A Suboptimal Estimation Algorithm for Vehicle Target Motion Parameters with Incomplete Measurement

Sujuan Chen[1, a]
[1]College of Automobile and Rail Transit, Nanjing Institute of Technology, Nanjing, 211167, China

Keywords: Detection probability, target tracking system, incomplete measurement, filtering algorithm.

Abstract: The detection probability of the radar of the vehicle target tracking system is often less than 1 during driving on urban roads, and the measurement data loss problem may occur. In this paper, the stability of the vehicle target tracking system is studied and the sufficient conditions are given for the stability of the mean-square exponent under incomplete measurement conditions. A suboptimal estimation algorithm for vehicle target tracking motion parameters under incomplete measurement conditions is given when the detection probability is known. The simulation results show that the proposed filtering algorithm is effective.

1 INTRODUCTION

With the development of social economy, the number of vehicles has increased rapidly, and the problem of road traffic safety has become increasingly prominent, which has become a global problem. A large number of casualties and property losses caused by traffic accidents every year in the world, Vehicle accident statistics show that the car rear-end accident is one of the most dangerous road traffic and one of the most dangerous accidents in the vehicle operation. And 91% of the rear-end collision accidents are caused by the driver's inattention. If the driver is reminded 0.5s in advance, the rear-end accident avoidance rate can reduced to 90%. The main threat to the driver is the other vehicles on the road. The purpose of car-assisted driving is to alert the driver to the driving environment and possible collisions with other vehicles. Therefore, real-time, accurate tracking and prediction of other vehicles on the road surface is required. In order to estimate the vehicle target motion parameters, it is first necessary to extract the dynamic target information from the road traffic scene, and then use the filtering algorithm to track the target vehicle to obtain the target operating parameters. All of this information can be measured by tracking the radar installed on the vehicle, but in complex traffic scenarios, the detection probability of the system is usually less than 1 due to obstacle obstruction, noise working environment and detection equipment failure, etc., so that the corresponding problem becomes an estimation problem with incomplete measurement. (SinopoliB, et.al, 2004) studied the critical detection probability problem of filter convergence under intermittent measurement, and the upper and lower bounds of the critical detection probability were obtained. BoersY et al. (Boers Y, et.al, 2006; Y. Boers, et.al, 2006; Fayad F, et.al, 2007) studied the modified Riccati equation and pointed out that the modified Riccati equation is the upper bound of the ideal estimated variance. The upper and lower bounds of the accuracy of the target tracking with a detection probability less than 1 are studied. In (Hongli Dong, et.al, 2010), variance-constrained Filtering for a class of H∞ nonlinear time-varying systems with multiple missing measurements was studied. These have laid a good foundation for further research on filtering problems under incomplete measurement.

The optimal filter based on a certain performance index is often not easy to establish In the actual vehicle target tracking system, therefore, some suboptimal filtering methods need to be found. The satisfactory filtering based on the covariance design method is based on the set of expected performance indicators composed of multiple performance indicators which meet the actual needs of the project as the objective function (Hongli Dong, et.al, 2010; Roland Hostettler and Petar M.D, 2015; X. Zhong, A, et.al, 2012; Sujuan Chen, et.al, 2012). Satisfactory filtering refers to designing a filter with a larger expected choice under the constraint

Chen, S.
A Suboptimal Estimation Algorithm for Vehicle Target Motion Parameters with Incomplete Measurement.
DOI: 10.5220/0008872301140120
In *Proceedings of 5th International Conference on Vehicle, Mechanical and Electrical Engineering (ICVMEE 2019)*, pages 114-120
ISBN: 978-989-758-412-1

constraint of a given error upper bound constraint. The given variance constraint is not necessarily the minimum variance, but can make each state variance satisfy the given variance constraint. Reference (Sujuan CHEN, et.al, 2012) studied the allowable sampling frequency based on the variance constraint of incomplete measurement.

From the above literatures, there are not many studies on the index constraint filtering problem under incomplete measurement. In this paper, a binary Bernoulli random variable is used to represent the data loss, and the sufficient conditions for the mean square exponential stability under the statistical significance of the system are given. A sub-optimal estimation algorithm with the given detection probability is given, so that the steady state value of the estimated error variance of each state component of the system is not greater than the respective predetermined variance constraint values.

The rest of the paper is organized as follows. Section 2 establishes the system model and introduces the Kalman estimation to deal with the proposed problem. Section 3 presents some lemmas and corollaries. In Section 4, the suboptimal filter algorithm with incomplete measurement is proposed. In Section 5, an example of vehicle target parameter estimation is given to verify the effectiveness of the proposed algorithm. A summary of our conclusions is given in Section 6.

2 SYSTEM MODEL AND PROBLEM FORMULATION

Consider the following discrete-time linear time-varying state-space system with additive Gaussian process and incomplete measurement. The state equation is given by

$$\boldsymbol{X}_{k+1} = \boldsymbol{A}\boldsymbol{X}_k + \boldsymbol{w}_k. \tag{1}$$

Where

k discrete-time index;

$\boldsymbol{X}_k \in R^n$ state vector;

A matrix of appropriate dimension;

$w_k \in R^n$ zero-mean white Gaussian process noise with covariance matrix $\sigma_w^2 > 0$.

The measurement equation with incomplete measurement is given by

$$\boldsymbol{Y}_k = d_k \boldsymbol{C}\boldsymbol{X}_k + \boldsymbol{v}_k. \tag{2}$$

Where $y_k \in R^m$ is the measured output at time k, and C is a known constant matrix. $v_k \in R^m$ is zero-mean white Gaussian measurement noise independent of w_k with zero mean and covariance matrices $\sigma_v^2 > 0$. The initial condition $X_0 \sim N(0, P_0)$, is uncorrelated with both w_k and v_k . The stochastic $d_k \in R$ which represents the arrival of the measurement at time k, is a Bernoulli distributed white sequence taking values on 0 and 1, with probability distribution:

$$\text{prob}\{d_k = 1\} = \lambda. \tag{3}$$

Consider the following filter for the system (1):

$$\hat{\boldsymbol{X}}_{k+1|k+1} = \boldsymbol{A}\hat{\boldsymbol{X}}_{k|k} + d_{k+1}\boldsymbol{K}\left(\boldsymbol{Y}_{k+1} - \boldsymbol{C}\boldsymbol{A}\hat{\boldsymbol{X}}_{k|k}\right). \tag{4}$$

Where K is the filter gain to be scheduled. Define the estimation error as $\boldsymbol{e}_{k+1} = \boldsymbol{X}_{k+1} - \hat{\boldsymbol{X}}_{k+1|k+1}$, then we can obtain

$$\begin{aligned} \boldsymbol{e}_{k+1} &= \boldsymbol{X}_{k+1} - \hat{\boldsymbol{X}}_{k+1|k+1} \\ &= \left(\boldsymbol{I} - d_{k+1}\boldsymbol{KC}\right)\boldsymbol{Ae}(k) \\ &\quad + \left(\boldsymbol{I} - d_{k+1}\boldsymbol{KC}\right)\boldsymbol{w}_k - d_{k+1}\boldsymbol{Kv}_{k+1} \\ &= \left(1 - d_{k+1}\right)\boldsymbol{Ae}(k) + d_{k+1}\left(\boldsymbol{I} - \boldsymbol{KC}\right)\boldsymbol{Ae}(k) \\ &\quad + \left(\boldsymbol{I} - d_{k+1}\boldsymbol{KC}\right)\boldsymbol{w}_k - d_{k+1}\boldsymbol{Kv}_{k+1}. \end{aligned} \tag{5}$$

Using the statistics of $\boldsymbol{w}_k, \boldsymbol{v}_k$ and P_0, the estimation error covariance obeys the following recursion:

$$\begin{aligned} \boldsymbol{P}_{k+1|k+1} &= (1 - d_{k+1})\left(\boldsymbol{AP}_{k|k}\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{Q}\right) + \\ & d_{k+1}\left(\left(\boldsymbol{I} - \boldsymbol{KC}\right)\left(\boldsymbol{AP}_{k|k}\boldsymbol{A}^{\mathrm{T}} + \boldsymbol{Q}\right)\left(\boldsymbol{I} - \boldsymbol{KC}\right) + \boldsymbol{KRK}^{\mathrm{T}}\right). \end{aligned} \tag{6}$$

Since the sequence $\{d_k\}$ is random, the equation (6) is stochastic and cannot be determined offline. Hence, only statistical properties can be deduced. Applying the mathematical expectation on d_{k+1} of equation (5), we can make the following error equation of statistical significance:

$$\begin{aligned} \bar{\boldsymbol{e}}_{k+1} &= \left(1 - \lambda\right)\boldsymbol{A}\bar{\boldsymbol{e}}(k) + \\ & \lambda\left(\boldsymbol{I} - \boldsymbol{KC}\right)\boldsymbol{A}\bar{\boldsymbol{e}}(k) + \\ & \left(\boldsymbol{I} - \lambda\boldsymbol{KC}\right)\boldsymbol{w}_k - \lambda\boldsymbol{Kv}_{k+1}. \end{aligned} \tag{7}$$

Define steady-state estimation error covariance as

$$\boldsymbol{P} := \lim_{k\to\infty} E_\lambda \left\{ \boldsymbol{e}_{k+1}\boldsymbol{e}_{k+1}^{\mathrm{T}} \middle| \boldsymbol{Y}_{k+1}, d_{k+1} \right\}. \quad (8)$$

Definition 1 (Sujuan CHEN, et.al, 2012). Given constants λ and $\boldsymbol{K}$, if the error system is Lyapunov progressively stable, then the error system is said to be progressively stable in statistical sense, and its corresponding steady-state error covariance $\boldsymbol{P}$ is satisfied.

$$\boldsymbol{P} = \hat{\boldsymbol{A}}\boldsymbol{P}\hat{\boldsymbol{A}}^{\mathrm{T}} + \boldsymbol{J}\boldsymbol{P}\boldsymbol{J}^{\mathrm{T}} + \boldsymbol{\Xi}_1. \quad (9)$$

Where

$$\hat{\boldsymbol{A}} := \sqrt{1-\lambda}\boldsymbol{A}, \quad (10)$$

$$\boldsymbol{J} := \sqrt{\lambda}\left(\boldsymbol{I}-\boldsymbol{KC}\right)\boldsymbol{A}, \quad (11)$$

$$\begin{aligned}\boldsymbol{\Xi}_1 := &\left(1-\lambda\right)\boldsymbol{Q} + \\ &\lambda\left(\boldsymbol{I}-\boldsymbol{KC}\right)\boldsymbol{Q}\left(\boldsymbol{I}-\boldsymbol{KC}\right)^{\mathrm{T}} + \\ &\lambda\boldsymbol{KRK}^{\mathrm{T}}.\end{aligned} \quad (12)$$

In this paper, our objective is to design the filter that, the following two problems are satisfied simultaneously:

Q1: the error system (6) of the statistical significance is exponentially mean-square stable;

Q2: the steady-state estimation error variance $\boldsymbol{P}$ satisfies $\left[\boldsymbol{P}\right]_{ii} \le \sigma_i^2 \left(i = 1,2,...,n\right)$.

Where $\left[\boldsymbol{P}\right]_{ii}$ is the diagonal element of $\boldsymbol{P}$, σ_i^2 stands for the pre-specified steady-state estimation error variance constraint on the i state, which is not smaller than a lower bound on the minimum estimation error variance.

Definition 2 (Z.Wang, et.al, 2006). If there is a constant value $\eta \ge 0, \tau \in (0,1)$, which satisfies the following inequality:

$$\begin{aligned}E\left\{\left\|\boldsymbol{e}_k\right\|^2\right\} \le \eta\tau^k E\left\{\left\|\boldsymbol{e}_0\right\|^2\right\}, \\ \forall \boldsymbol{e}_0 \in R^n, k \in R.\end{aligned} \quad (13)$$

Then, the system is stable with a mean square exponent for all possible measurement values lost.

3 STABILITY ANLYSIS AND THEORETICAL CRLB WITH INCOMPLETE MEASUREMENT

Lemma 1 (Sujuan CHEN, et.al, 2012). Given the parameters λ and $\boldsymbol{K}$, the following statements are equivalent

$$(1)\ \rho\left\{\hat{\boldsymbol{A}} \otimes \hat{\boldsymbol{A}} + \boldsymbol{J} \otimes \boldsymbol{J}\right\} < 1, \quad (14)$$

Or

$$\rho\left\{\hat{\boldsymbol{A}}^{\mathrm{T}} \otimes \hat{\boldsymbol{A}}^{\mathrm{T}} + \boldsymbol{J}^{\mathrm{T}} \otimes \boldsymbol{J}^{\mathrm{T}}\right\} < 1. \quad (15)$$

(2) There exists a positive definite matrix $\boldsymbol{P} > \boldsymbol{0}$, such that

$$\hat{\boldsymbol{A}}\boldsymbol{P}\hat{\boldsymbol{A}}^{\mathrm{T}} + \boldsymbol{J}\boldsymbol{P}\boldsymbol{J}^{\mathrm{T}} - \boldsymbol{P} < \boldsymbol{0}. \quad (16)$$

(3) There exists a positive definite matrix $\boldsymbol{P} > \boldsymbol{0}$, such that

$$\hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}\hat{\boldsymbol{A}} + \boldsymbol{J}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{J} - \boldsymbol{P} < \boldsymbol{0}. \quad (17)$$

(4) The system (6) of statistical significance is exponentially mean-square stable.

Where $\hat{\boldsymbol{A}}, \boldsymbol{J}$ are the same as Definition 1.

Corollary 1. Given the parameters λ and $\boldsymbol{K}$, if the system (6) is mean-square stable and

$$\hat{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{P}_1\hat{\boldsymbol{A}} + \boldsymbol{J}^{\mathrm{T}}\boldsymbol{P}_1\boldsymbol{J} - \boldsymbol{P}_1 < \boldsymbol{0}, \quad (18)$$

Holds, then $\boldsymbol{P}_1 \ge \boldsymbol{0}$.

Corollary 2. Given the parameters λ and $\boldsymbol{K}$, if there exists a positive definite matrix $\boldsymbol{P}_2 > \boldsymbol{0}$, such that the following inequality

$$\hat{\boldsymbol{A}}\boldsymbol{P}_2\hat{\boldsymbol{A}}^{\mathrm{T}} + \boldsymbol{J}\boldsymbol{P}_2\boldsymbol{J}^{\mathrm{T}} - \boldsymbol{P}_2 + \boldsymbol{\Xi}_1 < \boldsymbol{0}, \quad (19)$$

Holds, then the system (6) is exponential mean-square stable and satisfies $\boldsymbol{P} \le \boldsymbol{P}_2$.

A common method for evaluating the performance of a suboptimal filter is to perform a large number of simulations, or to compare with the lower bound of the ideal optimal performance, which characterizes the limits of the estimated performance of the filter. The CRLB is a lower bound on the estimation error variance. In the sense of second-order error performance indicators, the optimal linear mean square error can be expressed by the theoretical CRLB.

Suppose that $\hat{\boldsymbol{X}}_{k|k}$ is an unbiased state estimation of $\boldsymbol{X}_k$, $\boldsymbol{P}_{k|k}$ is the estimation error variance of $\boldsymbol{X}_{k|k}$, then we obtain

$$\boldsymbol{P}_{k|k} = E\left\{\left(\boldsymbol{X}_{k|k} - \boldsymbol{X}_k\right)\left(\boldsymbol{X}_{k|k} - \boldsymbol{X}_k\right)^{\mathrm{T}}\right\} \geq \boldsymbol{J}_k^{-1} \quad (20)$$

Where $\boldsymbol{J}_k$ is the Fisher Information Matrix. CRLB is the inverse of $\boldsymbol{J}_k$, that is, $\boldsymbol{C}_k = \boldsymbol{J}_k^{-1}$. From the conclusion of the literature (Sujuan CHEN, et.al, 2012), we know that the CRLB of vehicle tracking systems can be expressed as,

$$\boldsymbol{J}_{k+1} = \boldsymbol{D}_k^{22} - \boldsymbol{D}_k^{21}\left(\boldsymbol{J}_k + \boldsymbol{D}_k^{11}\right)^{-1}\boldsymbol{D}_k^{12} \quad (21)$$

Where,

$$\begin{aligned}
\boldsymbol{D}_k^{11} &= E\left[-\frac{\partial^2 \ln p\left(\boldsymbol{X}_k \,|\, \boldsymbol{X}_{k-1}\right)}{\partial \boldsymbol{X}_{k-1}^2}\right], \\
\boldsymbol{D}_k^{12} &= E\left[-\frac{\partial^2 \ln p\left(\boldsymbol{X}_k \,|\, \boldsymbol{X}_{k-1}\right)}{\partial \boldsymbol{X}_k \partial \boldsymbol{X}_{k-1}}\right], \\
\boldsymbol{D}_k^{21} &= E\left[-\frac{\partial^2 \ln p\left(\boldsymbol{X}_k \,|\, \boldsymbol{X}_{k-1}\right)}{\partial \boldsymbol{X}_{k-1} \partial \boldsymbol{X}_k}\right], \\
\boldsymbol{D}_k^{22} &= E\left[-\frac{\partial^2 \ln p\left(\boldsymbol{X}_k \,|\, \boldsymbol{X}_{k-1}\right)}{\partial \boldsymbol{X}_k^2}\right] + \\
&\quad E\left[-\frac{\partial^2 \ln p\left(\boldsymbol{Y}_k \,|\, \boldsymbol{X}_k\right)}{\partial \boldsymbol{X}_k^2}\right].
\end{aligned} \quad (22)$$

The system noise and measurement noise of the vehicle target state studied in this paper obey the Gaussian distribution, so the conditional probability density functions of the target motion state and the measurement noise satisfy:

$$\begin{aligned}
p\left(\boldsymbol{X}_k \,|\, \boldsymbol{X}_{k-1}\right) &= \frac{1}{\sqrt{2\pi|\boldsymbol{Q}|}} e^{\left\{-\frac{1}{2}\left[\boldsymbol{X}_k - \boldsymbol{A}\boldsymbol{X}_{k-1}\right]^{\mathrm{T}} \boldsymbol{Q}^{-1}\left[\boldsymbol{X}_k - \boldsymbol{A}\boldsymbol{X}_{k-1}\right]\right\}} \\
p\left(\boldsymbol{Y}_k \,|\, \boldsymbol{X}_k\right) &= \frac{1}{\sqrt{2\pi|\boldsymbol{R}|}} e^{\left\{-\frac{1}{2}\left[\boldsymbol{Y}_k - \boldsymbol{H}\boldsymbol{X}_k\right]^{\mathrm{T}} \left(\boldsymbol{R}_k^{m}\right)^{-1}\left[\boldsymbol{Y}_k - \boldsymbol{H}\boldsymbol{X}_k\right]\right\}}
\end{aligned} \quad (23)$$

Substituting equation (23) into equation (22), we have

$$\begin{aligned}
\boldsymbol{D}_k^{11} &= \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{A}, \\
\boldsymbol{D}_k^{12} &= -\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}^{-1}, \\
\boldsymbol{D}_k^{21} &= -\boldsymbol{Q}^{-1}\boldsymbol{A}, \\
\boldsymbol{D}_k^{22} &= \boldsymbol{Q}^{-1} + d_{k+1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}
\end{aligned} \quad (24)$$

Substituting equation (24) into (21) yields

$$\begin{aligned}
\boldsymbol{J}_{k+1} &= \boldsymbol{Q}^{-1} + d_{k+1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H} - \\
&\quad \boldsymbol{Q}^{-1}\boldsymbol{A}\left(\boldsymbol{J}_k + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}^{-1}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{Q}^{-1}
\end{aligned} \quad (25)$$

From the matrix inverse theorem, equation (25) can be rewritten as

$$\boldsymbol{J}_{k+1} = \left(\boldsymbol{Q} + \boldsymbol{A}\boldsymbol{J}_k^{-1}\boldsymbol{A}^{\mathrm{T}}\right)^{-1} + d_{k+1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}. \quad (26)$$

Since the sequence $\{d_k\}$ is random, the equation (26) is stochastic and cannot be determinedoffline. In order to avoid the computation complexity of CRLB, we only analyze the statistical properties.

Define

$$\bar{\boldsymbol{J}}_k = E_\lambda\left\{\boldsymbol{J}_k\right\}. \quad (27)$$

Applying the mathematical expectation on both sides of equation (26), we have

$$\bar{\boldsymbol{J}}_{k+1} = \left(\boldsymbol{Q} + \boldsymbol{A}\boldsymbol{J}_k^{-1}\boldsymbol{A}^{\mathrm{T}}\right)^{-1} + \lambda\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}. \quad (28)$$

Therefore, the recursion CRLB with incomplete measurement can be written in the following way

$$C_{k+1} = \left(\bar{\boldsymbol{J}}_{k+1}\right)^{-1} = \left(\left(\boldsymbol{Q} + \boldsymbol{A}\boldsymbol{J}_k^{-1}\boldsymbol{A}^{\mathrm{T}}\right)^{-1} + \lambda\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}^{-1}\boldsymbol{H}\right)^{-1}. \quad (29)$$

From the equation (29), we know that the performance of CRLB is impact of parameters $\lambda, \boldsymbol{Q}, \boldsymbol{R}$.

4 THE SUBOPTIMAL FILTER ALGORITHM WITH INCOMPLETE MEASUREMENT

Consider the operator

$$\Psi(\boldsymbol{P}_2,\boldsymbol{K}) = (1-\lambda)\boldsymbol{A}\boldsymbol{P}_2\boldsymbol{A}^{\mathrm{T}} + \lambda(\boldsymbol{I}-\boldsymbol{K}\boldsymbol{C})\boldsymbol{A}\boldsymbol{P}_2\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{I}-\boldsymbol{K}\boldsymbol{C})^{\mathrm{T}} - \boldsymbol{P}_2 + \boldsymbol{\Xi}_1. \quad (30)$$

From Corollary 2, we have

$$\Psi(\boldsymbol{P}_2,\boldsymbol{K}) < \boldsymbol{0}, \quad (31)$$

And $\boldsymbol{P} \le \boldsymbol{P}_2$, the system is mean-square exponential stable. If the $\sigma_i^2\left(i=1,2,...,n\right)$ are selected and satisfy

$$\left[\boldsymbol{P}_2\right]_{ii} \le \sigma_i^2\left(i=1,2,...,n\right). \quad (32)$$

Then $\left[\boldsymbol{P}\right]_{ii} \le \left[\boldsymbol{P}_2\right]_{ii} \le \sigma_i^2(i=1,2,...,n)$.

Looking for a set of filter gains which make inequality (31) holds and choosing the appropriate filter gains, which can estimate target state parameters with variance constraints.

Lemma 2 (Sujuan CHEN, et.al, 2012). If $(\boldsymbol{A},\boldsymbol{Q}^{1/2})$ is controllable and $(\boldsymbol{A},\boldsymbol{C})$ is detectable, then, there exist $\boldsymbol{Z}$ and $\boldsymbol{0} < \boldsymbol{Y} \le \boldsymbol{I}$ which make the following LMI (33) holds.

$$\Gamma_\lambda(\boldsymbol{Y},\boldsymbol{Z}) = \begin{bmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{21} & \Gamma_{22}\end{bmatrix} < \boldsymbol{0}. \quad (33)$$

Where

$\Gamma_{11} = -Y,$

$\Gamma_{12} = \Big[\sqrt{1-\lambda}\boldsymbol{YA} \quad \sqrt{\lambda}(\boldsymbol{YA}-\boldsymbol{ZCA}), \quad \sqrt{1-\lambda}\boldsymbol{Y} \quad \sqrt{\lambda}(\boldsymbol{Y}-\boldsymbol{ZC}) \quad \sqrt{\lambda}\boldsymbol{Z}\Big],$

$\Gamma_{21} = \Gamma_{12}^{\mathrm{T}},$

$\Gamma_{22} = diag\left\{-Y,-Y,-Q^{-1},-Q^{-1},-R^{-1}\right\};$

Remark 1. Given the error variance constraint $\boldsymbol{P}_{upper}$, and the following LMI (34) holds.

$$\boldsymbol{P}_2 < \boldsymbol{P}_{upper} \quad (34)$$

Where $\left[\boldsymbol{P}_{upper}\right]_{ii} = \sigma_i^2(i=1,2,...,n)$, $\left[\boldsymbol{P}_{upper}\right]_{ij} = \left[\boldsymbol{P}_2\right]_{ij}, (i \ne j)$. From the Shur complement decomposition, LMI (33) is equivalent to LMI (35),

$$\begin{bmatrix}-\boldsymbol{P}_{upper} & \boldsymbol{I}\\ \boldsymbol{I} & -\boldsymbol{Y}\end{bmatrix} < \boldsymbol{0} \quad (35)$$

We can get a set of parameters $(\boldsymbol{Y},\boldsymbol{Z})$ by solving a couple LMIs (33) and (35), Since $\boldsymbol{Y} = \boldsymbol{P}_2^{-1}, \boldsymbol{Z} = \boldsymbol{YK}$, then we can get a set of parameters (K,P). From Corollary 2, the required filter gain can satisfy the condition LMI (34)

The following is the filter design algorithm steps with variance constraints:

(1) Input detection probability λ, radar sampling interval T, acceleration intensity q, measurement noise variance $\boldsymbol{R}$, and initial state variance $\boldsymbol{P}_0$.

(2) $\boldsymbol{J}_0 = \boldsymbol{P}_0$, calculate the lower bound of CRLB $\boldsymbol{C}_k$.

(3) According to step 2, obtain the solution $\boldsymbol{C}_{crlb}$ of the steady-state CRLB and take the variance constraint $\boldsymbol{P}_{upper}$ so that it satisfies $\boldsymbol{C}_{crlb} < \boldsymbol{P}_{upper}$.

(4) Solving the linear matrix inequalities (33) and (35), a set of feasible solutions $(\boldsymbol{Y},\boldsymbol{Z})$ can be obtained. Then, the suboptimal gain parameters $\boldsymbol{K} = \boldsymbol{P}_2\boldsymbol{Z}$ can be obtained.

(5) Output the filter gain $\boldsymbol{K}$.

5 VEHICLE TARGET PARAMETER ESTIMATION EXAMPLE

In this section, we present an illustrative example in vehicle target parameter estimation to demonstrate the effectiveness of the proposed design algorithm.

Consider the system (1) and (2), the state consists of relative position, relative velocity along one axis. We can get the relative distance between front and rear cars from vehicle radar. Suppose the detection probability of radar is λ.

$$\boldsymbol{A} = \begin{bmatrix}1 & T\\ 0 & 1\end{bmatrix}, \quad \boldsymbol{Q} = q\begin{bmatrix}T^3/3 & T^2/2\\ T^2/2 & T\end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix}1 & 0\end{bmatrix},$$

The state is defined by: $\boldsymbol{X}_k = \begin{bmatrix}r_k & \dot{r}_k\end{bmatrix}^{\mathrm{T}}$, with initial value $\boldsymbol{X}_0 = \begin{bmatrix}100m & 10\,m/s\end{bmatrix}^{\mathrm{T}}$

The sampling interval $T = 0.05$, the system parameters are:

$\lambda = 0.9$; $T = 0.05s$; $q = 0.1$; $\boldsymbol{R} = 0.05^2(m^2)$

Using the filter algorithm, we can obtain

$$\boldsymbol{P}_{crlb} = \begin{bmatrix}1.0239 & 0.0270\\ 0.0270 & 1.0304\end{bmatrix}, \quad \boldsymbol{K}_{crlb} = \begin{bmatrix}1.0043\\ 0.0763\end{bmatrix} \quad (36)$$

Choosing the following matrix P_{upper} as the estimation error variances upper bounds, we employ Matlab LMI-Toolbox to find the filter parameters

$$P_{upper} = \begin{bmatrix} 1.0508 & 0.0542 \\ 0.0542 & 1.0578 \end{bmatrix} \quad (37)$$

From the step 4 of the filter algorithm, we can obtain the filter gain:

$$K = YZ = \begin{bmatrix} 0.5164 \\ 0.0521 \end{bmatrix} \quad (38)$$

Table 1 shows the filter gains obtained by the proposed method with different detection probabilities.

Table 1. The filter gain with different detection probability λ.

detection probability λ	filter gian K
0.6	$\begin{bmatrix} 0.9993 \\ 0.0894 \end{bmatrix}$
0.7	$\begin{bmatrix} 0.9987 \\ 0.0838 \end{bmatrix}$
0.8	$\begin{bmatrix} 0.9979 \\ 0.0795 \end{bmatrix}$
0.9	$\begin{bmatrix} 1.0043 \\ 0.0763 \end{bmatrix}$
1	$\begin{bmatrix} 1.0015 \\ 0.0736 \end{bmatrix}$

Through 500 Monte Carlo simulations, Fig 1 shows the comparison of the mean-square filtering error of the Kalman filtering and the proposed suboptimal filtering algorithm. It can be seen from Fig 1 that the proposed filtering algorithm can effectively estimate the state of the trajectory.

It can be seen from Fig. 1 that the proposed suboptimal filtering algorithm can meet the filter design requirements Q1 and Q2, and the filtering accuracy approaches CRLB. Fig.2 shows the proposed suboptimal filter error variance with different detection probabilities. It can be seen from Fig. 2 that the error mean square error becomes smaller as the detection probability increases.

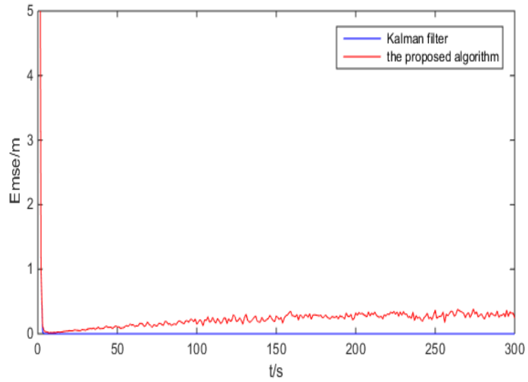

Figure 1. Comparison of Kalman filter error and proposed suboptimal filter error.

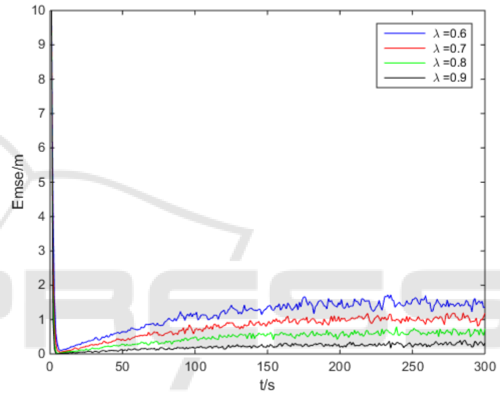

Figure 2. The proposed suboptimal filter error variance with different detection probabilities.

6 CONCLUSION

This paper studies the suboptimal estimation problem with incomplete measurement. under the variance constraint of linear discrete stochastic systems. The suboptimal filter gain can be designed according to the actual error variance index requirement of the vehicle target tracking system, so that the estimated error variance of the components of each state of the tracking system with incomplete measurement is not greater than the given variance constraint. Besides that, the influence of radar detection probability on the proposed filtering algorithm is also analyzed. The simulation results show that the error mean square error of the proposed suboptimal filtering decreases with the increase of the probability of detection.The algorithm proposed in this paper has certain engineering application value.

ACKKNOWLEDGEMENTS

This work was sponsored by Innovation Fun of Nanjing Institute of Technology (grant No. YKJ201434) and the Educational Commission of Jangsu Province of China (grant No.BE2017008-3), the author would also like to thank the reviewers for their corrections and helpful suggestions.

REFERENCES

SinopoliB, SchenatoL, FransceschettiLM, et a.l Kalman filtering with intermittent observations. IEEE Transactionson onAutomaticContro, 2004, 49(9): 1453-1464.

Boers Y, Driessen H. Results on the modified Riccati equation: target tracking applications. IEEE Transactions on Aerospace and Electronic Systems, 2006, 42(1): 379-384.

Y. Boers, H. Driessen. Modified Riccati equation and its application to target tracking. IEE Proc.-Radar Sonar Navig, 2006, 153(1):7-12.

Fayad F, Cherfaoui V. Tracking objects using a laser scanner in driving situation based on modeling target shape. 2007 Intelligent Vehicles Symposium. Istanbul, Turkey. 2007:44-49.

Hongli Dong, Zidong Wang, Daniel W. C. Ho and Huijun Gao. Variance-Constrained Filtering for a Class of H∞ Nonlinear Time-Varying Systems With Multiple Missing Measurements: The Finite-Horizon Case. IEEE Transactions on Signal Processing, 2010, 58(5):2534-2543.

Roland Hostettler and Petar M.D. Vehicle tracking based on fusion magnetometer and accelerometer sensor measurements with particle filtering. IEEE Transactions on Vehicular Technology, 2015, 6(11):4917-4928.

X. Zhong, A. Premkumar, and A. S. Madhukumar, Particle filtering and posterior Cramér-Rao bound for 2-D direction of arrival tracking using an acoustic vector sensor . IEEE Sensors. Journal, 2012, 12(2): 363–377.

Sujuan Chen, Guoqing Qi, Yinya Li, Andong Sheng. Suboptimal filtering with missing measurements and its application to target tracking, 2012 IET International Conference on Information Science and Control Engineering, 2012, 12: 821-825.

Sujuan CHEN, Guoqing QI and Andong SHENG. Allowable sampling frequency under error variance constraint of incomplete measurement. Control theory and application, 2012, 29(5): 629-634.

Z.Wang, F.Yang, D.W.C.Ho, X.Liu, Robust H∞ filtering for stochastic time-delay systems with missing measurements, IEEE Trans. Signal Process.54(7) (2006)2567–2578.