Integral Paraphrase of Physical Parameters of Non-uniformly Induced Medium in Optical Current Transformer

Dong Yin[1, a], Zhizhong Guo[1, b], Guoqing Zhang[1, c], Wenbin Yu[1, d], Guizhong Wang[2, e], Caiyun Mo[1, f]
[1]*School of Electrical Engineering and Automation Harbin Institute of Technology Harbin, China*
[2]*Research Institute of Industrial Technology Harbin Institute of Technology Zhangjiakou, China*

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Abstract: The three global physical quantities is presented to reveal optical sensing properties of a non-uniformly induced medium. The study found that, the sine function of average induction angle is the weighted integral of sine function of induction angle of optical path; the exponential function of average uneven angle is the weighted integral of exponential function of uneven angle of optical path; the global phase shift difference is the weighted integral of induced birefringence of optical path. The values verify that physical quantities have a specific but non-approximate integral relation with the corresponding physical quantity of optical path.

1 INTRODUCTION

The optical medium generates the optical effect under the action of the physical field. The physical field distributed along the optical path is sometimes uniform, but sometimes not. Uniform case is an exception of uneven cases, and the non-uniform physical field is more universal. The uneven physical field results in that the eigen coordinate system of optical medium changes along the optical path, unless the induction angle is constant. This optical medium is a non-uniformly induced medium (Xiao Zhihong, et al, 2017), and its Jones Matrix is obviously different from that of the uniform medium.

Jones Matrix describes the input and output relations of optical medium on the whole and its basic physical parameters belong to the whole medium (S.–Y.Lu and R. A. Chipman, 1994; Kyung S. Lee, 1999; S.–Y.Lu and R. A. Chipman, 1994; Sergey N. Savenkov et al, 2007; No ́e Ortega-Quijano et al, 2015; Alessandra Orlandini et al, 2001; H. Kogelnik, L. E. Nelson, J. P. Gordon, and R. M. Jopson, 2000). As for the non-uniformly induced medium, the induction angle, uneven angle and induced birefringence of the optical path cross section are distributed unevenly. Intrinsically, the global physical quantity depends on the physical quantity of optical path. The relations between average induction angle and induction angle of optical path, between average uneven angle and uneven angle of optical path and between global phase shift difference and induced birefringence of optical path are in close contact with each other.

Reference (Xiao Zhihong, et al, 2017) established the three-part analytical expression with the property of determinant of unitary matrix, laying a model basis for analyzing the optical effect of uneven physical field. In this paper, based on the research of reference (Xiao Zhihong, et al, 2017), the relational expression between global physical quantity and physical quantity of optical path is deduced with the Jones Matrix recurrence formula. The result indicates that the global physical quantity is the weighted integral of the corresponding physical quantity of optical path.

2 INDUCTION ANGLE PROPOSITION

A. The first recurrence relation of Jones Matrix

The non-uniformly induced medium is equally divided into n+1 infinitesimal element. The infinitesimal element Jones Matrix is

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$$\boldsymbol{J}_k = \begin{bmatrix} A_k & \not{k} jB_k \\ \hat{\not{k}} j\hat{B}_k & \hat{A}_k \end{bmatrix}, (\mathrm{k} = 0,1,2,\cdots,\mathrm{n}) \quad (1)$$

The infinitesimal element is sufficiently small, and is regarded as uniform. So,

$$A_k \in \mathbb{C}, B_k \in \mathbb{R}$$

And

$$\begin{aligned} A_k &= \cos\frac{d\varphi_k}{2} + jcos\alpha_k sin\frac{d\varphi_k}{2} \\ B_k &= sin\alpha_k sin\frac{d\varphi_k}{2} \end{aligned} \quad (2)$$

Where, $\mathbb{C}$ indicate the complex number field and $\mathbb{R}$ indicate the real number field; $\alpha_k = \alpha(z_k)$ is the induction angle of the optical path and $d\varphi_k = \varphi(z_k)$ is the infinitesimal element phase shift difference.

The cascade Jones Matrix constituted by the first $\mathrm{k}+1$ infinitesimal elements

$$\boldsymbol{J}_0^k = \prod_{i=0}^k \boldsymbol{J}_k = \begin{bmatrix} A_0^k & \not{k} jB_0^k \\ \hat{\not{k}} j\hat{B}_0^k & \hat{A}_0^k \end{bmatrix} \quad (3)$$

Where, $A_0^0 = A_0, B_0^0 = B_0$.

According to reference (Xiao Zhihong, et al, 2017), the three-element expression of the diagonal and non-diagonal elements of Jones Matrix J_0^k is indicated below

$$\begin{aligned} A_0^k &= \cos\frac{\varphi_0^k}{2} + jcos\alpha_0^k sin\frac{\varphi_0^k}{2} \\ B_0^k &= exp(j\sigma_0^k) sin\alpha_0^k sin\frac{\varphi_0^k}{2} \end{aligned} \quad (4)$$

Where α_0^k, σ_0^k and φ_0^k are respectively the average induction angle, average uneven angle and phase shift difference of Jones Matrix J_0^k.

It can be easily proved that the cascade Jones Matrix J_0^k accords with the recurrence relation

$$\begin{aligned} \mathrm{A}_0^k &= A_0^{k-1} A_k - B_0^{k-1} \hat{B}_k \\ \mathrm{B}_0^k &= A_0^{k-1} B_k + B_0^{k-1} \hat{A}_k \end{aligned} \quad (5)$$

Using three-element expression of J_0^k, (5) is rewritten into

$$\begin{aligned} A_0^k &= C_0^k + jc_0^k S_0^k \\ &= C_0^{k-1} C_k + j(c_0^{k-1} S_0^{k-1} C_k + c_k C_0^{k-1} S_k) \\ &\quad -(c_0^{k-1} c_k + exp(j\sigma_0^{k-1}) s_0^{k-1} s_k) S_0^{k-1} S_k \end{aligned} \quad (6)$$

$$\begin{aligned} B_0^k &= exp(j\sigma_0^k) s_0^k S_0^k \\ &= s_k (C_0^{k-1} + jc_0^{k-1} S_0^{k-1}) S_k \\ &\quad + s_0^{k-1} exp(j\sigma_0^{k-1})(C_k - jc_k S_k) S_0^{k-1} \end{aligned} \quad (7)$$

The following simple symbols are used for the abovementioned two formulas

$$\begin{aligned} & C_0^{k-1} = \cos\frac{\varphi_0^{k-1}}{2}, S_0^{k-1} = sin\frac{\varphi_0^{k-1}}{2} \\ & C_k = cos\frac{d\varphi_k}{2}, S_k = sin\frac{d\varphi_k}{2} \\ & c_0^{k-1} = cos\alpha_0^{k-1}, s_0^{k-1} = sin\alpha_0^{k-1} \\ & c_k = cos\alpha_k, s_k = sin\alpha_k \end{aligned} \quad (8)$$

B. Induction angle proposition

Induction angle proposition: for the non-uniformly induced medium, the sine function of average induction angle α is the weighted integral of the sine function of induction angle $\alpha(z)$ of the optical path.

$$sin\alpha = \int_0^L p(z) sin\alpha(z) dz \quad (9)$$

Where, $p(z)$ is the normalized integral coefficient, namely,

$$\int_0^L p(z) dz = 1 \quad (10)$$

Where, L is the length of optical path of the medium.

Prove: according to (7), the real part of non-diagonal element B_0^k has the following equivalent relationship

$$cos\sigma_0^k s_0^k S_0^k = C_0^{k-1} S_k s_k + (cos\sigma_0^{k-1} C_k + sin\sigma_0^{k-1} c_k S_k) S_0^{k-1} s_0^{k-1}$$

Note

$$P_k = C_0^{k-1} S_k (cos\sigma_0^k S_0^k)^{-1}$$

$$P_0^k = (cos\sigma_0^{k-1} C_k + sin\sigma_0^{k-1} c_k S_k) S_0^{k-1} (cos\sigma_0^k S_0^k)^{-1}$$

So, the real part of non-diagonal element B_0^k is rewritten into

$$\begin{aligned} sin\alpha_0^k = P_k sin\alpha_k &+ P_0^k P_{k-1} sin\alpha_{k-1} \\ &+ P_0^k P_0^{k-1} P_{k-2} sin\alpha_{k-2} + \cdots \\ &\cdot + P_0^k P_0^{k-1} \cdots P_0^1 P_0 sin\alpha_0 \end{aligned} \quad (11)$$

Where, $sin\alpha_0^{-1} = 0$, $P_0^0 = 1$.

The abovementioned formula means that the following formula holds.

$$sin\alpha_0^k = \sum_{i=0}^k P_i \prod_{j=i+1}^k P_0^j \; sin\alpha_i$$

Note

$$S_i = sin\frac{d\varphi_i}{2} = \frac{\tau_i}{2}d\varphi_i = \frac{\tau_i}{2}\frac{d\varphi_i}{dz}dz \qquad (12)$$

Where, τ_i is the coefficient which makes the equation hold and then put it into P_i is given as

$$P_i = C_0^{i-1}\left(cos\sigma_0^i S_0^i\right)^{-1} sin\frac{d\varphi_i}{2} = \frac{C_0^{i-1}\left(cos\sigma_0^i S_0^i\right)^{-1}\tau_i}{2}\frac{d\varphi_i}{dz}dz$$

So

$$sin\alpha_0^k = \sum_{i=0}^{k}\left(\frac{C_0^{i-1}\left(cos\sigma_0^i S_0^i\right)^{-1}\tau_i}{2}\frac{d\varphi_i}{dz}\prod_{j=i}^{k}P_0^{k-j}\right) sin\alpha_i dz$$

Note again

$$p_i = \frac{C_0^{i-1}\left(cos\sigma_0^i S_0^i\right)^{-1}\tau_i}{2}\frac{d\varphi_i}{dz}\prod_{j=i}^{k}P_0^{k-j}$$

There,

$$sin\alpha_0^k = \sum_{i=0}^{k} p_i sin\alpha_i\, dz$$

Due to

$$sin\alpha = \lim_{n\to\infty} sin\alpha_0^n = \lim_{n\to\infty}\sum_{i=0}^{n} p_i sin\alpha_i dz$$

So

$$sin\alpha = \int_0^L p(z) sin\alpha(z) dz$$

In case the medium is uniform, $sin\alpha(z) \equiv sin\alpha$, therefore

$$\int_0^L p(z)\boldsymbol{dz} = 1$$

The proposition holds.

3 PHASE SHIFT DIFFERENCE PROPOSITION

Phase shift difference proposition: For the non-uniformly induced medium, the global phase shift difference φ is the weighted integral of the induced birefringence $\Delta n(z)$ of optical path, namely

$$\varphi = \frac{2\pi}{\lambda}\int_0^L s(z)\Delta n(z)dz \qquad (13)$$

Where, λ is wave length; $\Delta n(z)$ is induced birefringence. The relation is met

$$\Delta n(z) = \frac{1}{n_o}\sqrt{\delta_\Delta^2(z) + \delta_w^2(z)} \qquad (14)$$

Where, n_o is the intrinsic birefringence index, $\delta_w(z)$ is the non-diagonal component of cross section induction tensor, $\delta_\Delta(z)$ is the average deviation of diagonal components of cross section induction tensor $\delta_a(z)$ and $\delta_b(z)$, Namely

$$\delta_\Delta(z) = \frac{\delta_a(z) - \delta_b(z)}{2} \qquad (15)$$

Prove: the real relation of (6) is

$$\cos\frac{\varphi_0^k}{2} = \cos\frac{\varphi_0^k + d\varphi_k}{2} - R_k sin\frac{d\varphi_k}{2} \qquad (16)$$

Wherein

$$R_k = (c_0^{k-1}c_k - cos\sigma_0^{k-1}s_0^{k-1}s_k - 1)S_0^{k-1}$$

The second item of (16) is a small quantity, meaning although

$$\cos\frac{\varphi_0^k}{2} \neq \cos\frac{\varphi_0^{k-1} + d\varphi_k}{2}$$

The coefficient S_k must exist. As a result, the following formula holds

$$\cos\frac{\varphi_0^k}{2} = \cos\frac{\varphi_0^{k-1} + S_k d\varphi_k}{2}$$

Therefore

$$\varphi_0^k = \varphi_0^{k-1} + S_k d\varphi_k$$

Where, $S_0 = 1$, $\varphi_0^{-1} = 0$. Moreover, when the medium is uniform, $S_k \equiv 1$.

The abovementioned formula predicts

$$\varphi_0^k = \sum_{i=0}^{k} S_i d\varphi_i$$

So

$$\varphi = \lim_{n\to\infty}\varphi_0^n = \lim_{n\to\infty}\sum_{i=0}^{n} S_i d\varphi_i$$

That is

$$\varphi = \int_0^L s(z)d\varphi(z)$$

Because of

$$d\varphi(z) = \frac{2\pi}{\lambda}\Delta n(z)dz$$

So

$$\varphi = \frac{2\pi}{\lambda}\int_0^L s(z)\Delta n(z)dz$$

The proposition holds.

Deduction 1: For the non-uniformly induced medium, the average induced birefringence and the induced birefringence of optical path satisfy the weighted integral relation

$$\Delta n = \frac{1}{\lambda}\int_0^L s(z)\Delta n(z)dz \tag{17}$$

Because of
$\varphi = \frac{2\pi L}{\lambda}\Delta n$,
It's substituted into (13) , deduction 1 is establishment,
Deduction 2: in case of uniform medium and approximately uniform medium,

$$\varphi = \frac{2\pi}{\lambda}\int_0^L \Delta n(z)dz \tag{18}$$

When the medium is uniform, $s(z) \equiv 1$. So, deduction 2 holds.

4 UNEVEN ANGLE PROPOSITION

A. The second recurrence relation of Jones Matrix
The optical medium is equally divided into 2n+1 infinitesimal elements. The adjacent infinitesimal elements of the Jones Matrix are multiplied with each other in succession to obtain the Jones Matrixes J_k of n+1 combined infinitesimal elements. Without loss of generality, the three-part expression of the Jones Matrix J_k element is expressed as follows, assuming the induction of combined infinitesimal element is uneven.

$$\begin{aligned} A_k \in \mathbb{C} &= \cos\frac{d\varphi_k}{2} + j cos\alpha_k sin\frac{d\varphi_k}{2} \\ B_k \in \mathbb{C} &= sin\alpha_k sin\frac{d\varphi_k}{2} exp(j\sigma_k) \end{aligned} \tag{19}$$

Where σ_k is the uneven angle of the combined infinitesimal element k, and its value is

$$\sigma_k = tan^{-1}\frac{sind\alpha_k}{sin\alpha_{k1}ctg\frac{d\varphi_{k2}}{2} + sin\alpha_{k2}ctg\frac{d\varphi_{k1}}{2}} \tag{20}$$

Where the subscripts $k1$ and $k2$ respectively indicate the first and second infinitesimal elements of the combined infinitesimal element k, and

$$d\alpha_k = \alpha_{k2} - \alpha_{k1} \tag{21}$$

Obviously, the uneven angle σ_k of combined infinitesimal element is the function of induction angle differential $d\alpha_k$.

The first $k+1$ combined infinitesimal element Jones Matrixes are multiplied in series to obtain the cascade Jones Matrix J_0^k. According to the recurrence relation of (5), the three-element expression form of Jones Matrix J_0^k can be obtained. The non-diagonal element is

$$\begin{aligned} B_0^k &= exp(j\sigma_0^k)s_0^k S_0^k \\ &= exp(j\sigma_k)s_k(C_0^{k-1} + jc_0^{k-1}S_0^{k-1})S_k \\ &\quad + exp(j\sigma_0^{k-1})\, s_0^{k-1}(C_k - jc_kS_k)S_0^{k-1} \end{aligned} \tag{22}$$

Where, the simple symbol of (8) is adopted.
B. Uneven angle proposition
Uneven angle proposition: For the non-uniformly induced medium, the exponential function of the average uneven angle σ is the weighted integral of the exponential function of uneven angle $\sigma(z)$ of optical path

$$exp(j\sigma) = \int_0^L q(z)exp\big(j\sigma(z)\big)dz \tag{23}$$

Where, $q(z)$ is the integral coefficient irrelevant to the uneven angle of optical path. The relation can be met.

$$\int_0^L q(z)dz = 1 \tag{24}$$

Prove: according to (22) , the non-diagonal element meets the relation

$$exp(j\sigma_0^k) = Q_k exp(j\sigma_k) + Q_0^k exp(j\sigma_0^{k-1})$$

Wherein

$$Q_k = s_k(C_0^{k-1} + jc_0^{k-1}S_0^{k-1})S_k(s_0^kS_0^k)^{-1}$$
$$Q_0^k = s_0^{k-1}(C_k - jc_kS_k)S_0^{k-1}(s_0^kS_0^k)^{-1}$$

Where $s_0^{-1} = 0$, $Q_0^0 = 1$.
This is similar to the verification of induction angle proposition, so

$$exp(j\sigma_0^k) = \sum_{i=0}^{k} Q_i \prod_{j=i+1}^{k} Q_0^j\, exp(j\sigma_i)$$

According to (12),

$$Q_i = \frac{1}{2}s_i\tau_i\big(C_0^{i-1} + jc_0^{i-1}S_0^{i-1}\big)\big(s_0^iS_0^i\big)^{-1}\frac{d\varphi_i}{dz}dz$$

Note

$$\begin{aligned} q_i &= \sum_{i=0}^{k}\frac{1}{2}s_i\tau_i\big(C_0^{i-1} \\ &\quad + jc_0^{i-1}S_0^{i-1}\big)\big(s_0^iS_0^i\big)^{-1}\frac{d\varphi_i}{dz}\prod_{j=i}^{k}Q_0^{k-j} \end{aligned}$$

So

$$exp(j\sigma_0^k) = \sum_{i=0}^{k} q_i\, exp(j\sigma_i)dz$$

Because

$exp(j\sigma) = \lim_{n\to\infty} exp(j\sigma_0^k) = \lim_{n\to\infty} \sum_{i=0}^{n} q_i\, exp(j\sigma_i)dz,$

$$exp(j\sigma) = \int_0^L q(z)exp\big(j\sigma(z)\big)dz$$

When the medium is uniform,

$$exp\big(j\sigma(z)\big) \equiv exp(j\sigma) \equiv 1$$

So

$$\int_0^L q(z)dz = 1$$

The proposition holds.

Deduction: The uneven angle proposition can be transformed into

$$\sigma = tan^{-1}\frac{\int_0^L q(z)sin\sigma(z)dz}{\int_0^L q(z)cos\sigma(z)dz} \tag{25}$$

Obviously, (21) can be written as

$$\begin{cases} cos\sigma = \int_0^L q(z)cos\sigma(z)dz \\ sin\sigma = \int_0^L q(z)sin\sigma(z)dz \end{cases}$$

Therefore, the deduction holds.

5 VALUE VERIFICATION

The numerical method is adopted to verify the abovementioned three propositions.

A. Verification method

If the changing rule of cross section induction tensor along optical path is known

$$\boldsymbol{\delta}(z) = \begin{bmatrix} \delta_a(z) & \hbar\delta_w(z) \\ \hat{\hbar}\delta_w(z) & \delta_b(z) \end{bmatrix}, (\hbar \in [1, j]) \tag{26}$$

Where, when $\hbar = 1$, $\boldsymbol{\delta}(z)$ is real symmetric, e.g., electric light, sound light, elastic light and other effects, when $\hbar = j$, $\boldsymbol{\delta}(z)$ is complex symmetric, e.g., magneto-optic effect.

According to the formula

$$\alpha(z) = \tan^{-1}\frac{\delta_w(z)}{\delta_a(z)-\delta_b(z)} \tag{27}$$

The induction angle of optical path is calculated.

According to (14), the induced birefringence $\Delta n(z)$ of optical path is calculated $\Delta n(z)$. Then, according to the formula

$$d\varphi_k = \frac{2\pi}{\lambda}\Delta n(z_k)dz \tag{28}$$

The phase shift difference $d\varphi_k$ of the infinitesimal element is calculated.

The n value will be large enough, and the medium optical path is equally divided into $n+1$ infinitesimal elements, namely

$$dz = \frac{L}{n+1}, (n \to \infty) \tag{29}$$

When the uneven angle proposition is verified, the number of infinitesimal elements needs to be doubled so that the infinitesimal element uneven angle can be calculated according to (20).

Starting from the first infinitesimal element, all infinitesimal elements are calculated. When the kth infinitesimal element is calculated, the Jones Matrix is

$$\boldsymbol{J}_0^k = \boldsymbol{J}_0^{k-1}\boldsymbol{J}_k \tag{30}$$

So, the average induction angle α_0^{k-1}, average uneven angle σ_0^{k-1} and phase shift difference φ_0^{k-1} of Jones MatrixJ_0^{k-1} are already known. According to the formulas

$$\alpha_0^k = tan^{-1}\frac{|B_0^k|}{Im(A_0^k)} \tag{31}$$

$$\sigma_0^k = tan^{-1}\frac{Im(B_0^k)}{Re(B_0^k)} \tag{32}$$

$$\varphi_0^k = 2tan^{-1}\frac{\sqrt{Im^2(A_0^k)+|B_0^k|^2}}{Re(A_0^k)} \tag{33}$$

The average induction angle α_0^k, average uneven angle σ_0^k and phase shift difference φ_0^k of Jones Matrix J_0^k are obtained. So, because of

$$sin\alpha_0^k = sin\alpha_0^{k-1} + p(z_k)sin\alpha_k dz$$

$$exp(j\sigma_0^k) = exp(j\sigma_0^{k-1}) + q(z_k)exp(j\sigma_k)dz$$

$$\varphi_0^k = \varphi_0^{k-1} + s(z_k)d\varphi_k$$

These are obtained

$$
\begin{aligned}
p(z_k) &= \frac{sin\alpha_0^k - sin\alpha_0^{k-1}}{sin\alpha_k dz} \\
q(z_k) &= \frac{exp(j\sigma_0^k) - exp(j\sigma_0^{k-1})}{exp(j\sigma_k)dz} \\
s(z_k) &= \frac{\varphi_0^k - \varphi_0^{k-1}}{d\varphi_k}
\end{aligned}
\quad (34)
$$

The abovementioned traverse calculation starts from the starting end of medium and lasts until the end. Therefore, three weight coefficient series of the whole optical path can be obtained

$$
\begin{aligned}
p(z) &\leftrightarrow [p(z_0), p(z_1), p(z_2), \cdots, p(z_n)] \\
q(z) &\leftrightarrow [q(z_0), q(z_1), q(z_2), \cdots, q(z_n)] \\
s(z) &\leftrightarrow [s(z_0), s(z_1), s(z_2), \cdots, s(z_n)]
\end{aligned}
\quad (35)
$$

The existence of the above-mentioned three weight coefficient sequences is a numerical verification for the induction angle proposition, the uneven angle proposition and the phase-shift difference proposition.

B. Verification case

The DOCT simulation model was created with ANSOFT software of infinite element analysis to conduct a simulated analysis on the integral paraphrase and physical parameters of non-uniformly induced material parameters. The initial condition is set as: magneto-optic glass length is 0.05m, the vertical distance from its structural center to conductor is 0.02m, and the linear birefringence of magneto-optic glass is 3 °/cm.

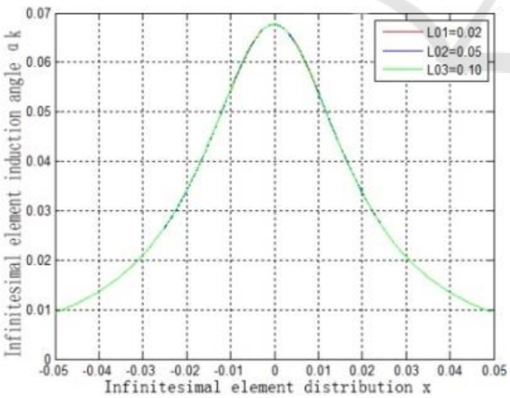


Fig 1. Infinitesimal Element Induction Angles with Different Lengths.

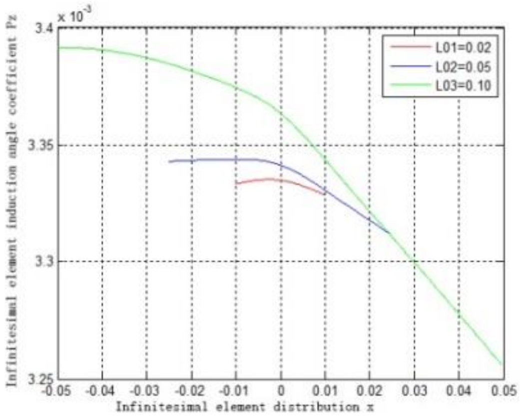


Fig 2. Weighting Coefficients of Average Induction Angles with Different Lengths.

As shown in Fig. 1, the infinitesimal element induction angles at a same point of magneto-optic glass with different lengths are the same, indicating the induction angle is the function of the space magnetic field, only depends on the space position, and takes on the symmetric feature of an even function around the conductor. When the infinitesimal element is vertical to the conductor center, the induction angle has a maximum value. As shown in Fig. 2, the weighting coefficient of infinitesimal element induction angle is a monotonic decreasing function. When magneto-optic glass $L = 0.05$, the infinitesimal element weight coefficient of starting end is 1% more than that of terminal end; when $L = 0.10$, the starting end is 4.5% larger than the terminal end, and it tends to occur that the difference between the weight coefficients of the starting and terminal ends gradually increases as L increases. When the horizontal ordinate of the selected conductor center is x =0, the absolute value of the derivative of the infinitesimal element weighting coefficient on the left of magneto-optic glass is smaller that of the right of the same, indicating that the induction effect of the infinitesimal induction angle on the integrity tends to decline as the space position deviates.

(2) Infinitesimal element uneven angle and weighting coefficient

The magneto-optic glass of L=0.02m, 0.05m and 0.10m are selected to calculate the infinitesimal element induction angles of different positions and their weighting coefficients. The simulation result is shown in Fig.3 to Fig. 6.

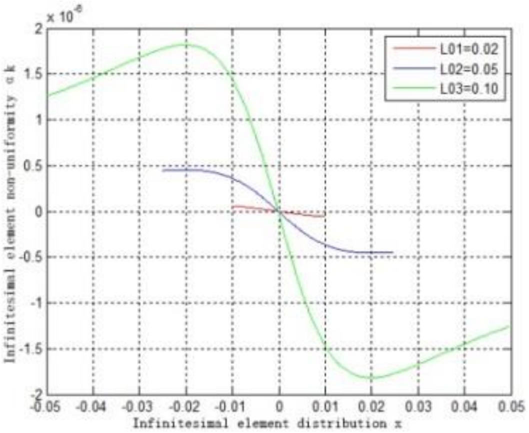
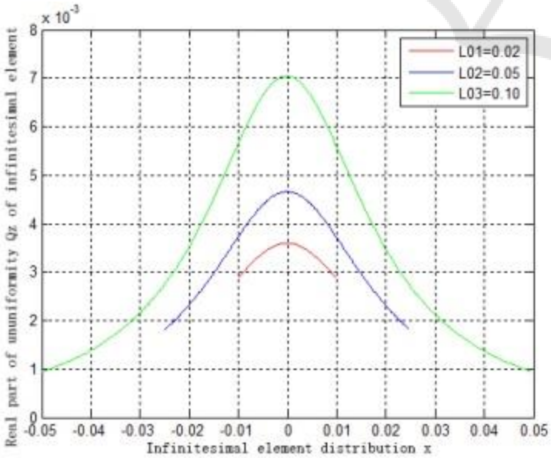
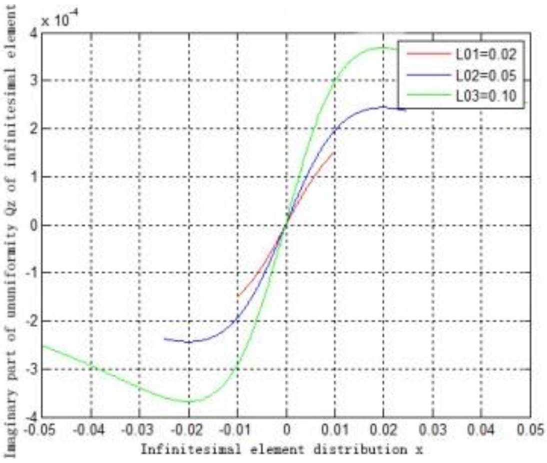


Fig 3. Unevenness of Infinitesimal Elements with Different Lengths.

As shown in Fig. 3, the infinitesimal element uneven angle increases as the deviation position increases. And the infinitesimal element uneven angle is 0, when the infinitesimal element is vertical to the conductor center. Different from infinitesimal element induction angle, infinitesimal element uneven angle is affected by the sensor element length except for depending on space position. . For the infinitesimal element of the selected position $x = 0.01\mathrm{m}$, the infinitesimal element uneven angle of sensor length $L = 0.10\mathrm{m}$ is 60% larger than $L = 0.05\mathrm{m}$, indicating that the sensor element length magnifies the non-uniformity of the infinitesimal element.



(a) Real Part of Weight Coefficient



(b) Imaginary Part of Weight Coefficient

Fig 4. Average Uneven Angle Weighting Coefficient with Different-Lengths.

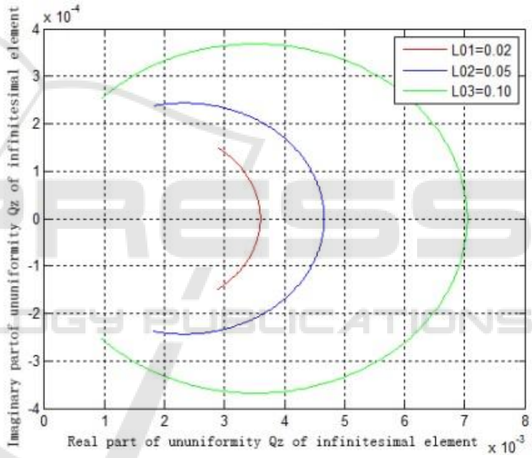


Fig 5. Average Uneven Angle Weighting Coefficient of Different Lengths (complex plane).

Judging from the deduction of (34), the weight coefficient of infinitesimal element uneven angle is complex. As shown in Fig. 4 and Fig. 5, the real and imaginary parts of average uneven angle weighting coefficient of different lengths takes on the symmetric features of an even function and an odd function. They increase the change degree of weight coefficient as the sensor element length increases.

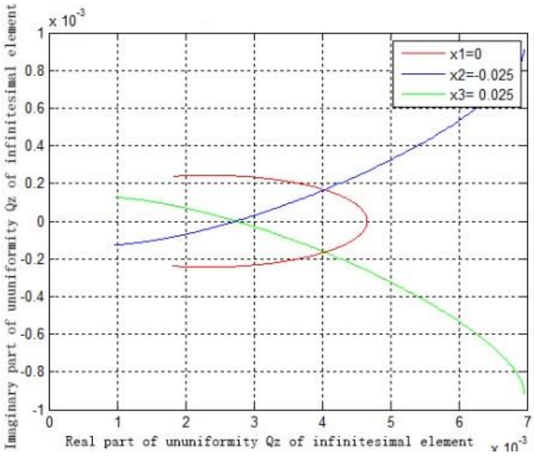


Fig 6. Average Uneven Angle Weighting Coefficient of Different Deviation Layouts (complex plane).

The selected sensor element lengths are the same, the simulation curve of all infinitesimal element uneven angle weighting coefficients in case of different deviation positions is shown in Fig. 6. With the sensor element center deviating towards the left or right, the non-uniformly induced degree increases, the real and imaginary parts of the non-uniform weighting coefficient no longer have the symmetric feature but instead the monotonic feature, and tend to increase progressively on the complex plane.

6 CONCLUTION

The whole physical quantity of non-uniformly induced medium depends on the physical quantity of optical path, and accords with the specific but non-approximate weighted integral relations. Specifically:

(1) The sine function of average induction angle α is the weighted integral of the sine function of induction angle $\alpha(z)$ of the optical path.

(2) The exponential function of average uneven angle σ is the weighted integral of the exponential function of uneven angle $\sigma(z)$ of optical path.

(3) The global phase shift difference φ is the weighted integral of induced birefringence$\Delta n(z)$ of optical path, i.e., the average induced birefringence Δn is the weighted integral of induced birefringence $\Delta n(z)$of optical path.

REFERENCES

Alessandra Orlandini et al, “A Simple and Useful Model for Jones Matrixto Evaluate Higher Order Polarization-ModeDispersion Effects,” IEEE photonics technology letters13 (11), 1176–1178 (2001).

H. Kogelnik, L. E. Nelson, J. P. Gordon, and R. M. Jopson, “Jones matrix for second-order polarization mode dispersion,”Opt. Lett. 25, 19–21 (2000).

Kyung S. Lee, “New compensation method for bulk optical sensors with multiple birefringences,”Appl. Opt. 28(11), 2001–2011 (1999).

Noé Ortega-Quijano et al, “Generalized Jones matrix method for homogeneous biaxial samples,” Opt. Soc. Am. A 23, 20428–20438 (2015).

S.–Y.Lu and R. A. Chipman, “Homogeneous and inhomogeneousJones matrices”. Opt. Soc. Am. An 11, 766–773 (1994).

S.–Y.Lu and R. A. Chipman, “Homogeneous and inhomogeneous Jones matrices”. Opt. Soc. Am. An 11, 766–773 (1994).

Sergey N. Savenkov et al, “Eigen analysis of dichroic, birefringent, and degenerate polarization elements a Jones-calculus study,” Appl. Opt. 46(20), 6700–6709 (2007).

Xiao Zhihong, Guo Zhizhong, Zhang Guoqing, Yu Wenbin, Yu Tongwei, Li Zhiliang. “Study on the Faraday optical rotation effect of inhomogeneous magnetic field” Proceedings of the CSEE, 2017, 37(08):2426-2436.