectors of velocities of the bodies in figure 6 and figure 7 describes in GeoGebra as follow: `V1=Vector(A, A3)`, where `A3=(x(A1), y(A2))`, `A1=(x(A)+mst*y(Point( numericalIntegral3, c)), y(A))` or `A1=(x(A) + mst*vx1,y(A))`, `A2=(x(A),y(A)+mst*y(Point (numericalIntegral4,c)))` or `A2=(x(A),y(A) +mst*vy1);` `V2=Vector(B,B3)`, where `B3=(x(B1),y(B2))`, `B1=(x(B)+mst*y(Point (numericalIntegral7, c)), y(B))` or `B1= (x(B)+mst*vx2,y(B))`, `B2=(x(B),y(B)+mst *y(Point(numericalIntegral8,c)))` or `B2=`

`(x(B),y(B)+mst*vy2); V3=Vector(C,C3)`, where `C3=(x(C1),y(C2))`, `C1=(x(C)+mst*y(Point (numericalIntegral11, c)), y(C))` or `C1= (x(C)+mst*vx3,y(C))`, `C2=(x(C),y(C)+mst*y (Point(numericalIntegral12,c)))` or `C2= (x(C),y(C)+mst*vy3);`

`mst` – scale factor for displaying vectors used for convenience because in normal scale they can be disproportionate in size – some much larger than others (in figure 5, figure 6 and figure 7 value of `mst = 0.1`).

Vectors of forces acting on the bodies and their resultants for three different models are represented in figure 6 and figure 7 (in order of construction). Indices 1, 2 and 3 are referring to models MM I, MM II and MM III respectively:
`F1=Vector(A,F1_1)`, where `F1_1=(x(A),y(A) -mst*m*g);`
`F2=Vector(B,F2_3)`, where `F2_3=F2_1 + F2_2-B`, `F2_1=(x(B),y(B)-mst*m*g)`,
`F2_2=(x(B) + mst*F2x_r, y(B) + mst*F2y_r)`,
`F2x_r= -k_2*vx2*sqrt(vx2^2 + vy2^2)`,
`F2y_r= -k_2*vy2*sqrt(vx2^2 + vy2^2);`
`F2_T= Vector(B,F2_1); F2_r= Vector(B,F2_2);`
`F3= Vector(C, F3_5)`, where `F3_5=F3_3+F3_4-C`,
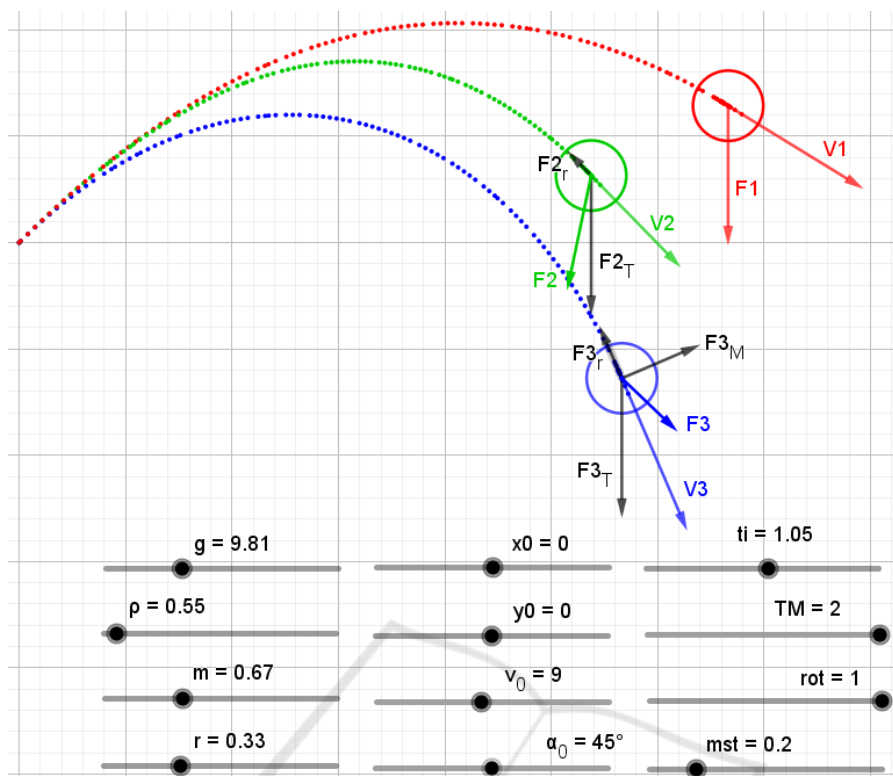`F3_3= F3_1 + F3_2-C`,
`F3_4= (x(C) + mst*F3x_M, y(C) + mst*F3y_M)`,

Figure 6: Fragments of trajectories of bodies plotted in GeoGebra for MM I – MM III with `rot=+1`.

```
F3_1= (x(C), y(C) - mst*m*g),
F3_2= (x(C) + mst*F3x_r,y(C) + mst*F3y_r),
F3x_r=-k_2*vx3*sqrt(vx3^2 + vy3^2),
F3y_r=-k_2*vy3*sqrt(vx3^2 + vy3^2),
F3x_M=rot*(-k_3)*vy3*sqrt(vx3^2 + vy3^2),
F3y_M=rot*k_3*vx3*sqrt(vx3^2 + vy3^2).
```

Vectors of the gravity force $F3_T$, the force of air resistance $F3_r$ and the Magnus force $F3_M$ for model M III are represented as follows:

```
F3_T=Vector(C,F3_1),
F3_r=Vector(C,F3_2),
F3_M=Vector(C,F3_4).
```

Thus, the students implements the mathematical models MM I, MM II, MM III in the GeoGebra environment with the help of this simple mathematical apparatus based on the method of coordinates and elementary vector algebra.

Figure 8, figure 9 and figure 10 shows the dynamic vector diagrams which our students are plotting in GeoGebra and studying by computer simulation. On them in dynamics are displayed the vector diagrams of the velocities of bodies and their projections in different phases of motion according to the models MM I, MM II, MM III. These diagrams are dynamic and interactive because they automatically changes if you use the sliders shown in

the figure to set new values for the model inputs. At moment represented results at time `ti = 1.04 s` for the following values of the initial parameters: `g=9.81, ro=1.213, m=0.68, r=0.12\verb, x0=0, y0=0, v0=9.3, alpha0=45, rot=1, mst=0.1.`

The using of dynamic vector diagrams for visualizing dynamic characteristics to study and analyze mechanical movement when teaching students the basics of mathematical modeling is the main difference between this studies from the previous ones. The computer simulation and the studying dynamic vector diagrams of velocities of bodies allows for students interactively observe changes that occur with vectors of velocities depending on changes of values of various model parameters and study and compare the dynamic characteristics of motion of the bodies in different phases of its movement.

The students simultaneously study and analyze the corresponding plots of velocities of the bodies and their projections on the coordinate axes similar to those shown in figure 11 and figure 12. In contrast to vector diagrams these plots better shows how changes the absolute value of velocity of the body. The analysis allow to us to find the direction of movement of body at different times. At the same time dynamic vector diagrams of velocities give us a clear visual
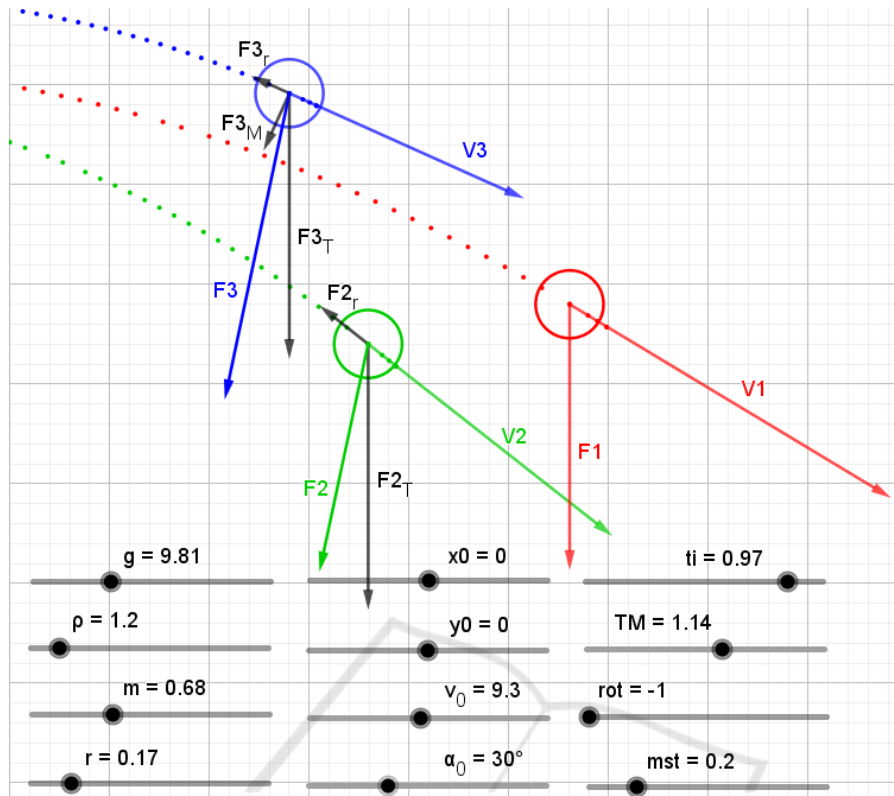
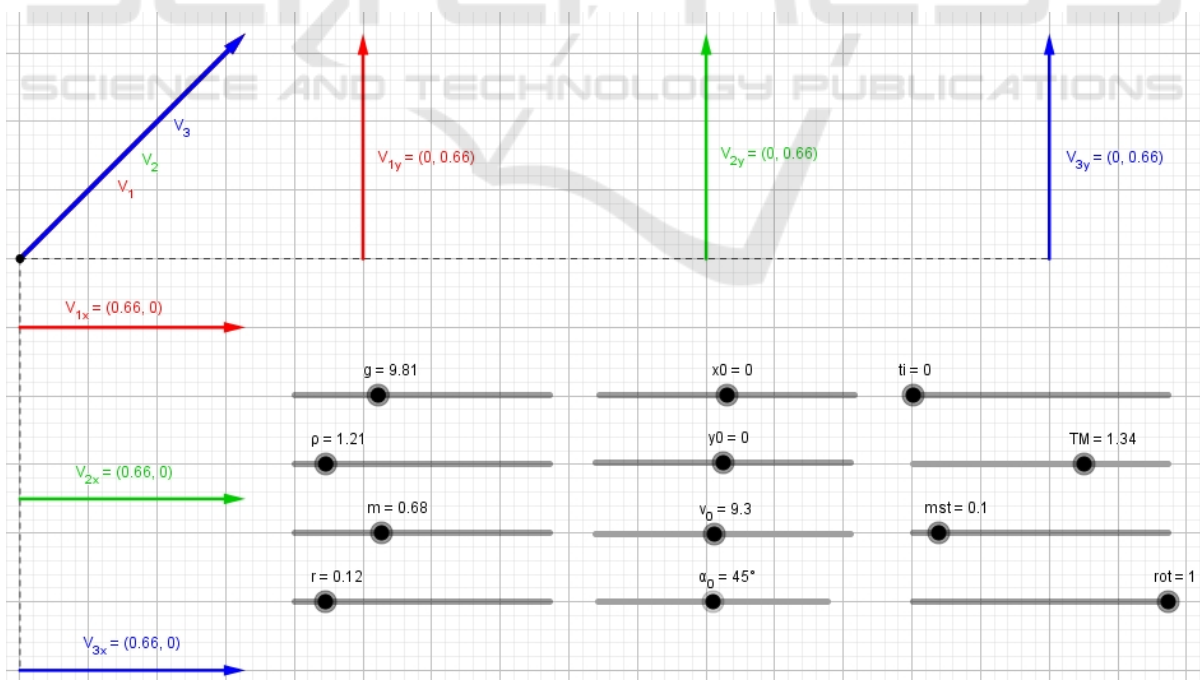Figure 7: Fragments of trajectories of bodies plotted in GeoGebra for MM I – MM III with `rot=-1`.

Figure 8: The dynamic vector diagram of velocities of bodies plotted in GeoGebra for MM I – MM III+ (clockwise rotation) at start of movement: ti=0, $rot = +1$.
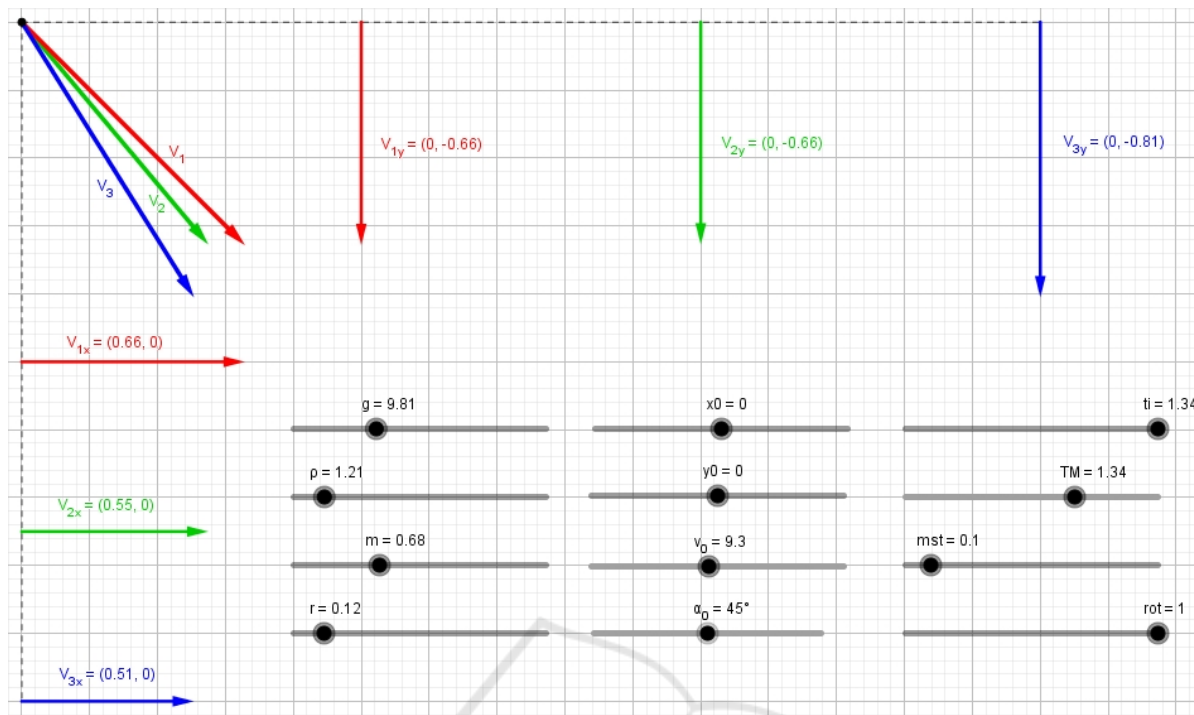
345

Figure 9: The dynamic vector diagram of velocities of bodies plotted in GeoGebra for MM I – MM III+ (clockwise rotation) at the end of motion: `ti=1.34`, `rot=+1`.
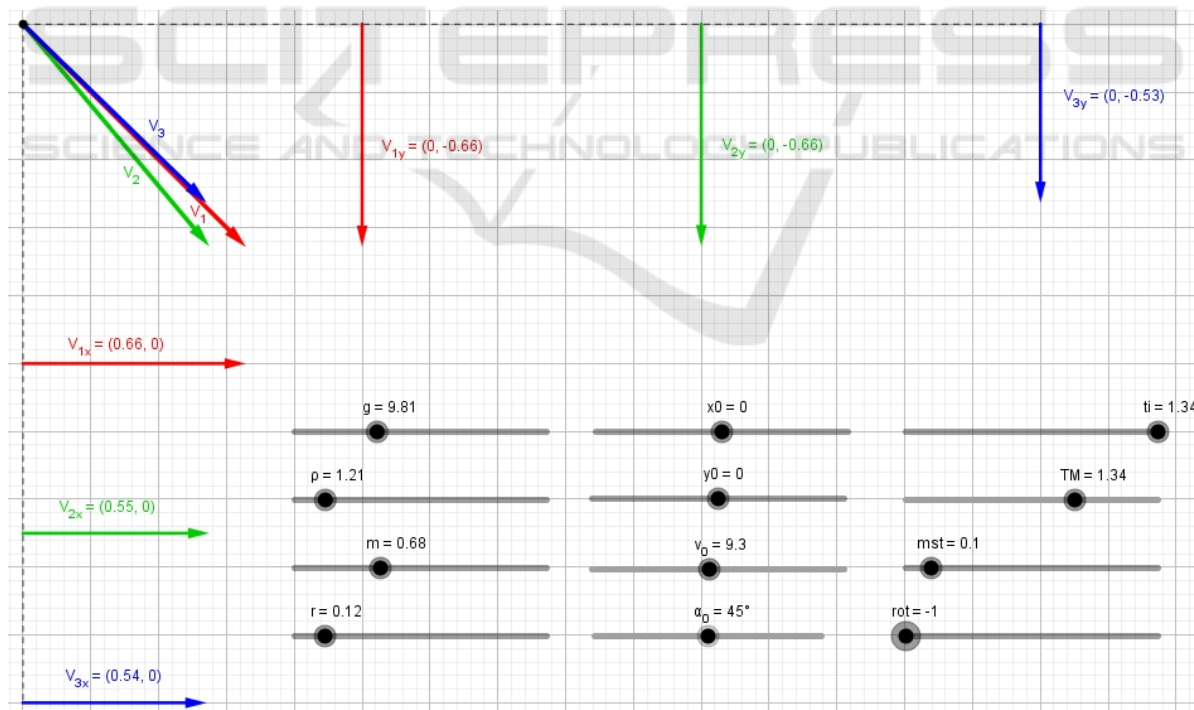

Figure 10: The dynamic vector diagram of velocities of bodies plotted in GeoGebra for MM I – MM III- (counterclockwise rotation) at the end of motion: `ti=1.34`, `rot=-1`.

representation not only of the absolute value of the body's velocity and its projections (this can be judged by the length of the corresponding vectors) but also of the direction of motion of the body (this can be seen

directly in the direction of the vector of body velocity).

Figure 11 allow us to compare the plots of the horizontal projections of velocity of the body $vx1(t)$, $vx2(t)$, $vx3(t)$ for the three different models MM I, MM II and MM III for two cases: when $rot=+1$ (clockwise rotation) and $rot=-1$ (counterclockwise rotation). Studying these plots provides for the students a lot of useful information about the features of body movement thrown at an angle to the horizon under different conditions. In particular, these plots are shows clearly that when rotated clockwise the body at the beginning of the movement accelerates in the horizontal direction and then slows down. On the contrary, for $rot=-1$ the body slows down at first, but somewhat accelerates at the end of the movement.

The GeoGebra allows us to build similar plots and diagrams for all functions that are the solution of the models. Performing their interactive comparisons at various values of the parameters of the models we can quickly conduct a mini-study to establish the all necessary facts or identify the patterns. For example, the Fig. 12 shows the plots of the functions $y1(t)$, $y2(t)$, $y3(t)$ for the MM I, MM II, MM III when $rot=1$ and $rot=-1$. Changing the time of simulation TM we can quickly set the time point when the body touches the surface of the earth (determine the flight time). As you see, for identical input parameters of the model, the body rotating clockwise will fall to the surface of the earth in 1.14 s and the body rotating counterclockwise will be flying 1.46 s. This technique ensures the effective formation of research competencies of the students.

Similarly, our students study and analyze the forces acting on the body during movement and their resultants, as well as their projections. Performing computer simulations in Excel and GeoGebra based on the created mathematical models they observe and analyze, for example, how these vector quantities change at different points in time, what are the tendencies of these changes, what they mean and what they indicate. The corresponding illustrations using GeoGebra will be given below. This approach contributes to formation of the analytical abilities and development of skills of graphic analysis of students.

2.5 Performing the Learning Exercises with Excel and GeoGebra (Group EG)

The experimental group EG performed the same learning exercises using simultaneously Excel and GeoGebra in accordance with methodology described above.

The participants of this group used Excel (if feel it necessary or expedient, in particular, for numerical calculations or for presenting the results in tabular form) or the GeoGebra software for visual representation and analysis the dynamic characteristics in form of plots and dynamic vector diagrams. As example of these works you may see the vector diagrams of the forces acting on the body plotting according to the models M I, MM II and MM III in figure 13 and figure 14.

3 RESULTS

The table 1 shows the results of final learning outcomes of students of the Faculty of Engineering and Technology of the agricultural university at the end of the experiment described in this article: training the basics of mathematical modeling using visualization of the dynamic characteristics of mechanical movement in the form of dynamic vector diagrams. The final learning outcomes of the students of groups E, G and EG were evaluated on the results of solving a set of typical learning problems or individual independent works. These results are presented here on a 100-point scale (Flehantov and Ovsiienko, 2019).

The primary statistical data processing results of the experiment (table 2) are showed that the average scores in all groups (mean) are different. The means for groups E and G close to each other (73.36 and 73.85) but both lower than the mean of group EG (78.25). The mean values in groups E and G almost coincide with similar indicators in the same groups of the previous years: MeanE=73.36 (2019) vs. 73.4 (2018); MeanG=73.85 (2019) vs. 73.7 (2018). At the same time the mean value in group EG-2019 exceed the corresponding figure in group EG-2018: MeanEG=78.25 (2019) vs. 77.4 (2018). The result of Shapiro-Wilk test shows the trust of hypothesis about normal data distribution in all groups.

Analysis of variance (ANOVA) showed a statistically significant difference in average values of learning outcomes (Score/Point/Bal) in all groups ($F = 4.678693$; $p = 0.010613$) (table 3). The post-hoc comparison for means of groups E vs. G, E vs. EG, G vs. EG shows that the difference between the means of groups E and G is within the statistical error (table 4). The pair-wise post-hoc comparisons results indicate the statistical significance of the difference between the mean for group EG and the mean groups E and G.
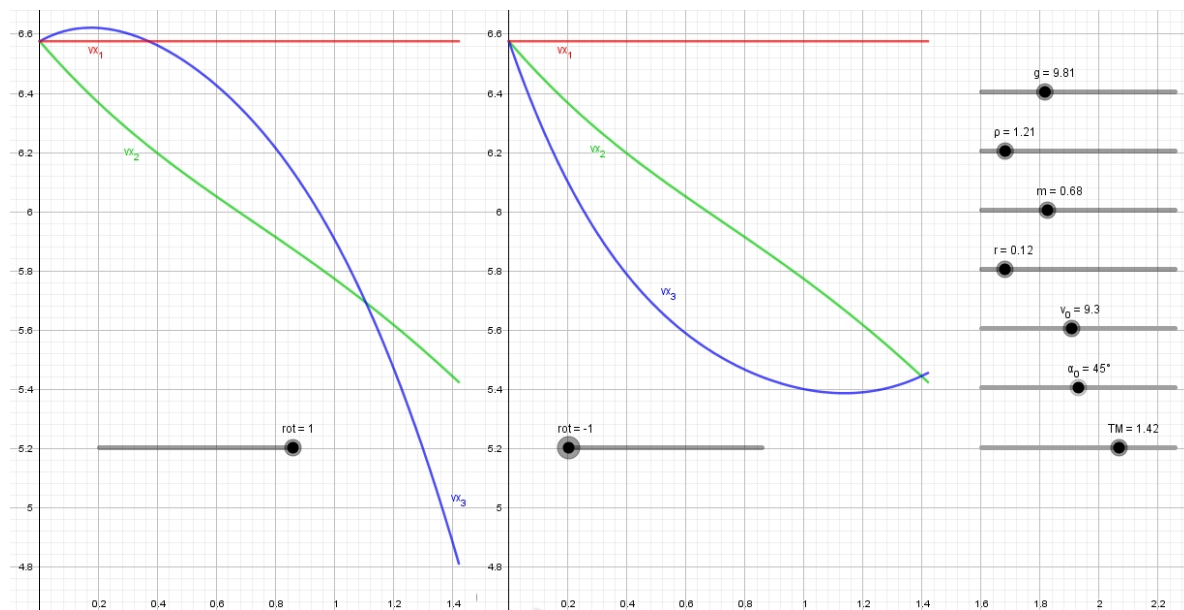
Figure 11: The visual study in GeoGebra: how changes the projection v_x for math models MM I, MM II, MM III+ (*rot* $= +1$, clockwise rotation) and MM III- (`rot=-1`, counterclockwise rotation).
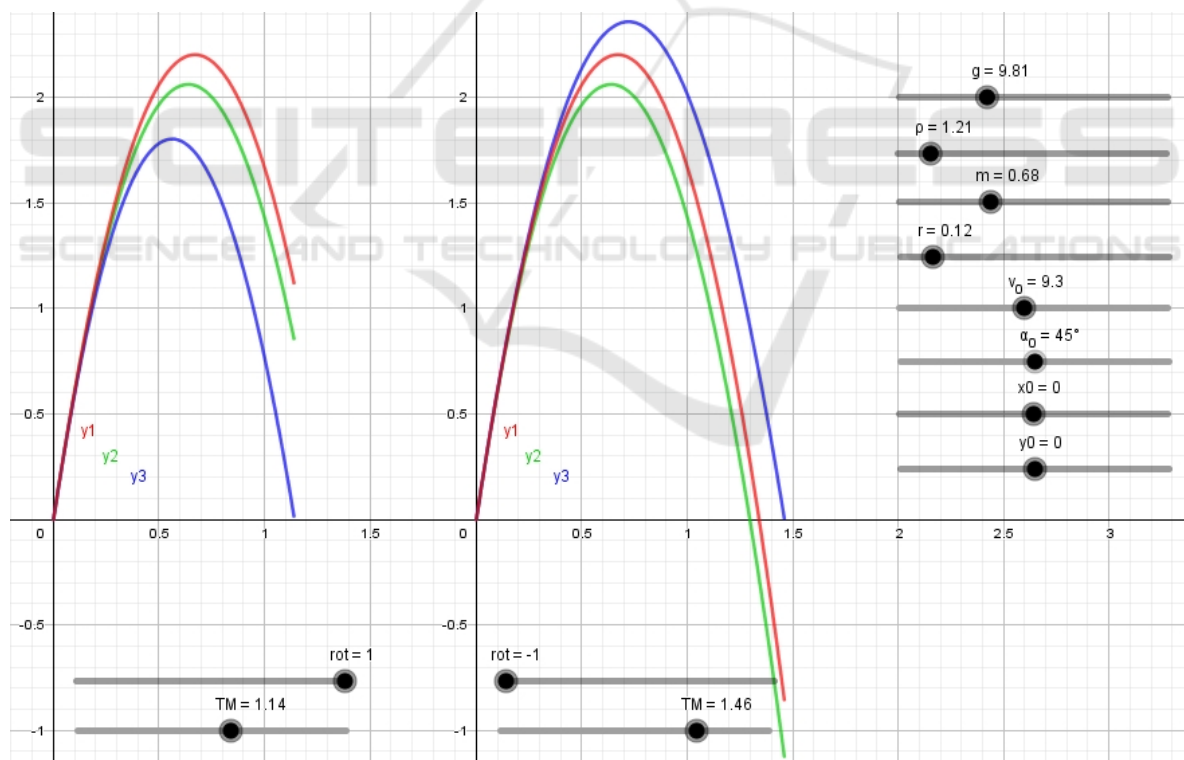
Figure 12: The visual study in GeoGebra: how long is the spherical body flight in air according to different math models if the body rotate clockwise and counterclockwise? (`rot=+1` and `rot=-1`).

4 DISCUSSION

In this article we made compare the earlier results we obtained when our students at first used Excel spread-sheets as the tool for solving learning exercises and then the GeoGebra dynamic geometry system was added. At the same time during the training sessions

Figure 13: The dynamic vector diagram of X-component F_x of the resultant of the forces acting on the body in the four cases of flight: `ti = 0, rot = 1`; `ti = TM, rot = 1`; `ti = 0, rot = -1`; `ti = TM, rot = -1`.

Table 1: Final learning outcomes of students after the experiment.
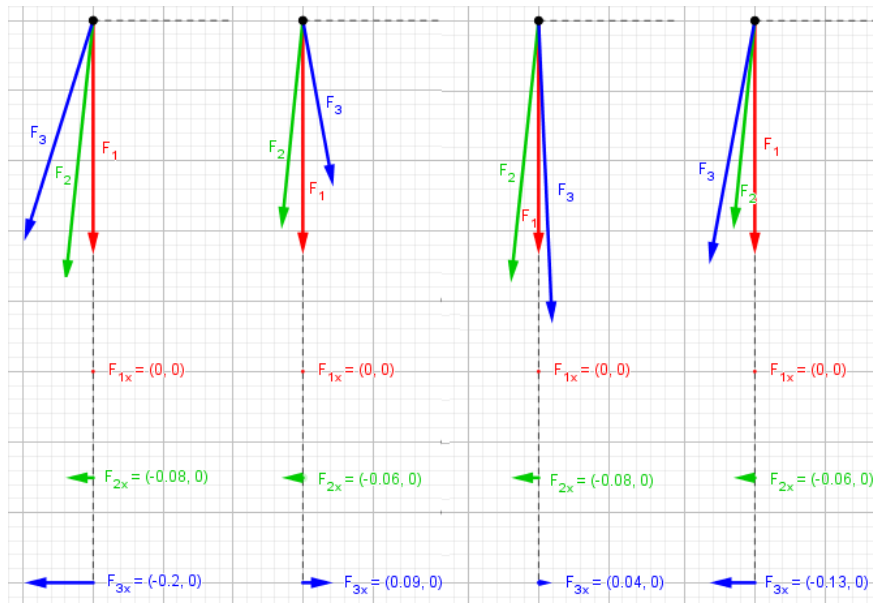
Score	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-95
Group E	1	2	6	9	14	10	7	4	-
Group G	-	3	6	10	13	15	8	3	1
Group EG	-	1	3	5	10	15	9	7	5

Table 2: Primary statistical data processing results.

Group	Valid N	Mean	Conf.-95%	Conf.+95%	Median	Mode
E	53	73.36	71.16	75.55	74	Multiple
G	59	73.85	71.74	75.95	74	76
EG	55	78.25	75.94	80.57	77	76

Group	Freq. Mode	Min	Max	SD	Shapiro-Wilk	test
E	5	55	90	7.96	W=0.98823	p=0.87840
G	5	56	91	8.08	W=0.98749	p=0.80486
EG	6	60	95	8.56	W=0.98158	p=0.55809

Table 3: ANOVA results.

	SS	df	MS	SS_Err	df_Err	MS_Err	F	p
Point	1249.818	2	624.9091	10588.73	164	64.56544	4.678693	0.010613

we consistently and systematically studied how various methods and techniques of visualizing the results of modeling affect the results of educational achievements of students who study at the agrarian university (the specifics of teaching this group of students to mathematical disciplines were discussed by us earlier).

Based on the above we conclude that visualization of the dynamic characteristics of mechanical movement in the form of dynamic vector diagrams using the GeoGebra software allow to improve the educational achievements of students of the agrarian university when studying the basics of mathematical modeling.

There are several possible reasons for this result. At first, since implementation of mathematical models in MS Excel, in fact, is a stepwise reproduction of the sequence of the mathematical operations by means of spreadsheets, than their use contributes to a better understanding of the students of the technical
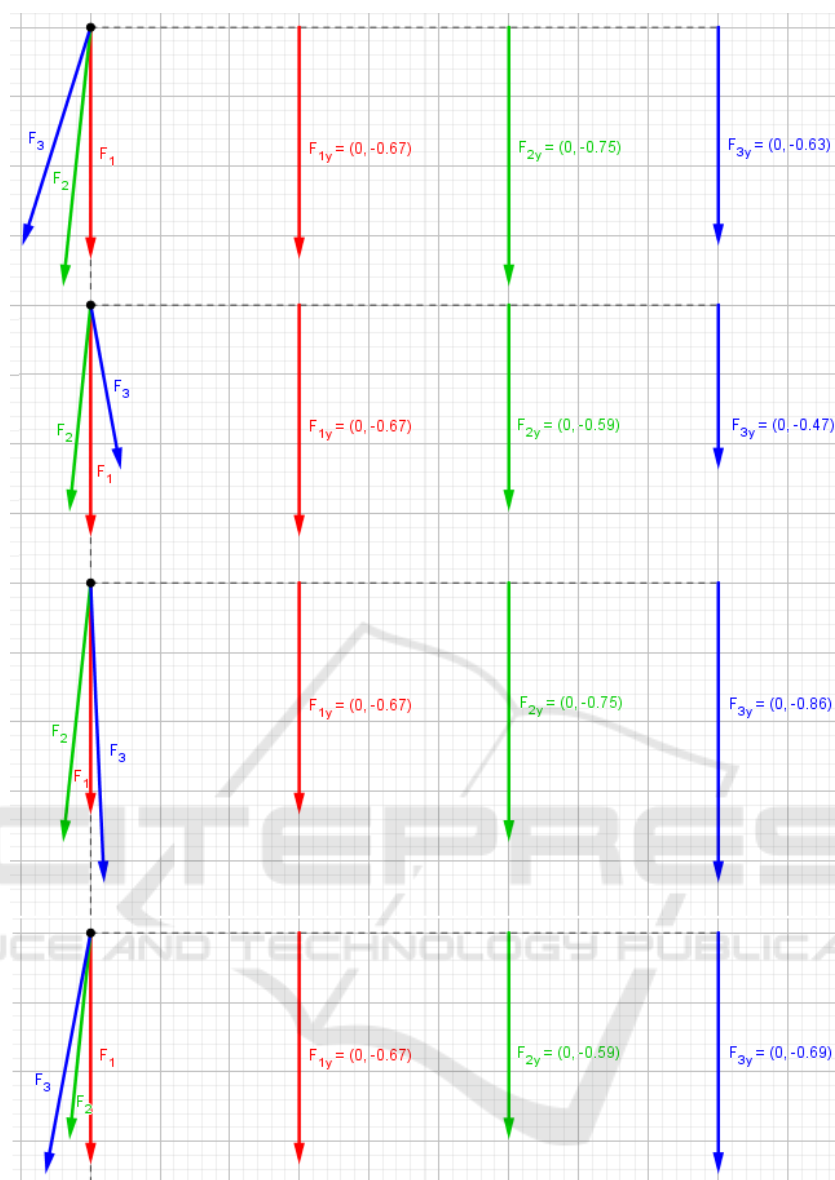
Figure 14: The dynamic vector diagram of Y-component F_y of the resultant of the forces acting on the body in four cases (from top to right): `ti = 0, rot = 1`; `ti = TM, rot = 1`; `ti = 0, rot = -1`; `ti = TM, rot = -1`.

component of mathematical modeling, provides them with the opportunity to diagnose calculations by itself, and also creates additional didactic benefits for the teacher, such as being able to identify and discuss with students all the intermediate effects and simulation outcomes that may remain invisible (hidden) when using professional computer math systems such as Mathcad (Flehantov and Antonets, 2017), Mathematica, MATLAB or Maple, etc.

In addition, visualization of the trajectory of mechanical motion of bodies in the form of Excel charts provides the formation of intuitive ideas about how the characteristics of movement of bodies changes depending on the initial conditions and other parameters. This allows to students to acquire the skills of consciously adjusting input parameters to achieve the desired simulation result. However the standard Excel features do not allow them to visualize the changes in the instantaneous velocities and the direction of motion of bodies along their trajectories. Therefore using of Excel in training does not create sufficient conditions for the formation of skills to analyze the dynamic characteristics of movement based on numerical data.

Table 4: Pair-wise post-hoc comparisons results.

Pair-wise post-hoc comparisons of means	E vs. G	E vs. EG	G vs. EG
LSD-test	p>0.9468	p<0.0262	p<0.0120
Duncan-test	p>0.9475	p<0.0135	p<0.0151
Tukey HSD for unequal N test	p>0.9977	p<0.0061	0.0048

On the other hand the GeoGebra allows to the students to form their intuitive spatial perceptions faster by visual analyzing dynamic motion characteristics. In our view this is made possible primarily by the dynamic visualization of vector characteristics of motion. We observed that after completing the proposed learning exercises with the help of GeoGebra the students more easy formulated the meaningful answers to questions of qualitative evaluation. Studying the dynamic plots such as figure 11 in an interactive mode allows them quickly and accurately answer the questions such as: "How does the change the direction of rotation will affect the linear velocity of the body?", "At what points in time the velocity of the body will in airless space be equal to the speed of the body in air?" and so on. However, the students who were working in Excel environment were better at answering questions about the quantitative characteristics of the models.

That's why as in our previous study (Flehantov and Ovsiienko, 2019, 2016) we are of the opinion that using only GeoGebra to study the basics of modeling creates the inconvenience to evaluate the simulation results in a numerical dimension. Therefore it does not provide a sufficient level of skills to numerically evaluate the characteristics of the phenomenon or the simulated process. At the same time the wide using of GeoGebra during BMM training contributes to the effectively formation of intuitive spatial representations that are very important for engineering professionals. Because, as in the previous study, there is no significant difference between final learning outcomes in groups E and G, this could mean that simultaneous use of Excel and GeoGebra compensates for these shortcomings and therefore provides the best educational achievements. At the same time, the using of dynamic vector diagrams in GeoGebra environment to learning the basics of mathematical modeling further up these results.

5 CONCLUSION

The analysis of the data we collected shows that the students who simultaneously used GeoGebra and Excel at the highest level of learning difficulty in dynamic vector diagram visualization mode demon-

strated a better understanding of mathematical modeling problems and improved their ability to use mathematical modeling to solve problems. At the same time, we found no statistically significant difference in learning outcomes between groups of students who used only Excel or only GeoGebra separately during BMM training. In addition, we noticed that students who used Excel for computer modeling responded better to quantitative questions of exam. The students who performed their learning exercises exclusively in the GeoGebra environment were much better at dealing with quality questions.

Results of this study we get conclusion that the simultaneous use of Excel and GeoGebra with vizualisation mode by dynamic vector diagrams demonstrated improved the academic achievement of students with BMM. This is indicated by the statistically significant difference between the average results of students' academic achievement, shown in table 2 confirmed with the results of table 3 and table 4.

So, this research show that the proper use of appropriate software to visualize the dynamic characteristics of mathematical models in the context of the teaching BMM at an agrarian university is an effective way to improve student performance. We were concluded that the hypothesis that visualization of the characteristics of mechanical movement by the dynamic vector diagrams can to improve the educational achievements of students is true. However the assumption that visualizing the results of modeling using GeoGebra in conjunction with a differentiated approach to learning creates additional conditions for improving students' knowledge, taking into account the specifics of their vocational training requires further verification.

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