Students' Understanding of the Equal Sign: A Case in Suburban School

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Abstract: Mathematics has an essential role in education and to get employment opportunities then algebraic topics are one of the fundamental topics of efforts to reform mathematics education. Understanding algebra is related to understanding the basic concepts of equality. There are not many students understand the meaning of the equal sign. Therefore, this article is aimed to examine students' understanding of the meaning of the equal sign in solving an equation. Knowing the students' understanding deeply, we used this research by choosing the qualitative method. The research carried out the 22 students, the second-semester secondary school, who have learned the material of algebraic form and its operations, the seventh grade of SMP in Pekanbaru, Riau. The data provided in this study shows that many students do not have a sophisticated understanding of the meaning of the equal sign. Most students understand the equal sign as a result of a problem, such as solving a problem likely a count from the left side to the right side. The view of the students' misunderstanding about the equal sign causes that the students perceive the meaning of the equal sign as the result of operations, not a symbol of mathematical equality.

1 INTRODUCTION

In recent years, many mathematics education communities have been interested in investigating students' difficulties in learning algebra (Gutiérrez et al., 2016). Algebra is a fundamental concept for understanding high and advanced school mathematics either for their daily lives or for employment.

Much research has done in investigating students' difficulties in learning algebra, especially for middle school students, but algebra problems still provide exciting opportunities to be examined in many aspects. Lately, the theme of early algebraic thinking has become an interesting one. This theme indicates that learning algebra, not only for students in middle school but also students, in primary school, can learn.

Research in early algebra began with the movement research of Carraher and Schliemann about how young students were able to start learning algebra in elementary school (Carraher, 2007; Kieran et al., 2018). Algebra in middle school usually has been a difficult subject for the students. The transition from arithmetic to algebraic thinking give the difficulties problem to the students. From the research Carraher and Schliemann, they showed that the young students could learn algebra earlier. The

students are given a problem in their learning topic, like numbers, then they could solve the problem algebraically. However, their idea research gave difference against doing the concept of a traditional curriculum in which states that algebra can be learned if students already have sufficient knowledge in arithmetic. The students, in elementary school, learn about numbers and operations, but, in secondary school, they will learn about algebra (Van Amerom, 2003; Chimoni et al., 2018). If the students who are already proficient in mathematics, it will easy to learn algebra, but the fact that occurs, the student's prior experience in arithmetic is not enough to help students learn algebra, even make students afraid of learning algebra.

There are two ideas for algebra, the basic concept for developing algebraic reasoning, that is variable and equality (Knuth et al., 2005; Ertekin, 2017). In this article, we focus on the discussion of the concept of equality. Equality is an essential concept for algebra achievement (Van Amerom, 2003; Chimoni et al., 2018). In elementary school, the students have learned about equality. They learn about finding the value of an unknown number. Then in the next level, the student will learn how to connect the two mathematical expressions that have the same value.

The symbol that became a necessary concept is the equal sign. Most of the student has the complicated meaning of the equal sign. They generally perceive that the equal sign as a result of arithmetical operation rather than mathematical equality (Kieran, 1981; Stephens et al., 2013; Vermeulen and Meyer, 2017). The famous research founded by Falkner, Levi, and (Falkner et al., 1999) showed that most of the students perceive the meaning of the equal sign as an operational symbol, calculating the number from left side to right side. Most of the students do not recognize the meaning of the equal sign as a relational symbol.

The usage of the equal sign in mathematics has four primary categories, such as an answer of the operation, quantities equality, an expression that is right for all value of the variable, and an expression that express the new variable (Warren, 2006; Ertekin, 2017; Vermeulen and Meyer, 2017).

In Indonesia, the research on the meaning equal sign does not take in much discussion in the area of middle school. Whereas the understanding of the equal sign is an essential concept in algebra learning, it is necessary to advance mathematical concepts. Indonesia Curriculum does not mention explicitly about the meaning of the equal sign. Though the meaning the equal sign is the basic concept for the learning mathematics. In this paper, we drew the students' understanding of the equal sign with examine their worksheet and thinking by interview. However, the students have learned algebra topics, their thinking of the meaning the equal sign describe how they were understanding of the equal sign.

2 RESEARCH METHOD

This research used a qualitative methodology and descriptive analysis, based on the participant observation, interview, and worksheet (Creswell and Creswell, 2017).The task was about algebra domain, and the participants have already learned in the first semester in grade seven. The participant was from a private middle school in Pekanbaru. The school has a well-resourced in the suburban area of Pekanbaru City. Its teacher is a very enthusiastic person and proper motivation. The position of the researcher in this research was observer and researcher directly while the research was doing. Because this research was a qualitative approach, the researcher was a dominant aspect to control the process of the research, and teacher became a partner and together decided on examining the work of students and selecting students to be interviewed. This study took part in

22 students (aged 13 to 14). In general, the students have a different academic, and their daily life was surrounding in that school.

Data for this research was taken from students' answer sheet, such as the written document that the students' produces, and the audio record while interviewing the participants. The participants were given the essay test; then, they wrote the answer in the blank paper as a document for the researcher. After they finished the answer, the researcher examined the answer sheet and discussed with the teacher how the result of the students' worksheet. We chose three participants who solve the problem correctly and three participants who solve the problem incorrectly. The participants also were students who were able to communicate when the researcher conducted interviews and evaluated students' understanding in solving problems.

The data were analyzed by description for the document answer sheet. We collected all document and the code based on the student answer and what their understanding of the equal sign while solving the equation. The audio record, the result of the students' interviewing, were analyzed in transcripts, and we chose what students' the misunderstanding and the thinking of the equal sign.

3 RESULTS AND DISCUSSION

As it already mentions the concept of the equal sign, that is, the equality shows two mathematical expressions that have the same value. In the first question in this study, we present simple equations that are similar assignments in the students' elementary school. The task is the following "Find the value of this number sentence $30 = \Delta + 20$". Most of the students solve the problem by subtraction operation. The students claim that the first time while seeing this problem as strange. The students are seldom solving this problem. They say that they usually solve the problem like $\Delta + 20 = 30$. So, it is a simple problem to solve.

However, it is a strange form for themselves; they can solve and find a solution. There are 19 students get the correct answer of the 22 seventh grade students in this class. Three students have an incorrect answer in seeing the symbol triangle. The mistake of the students is when they move the triangle symbols from the right side to the left side then do the addition $30 + 20 = 50$.

The correct students find the solution by using subtraction operation as $30 - 20 = \Delta$". However, initial their thinking is confused; they move the

triangle from the right side to the left side. It emerges that in their thinking, the symbol should put it on the left side. The student is usual in solving the equation by counting from the left side to the right side. The student's answer sheet was categorized as operational level in which the student's ides that do the operation number like addition or subtraction (Knuth et al., 2008), when the students are asked the meaning of the equal sign in that problem, student answer that it is as a sign the answer to the problem. The researcher has indicated that the operational interpretation of the students; it is mostly from their experience influences in prior knowledge in elementary school.

The usage of the equal sign is in this case as an answer to the operation. The students are solving the problem in this case for one unknown number. The students think that it is a simple problem and more accessible to solve it. Most research shows that the students are not troubled while solving equation $3 + 4 = ?$ but trouble in solving in $8 = 4 + ...$ research found that proportion students to understanding the function of the equal sign as relational only 37% of the students at the same time just 48% of the students know the equal sign as an operational (Asquith et al., 2007).

As a result of this case, the students link their thinking about the equal sign with the operational aspect in arithmetic. The students perform the equal sign as a symbol to get a final answer from their calculation.

Another case that the student says while interviewing. The student is confused while moving the symbol and number from the left side to the right side, and vice versa. If the positive numbers move from the left side to the right side, the positive numbers will become the negative numbers. They put the triangle on the left side, but they do not give the negative sign in the triangle symbol. They do the addition of number $\Delta = 30 + 20$. It is incorrect one in this task. The inaccuracy of the students is one of which causes the wrong answer.

While interviewing happening, another student tries to solve the other problem. We find the unusual case of his answer sheet and try to investigate what he did. That is his handwriting.
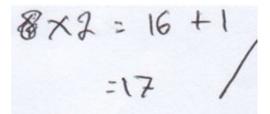

Figure 1: The student put the equal sign as a result.

In figure 1 show that the student put the equal signs as a result of $16 + 1$. He does not think that the meaning of the equal sign is the equivalence of two mathematical expressions. In this case, the expression of 8×2 and $16 + 1$.

They think the equal sign is the result of what they are doing and finishing the problem. When we are expanding the students writing, that is $8 \times 2 = 16 + 1 = 17$, of course, it became the wrong expression. This way is that most of the students write while solving the problem. The impact of these mistakes that these students always make will give forthcoming aspects in understanding the meaning of the equal sign (Vermeulen and Meyer, 2017).

Next, we look at the third equation that it is given to the students. The task is, how do we solve this statement to be true? This is the number sentence problem $\Diamond + 4 = \nabla + 1$. This problem is two mathematical expressions that have two unknown numbers. The student should find the two unknown numbers, then the sum of the numbers should equal between in the left side and the right side.

The student answer sheet seems to contradict from the researcher thinking. He gives a unique answer from that equation. He put five on the left side and two on the right side. Firstly when we examine the answer sheet, we do not understand the meaning of this answer. After that, we interview the student for knowing this reason.

Figure 2: The Student is misunderstanding while solving equality task.

From figure 2, we directly say that it is a wrong statement. We will not think why the student chooses those numbers. In interviews, the student shows the reason that he writes in solving this equation. The transcript between researcher and students will be shown in this following (R is a researcher, and S is a student) :

R: Is 5 + 4 the same as 2 + 1?.

S: No.

R: Why do you put 5 and 2 in that the geometric form?

S: Because? (silence and scratch the paper)

S: I think that 5 add 1 equal to 6 and 4 add 2 equals to 6. So, the answer is 5 and 2.

R: Ok, let us see the question again!

S: (See the problem and try to change the answer).

R: Why do you change this position?

S: Because 5 + 1 = 6 and 2 + 4 = 6.

R: Are you sure? Can you explain it?

S: Hmm, I think this a correct one (he point his script)

R: What do you think about the equal sign?

S: The equal sign stands for the result.

R: Can you explain it?

S: Yach, it is adding 5 + 1 = 6 dan 2 + 4 = 6. So, the left side and the right equal to 6

R: Is there other meaning that you know about the equal sign?

S: No.

This student sees the equal sign as the result of the operation of the number. Nevertheless, in his answer sheet, initially, he does not make aware of the equal sign between two mathematical expressions. He does the calculation to find the value; then he decides that the task is a true statement.

From the results of this research, Many students view the equal sign as an operational symbol and the result of the calculation of numbers (Stephens et al., 2013) They fail to see the equal signs as the symbols of the equivalence between the left side or the right side (Kieran, 1990; Knuth et al., 2008). The other student thinks that there is no relation between the two statements either in left or right. The students cannot be able to explain why this number sentence is correct or not, and why they do the calculation by adding between two number.

Concerning the correct answer of the students, they are correctly in solving the equation, but they are not correct the meaning of the equal sign (Knuth et al., 2008). Most students did not seem to understand the usage of the equal sign. The concept of the equal sign is necessary to lead the student in learning algebra either in middle school or the advance school. The problem seems the simple equation, but it has a goal to find the students' understanding. The students are easier to find the one unknown number, but it is challenging to find more unknown numbers (Falkner et al., 1999; Warren, 2003; Ertekin, 2017).The limited experience of the student in solving the problem makes this task to be a difficulty. For a reason, the students should typically use the meaning of the equal sign in various mathematical problems.

The next research of this topic, the researcher thinks about the students in the city area. The question

for the researcher is, "Is there the same problem for the city school in Riau province?". It will become the next research for us to know the problem in Pekanbaru city. The larger target for the research could help to find more problem.

4 CONCLUSIONS

Based on the discussion that students show in their responses while solving the problem, indicate that the student has learned and see the equal sign that is a result activity from a statement in left to right. Most of the student does not see the relation between two statements in the right and left. In this research, we cannot conclude that the students in the suburban cannot understand the equality domain. In this aspect, it is just looking at the view of how the understanding of student while solving the equation.

As we reflect that in our background in this paper, this is the notion of equality and the equal sign in the middle classroom, and we should continue the other aspect what the student interest and excited while solving the task and bring in-depth discussion. The students understanding the equal sign should be developed as they learn about numbers and operations. This understanding will give the student the reflection about the equation and will lead them to learn learning algebra in the next grade.

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