

# Trajectory Tracking and Formation Control of Multiple Mobile Robot based on Leader Follower Approach: Comparing Constant and Non-constant Velocity of the Leader

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**Keywords:** formation control, tracking trajectory, leader follower, mobile robot.

**Abstract:** Formation control with leader follower approach and tracking trajectory with comparing constant and non-constant velocity of the leader is proposed in this paper. Non-constant velocity aim to solve the problem of time requirement to achieve asymptotic tracking error in velocity constant issue of the leader. Controller is designed based on mobile robot kinematics model. Mobile robot used similar type of model and characteristics. Trajectory information used to control the leader's velocity. Position and velocity of the leader used to determine the follower movement velocity. Follower tracks the virtual position to make a rigid formation. The experiment was carried out using two scenarios. First, leader moves linearly toward a destination point, second, leader moves along a circular trajectory. Each scenario analyzed and compared between constant and non-constant velocity of the leader. The experimental results show leader's velocity with a non-constant value has a slightly slower than constant velocity to achieve formation error that is close to zero. However, the leader's velocity with a non-constant value has a faster time to achieve tracking trajectory error that is close to zero.

## 1 INTRODUCTION

Formation control of mobile robot is one of the many research topics currently conducted. That is because of its ability to perform complex tasks with high efficiency and reliability. A rescue mission, moving large objects, hunting, forming a satellite formation and clustering is too difficult or impossible for a single robot. Formation controls are also used in completing agricultural task (Cartade et al., 2004), supervisory assignment (Tang and Özgüner, 2005) and transportation (Loianno and Kumar, 2018). The main purpose of formation control is to move to follow the trajectory and maintain the formation. Formation controls have the challenge that each agent usually cannot rely on centralized coordination and must use local information to achieve the desired formation. The category of formation control is divided into centralized (De la Cruz et al., 2006) and decentralized (Li, Er and Zhang, 2017). It is centralized if all controls and monitors are performed by a centralized processor. Decentralized

when all robots have local control over the task. There are several strategies to implemented the formation control of a group of mobile robots, e.g., behaviour-based control (Droge, 2015), virtual structure (Cao and Liu, 2012), leader-follower (Loria, Dardemir and Alvarez Jarquin, 2016, Ghamry and Zhang, 2015), relative position-based (Dimarogonas and Kyriakopoulos, 2008), artificial potentials field (Ying and Xu, 2015) and graph theory (Han et al., 2012).

One of the most popular formation controls is the leader follower approach. The approach consists of one robot as a leader and some robots as followers. For example, there is one robot as a leader following a trajectory and then another robot as a follower who follows a leader with a predetermined position and orientation. The advantage of the leader follower approach is the effectiveness in controlling a group of mobile robots by simply assigning a single trajectory to the leader only. Some previous research on leader follower usually determines the velocity of leader's movement constantly, for example by (Choi, Choi and Chung, 2012). Experiments conducted

using two robots, one as a leader and other as a follower. The controller method used by the follower is PID. In other studies that provide a constant value to the leader is performed by (Li and Xiao, 2005, Obayashi et al., 2017, Guo et al., 2017). The issue in the trajectory tracking and formation control problem by setting the leader's velocity constantly is need a longer time to reach the asymptotic tracking error.

Formation control with leader follower approach and tracking trajectory with comparing constant and non-constant velocity of the leader is proposed in this paper. Non-constant velocity aim to solve the problem of time requirement to achieve asymptotic tracking error in leader velocity constant issue. Trajectory information is used to control leader's velocity, position and leader's velocity used to control follower's movement velocity. Follower will tracks a predetermined virtual position to form a formation, while the leader tracks a point or trajectory. Direct kinematic used to control design and implemented to all robots. The objective to be achieved in this research is to generate optimal time to achieve asymptotic tracking error, either formation error or trajectory tracking error.

## 2 ROBOT MODEL

In this section the mobile robot dynamics and kinematics models based on the research by (De la Cruz et al., 2006) are described. Type of mobile robot use differential drive. The movement of a mobile robot wheeled is influenced by the movement of each wheel. Mobile robot used in this research is illustrated as in Figure 1.

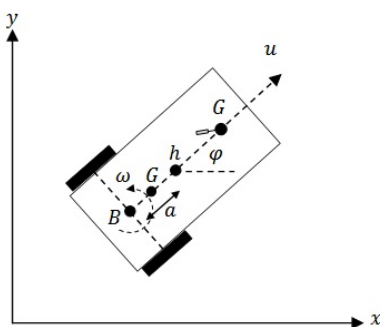


Figure 1. Model of mobile robot

$u$  and  $\omega$  are linear and angular velocity;  $\varphi$  is orientation of the mobile robot;  $a$  is distance between center point of the wheel axis and local coordinates;  $G$  is center of robot mass;  $B$  is center

point of the wheel axis;  $C$  is castor wheels; and  $h$  is position robot in the global coordinate. The mathematical model is completely written as;

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos \varphi & -\alpha \sin \varphi \\ \sin \varphi & \alpha \cos \varphi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \\ 0 \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} \dot{u} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \frac{\theta_3}{\theta_1} \omega^2 & -\frac{\theta_4}{\theta_1} u \\ -\frac{\theta_5}{\theta_2} u \omega & -\frac{\theta_6}{\theta_2} \omega \end{bmatrix} + \begin{bmatrix} \frac{1}{\theta_1} & 0 \\ 0 & \frac{1}{\theta_2} \end{bmatrix} \dots \dots \begin{bmatrix} u_r \\ \omega_r \end{bmatrix} + \begin{bmatrix} \delta_u \\ \delta_\omega \end{bmatrix} \quad (2)$$

$$\theta = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \theta_4 \quad \theta_5 \quad \theta_6]^T \quad (3)$$

$$\delta = [\delta_x \quad \delta_y \quad 0 \quad \delta_u \quad \delta_\omega]^T \quad (4)$$

Equation (1) is a kinematics model and (2) is a dynamics model of the mobile robot. Equation (3) is a vector of identified parameter and (4) is an uncertainty parameter that occurs in the robot while movement. Variable  $\theta$  is related to physical robot and refers to the research conducted by (Martins et al., 2008). Variable  $\delta_x, \delta_y$  are slip velocity and orientation functions of the mobile robot, while  $\delta_u, \delta_\omega$  is a physically caused disturbance function such as mass, inertia, wheel and tire diameters, motor and servos parameters, power on wheels, and others . Parameter  $\delta$  is a disturbance that occurs and affects the movement of the mobile robot.

## 3 LEADER-FOLLOWER FORMATION CONTROL

In the leader follower approach, agent or robot is divided into two types of roles, i.e. one role as leader and the other as follower. To form and maintain the formation, follower must know position to the leader. Follower must always keep the distance and angle error of the leader equal to zero, in other words, robot must always go to the specified reference point  $(x_{dF}, y_{dF})$ . Reference point is always rigid to leader, and are called as a virtual position. Illustration of leader follower approach shown as Figure 2, where  $dist_r$  and  $\theta_r$  are distance and angle reference respectively;  $(x_L, x_L)$  is currentt position of

the leader;  $\varphi$  is heading of the leader,  $(x_{dL}, y_{dL})$  is position reference of the follower;  $(x_F, y_F)$  is current position of the follower; and  $e_x$  and  $e_y$  position error in  $x$  and  $y$  axis respectively. Based on Figure 2, distance reference and angle reference can be defined in Equation (5) and (6), respectively, and virtual position can be obtained using Equation (7) and (8).

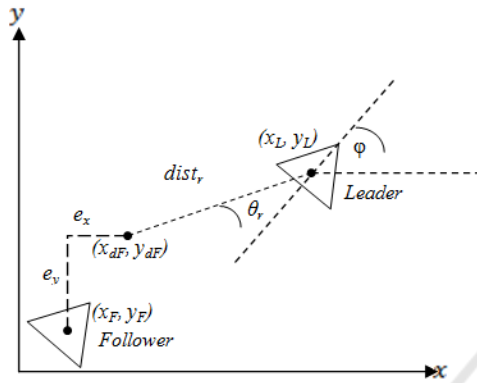


Figure 2. Leader follower model

$$dist_r > 0 \quad (5)$$

$$-\frac{\pi}{2} < \theta_r < \frac{\pi}{2} \quad (6)$$

$$x_{dF} = (x_L - (dist_r * \cos(\varphi_L + \theta_{rF}))) \quad (7)$$

$$y_{dF} = (y_L - (dist_r * \sin(\varphi_L + \theta_{rF}))) \quad (8)$$

#### 4 CONTROL DESIGN

Motion control is designed based on the kinematic model of mobile robot. Type of decentralized control is used in this study, so each mobile robot use local control. Assuming there is no slip on the wheel and a disruption to the robot dynamics, the kinematic equation is written as follows;

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} \cos \varphi & -\alpha \sin \varphi \\ \sin \varphi & \alpha \cos \varphi \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ \omega \end{bmatrix} \quad (9)$$

Assuming the specifications of all robots are similar, then the control laws proposed for all mobile robots are written in Equation (10).

$$\begin{bmatrix} u_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos \varphi & \sin \varphi \\ -\frac{1}{a} \sin \varphi & \frac{1}{a} \cos \varphi \end{bmatrix} \dots \quad (10)$$

$$\begin{bmatrix} \dot{x}_d + s_x \tanh\left(\frac{k_x}{s_x} e_{x_i}\right) \\ \dot{y}_d + s_y \tanh\left(\frac{k_y}{s_y} e_{y_i}\right) \end{bmatrix}$$

Where;  $\dot{x}_d, \dot{y}_d$  is desired velocity,  $(e_{xi}, e_{yi})$  the position error,  $(k_x, k_y)$  is gain controller where  $k_x > 0$  and  $k_y > 0$ ,  $(s_x, s_y)$  is saturation of the robot;  $(x_d, y_d)$  is desired position;  $(x_i, y_i)$  is current position of the robot; and  $i$  is index of the robot. Distance error on the  $x$  and  $y$  axis is calculated using Equation (11) and (12), respectively.

$$e_{x_i} = x_{d_i} - x_i \quad (11)$$

$$e_{y_i} = y_{d_i} - y_i \quad (12)$$

#### 5 EXPERIMENTAL RESULTS

Experiments conducted to verify and determine the effectiveness of the control design. Experiments uses three mobile robots and the desired formation is a triangular shape as shown in Figure 3. The robot parameters used in the experiment are shown in Table 1. Experiment is comparing constant and non-constant velocity of the leader while perform formation and tracking trajectory. The experiments were conducted in two scenarios. First, leader moves linearly to a specified point. Second, leader following a circular trajectory.

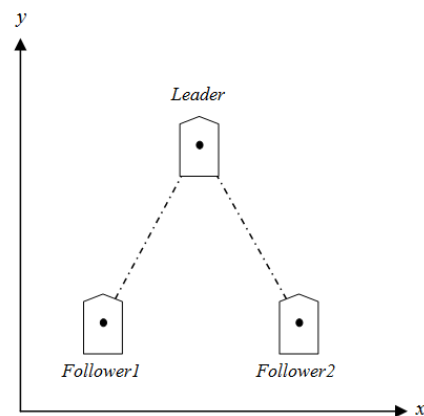


Figure 3. Triangular shape formation

Table 1: Parameters of the mobile robot

Parameter	Leader	Follower1	Follower2
$k_x$	0.5	0.5	0.5
$k_y$	0.5	0.5	0.5
$s_x$	0.4	0.4	0.4
$s_y$	0.4	0.4	0.4
$\alpha$	0.2 m	0.2 m	0.2 m

### 5.1 Linear Movement

In this scenario, the leader is moves to the specified point, and then the follower moves to construct a predetermined formation. Destination point of the leader is  $x=6$  and  $y=0$ . Initial position of each robot has not yet formed a triangle formation. The initial values of the positions of each robot and the formation parameters are shown in Table 2. The leader's velocity with constant value is 0.01 m/s. For simplicity, we agree that  $a$  is leader's velocity with constant value and  $b$  is leader's velocity with non-constant value or leader's velocity with controller value.

Table 2: Initial position and formation parameters

Parameter	Leader	Follower1	Follower2
$x$ initial	1	0.5	0
$y$ initial	0	0	0.5
$\phi$ initial	0	0	0
$dist_r$	-	0.5 m	0.5 m
$\theta_{rF}$	-	0.785 rad	-0.785 rad

In the linear movement experiments, formation error of  $a$  is more faster than  $b$  to close to zero as shown in Figure 4. In the Figure,  $a$  is constant velocity and  $b$  is non-constant velocity of the leader respectively. The steady state error of the robot during make a formation has close to zero in  $s = 4$  in  $a$ , whereas in  $b$  close to zero in  $s = 32$ . Formation error for each follower is obtained using (13). Tracking goal position error with a non-constant velocity of the leader more faster than constant velocity to close to zero as shown in Figure 5. Steady-state error of tracking goal position occurred at  $s = 500$  in  $a$ , whereas in  $b$  occurred at  $s = 40$ . The experiment shows that in  $a$ , formation is faster to rigid, but takes longer time to reach destination point. In otherwise,  $a$  need longer time to reach rigid formation, but  $a$  faster to get to the destination

$$dist = \sqrt{e_x^2 + e_y^2} \tag{13}$$

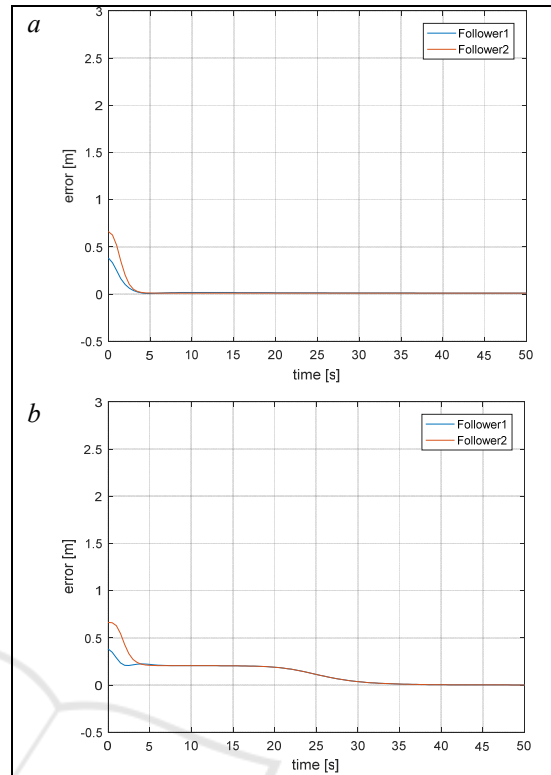


Figure 4. Formation error

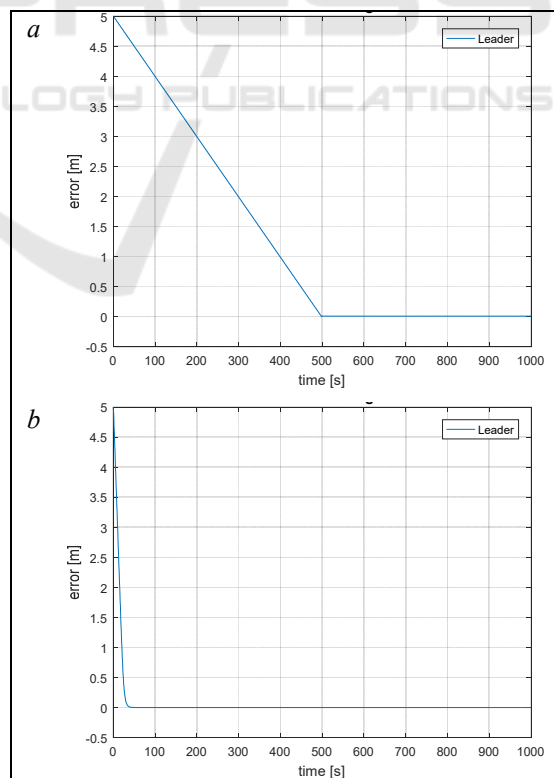


Figure 5. Tracking goal position error

The linear and angular velocities of each robot are shown in Figure 6, whereas the robots trajectory are shown in Figure 7. Linear velocity each follower of *a* and *b* close to leader in  $s=4$ . Angular velocity of *a* close to the leader in  $s=5$ , *b* close to the leader in  $s=7$ . *b* before reaching steady state in linear velocity, there is an overshoot at  $s=2$  for follower1 and at  $s=3$  for follower2. Angular velocity in *b*, before reaching steady state, there is an overshoot at  $s=4$  for follower2 and at  $s=3$  for follower2.

## 5.2 Trajectory Tracking

In the second scenario, the leader's movement is controlled to follow the circle trajectory. While leader moves to follow trajectory, follower is controlled to formed a triangle formation. The initial position of each robot used first scenario experiment parameters. Initial position values of each robot and formation parameters are shown in Table 2. The leader's velocity value is  $0.01 \text{ m/s}$  for experiments with leader having constant values.

According to this experiment, formation errors of each follower converge to zero at  $s=4$  in *a*, while *b* close to zero at  $s=16$ . The trajectory tracking errors has close to zero at  $s=40$  and at  $s=800$  for *a* and *b* respectively. This indicates that the formation has been more quickly achieved if the leader is given constant velocity, but trajectory tracking error is very slow close to zero. whereas in *b*, formation is established slowly, but trajectory tracking error is faster close to zero. Comparison of distance error during the formation established in this experiment is shown in Figure 8, while the comparison of trajectory tracking error is shown in Figure 9.

The linear and angular velocity of each robot are shown in Figure 10. Linear velocity of *a* close to leader in  $s=4$ , while *b* in  $s=20$ . Angular velocity of *a* close to the leader in  $s=31$ , *b* close to the leader in  $s=15$ . *b* before reaching steady state in linear velocity, there is an overshoot at  $s=2$  for follower1 and follower2. After the overshoot, follower2 is slower than follower1 to reach steady state. Angular velocity in *b*, before reaching steady state, there is an overshoot at  $s=4$  for follower1 and follower2. Since  $s=2$ , angular velocity of all followers has the same value.

Figure 11 shows the trajectory of leader and followers in the second experiment. In this experiment, it can be seen that the robot formation with leader follower approach is rigidly.

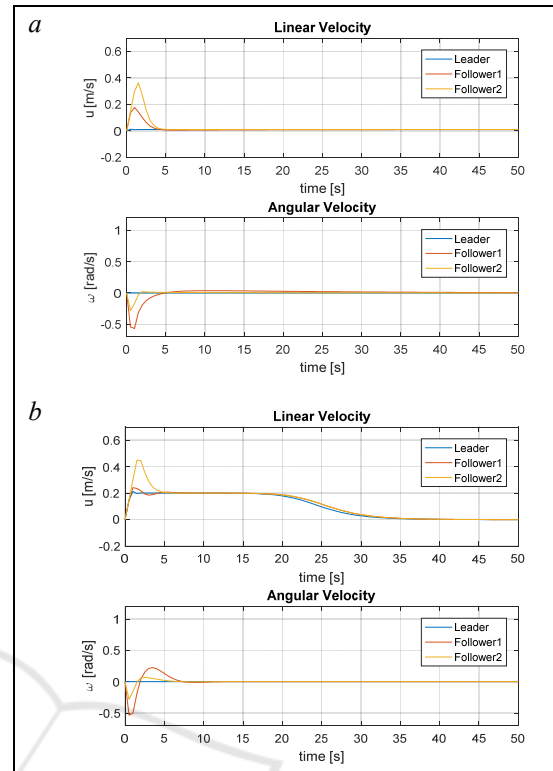


Figure 6. Linear and angular velocity of the robot

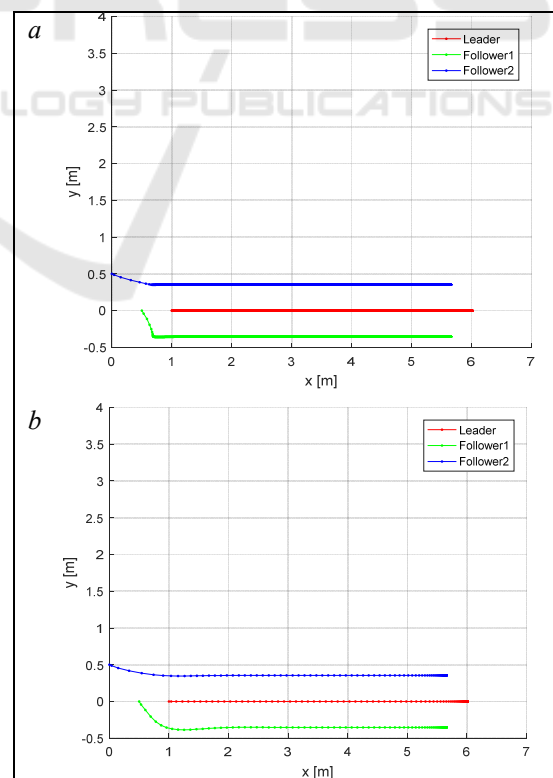


Figure 7. Trajectory of the robot

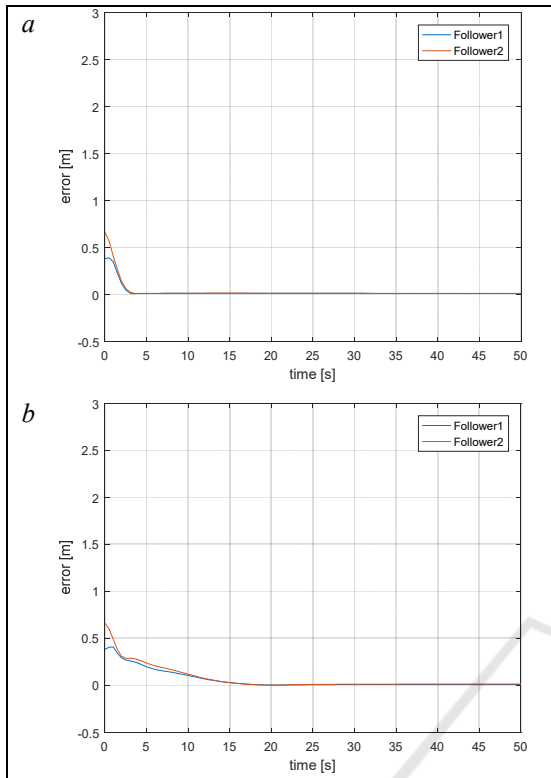


Figure 8. Formation error

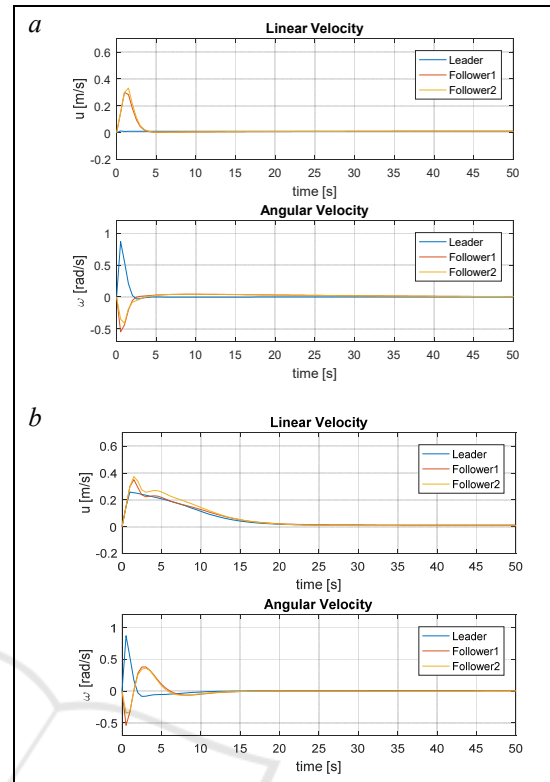


Figure 10. Linear and angular velocity of the robot

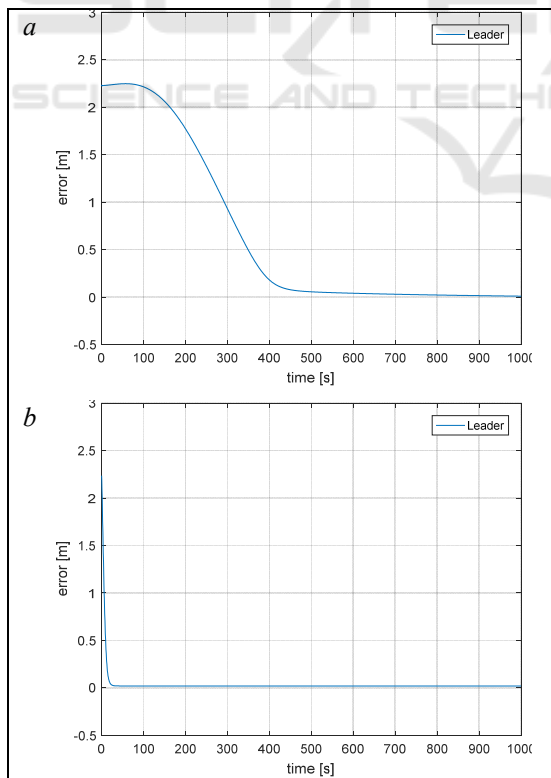


Figure 9. Tracking trajectory error

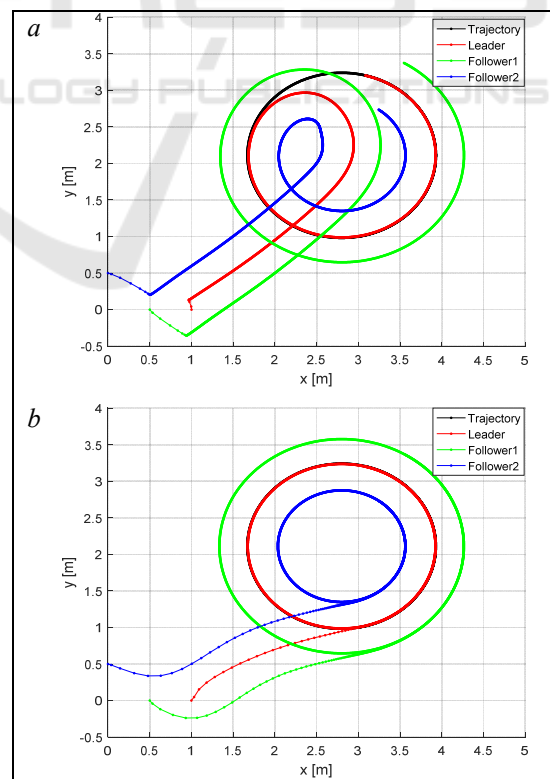


Figure 11. Trajectory of the robot

## 6 CONCLUSIONS

Experiments on formation control with leader follower approach and tracking trajectory by a group of mobile robot were performed with two scenarios. The experiment also compared the results between the constant and non-constant leader velocity. The formation used in the experiment is a triangular shape. The experimental results show that the leader with non-constant velocity has a slower than that of the constant velocity to established formation, i.e. at  $s = 32$  and  $s = 4$  for the first scenario, and  $s = 16$  and  $s = 4$  for second scenario, respectively. However, the leader with non-constant velocity has a faster to achieve trajectory tracking error close to zero, i.e.,  $s = 40$  and  $s = 500$  for the first scenario, and  $s = 40$  and  $s = 800$  for second scenario, respectively. In the future research, experiments can be implemented to robots that work in 3D plane like quad-rotor.

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