The Problem of Finding the Best Strategy for Progress Computation in Adaptive Web Surveys

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Abstract:

Web surveys are typical web information systems. As part of the interface of a survey, progress indicators inform the participants about their state of completion. Different studies suggest that the progress indicator in web surveys has an impact on the dropout and answer behaviour of the participants. Therefore, researchers should invest some time in finding the right indicator for their surveys. But the calculation of the progress is sometimes more difficult than expected, especially, in surveys with a lot of branches. Current related work explains the progress computation in such cases based on different prediction strategies. However, the performance of these strategies varies for different surveys. In this position paper, we explain how to compare those strategies. The chosen Root Mean Square Error measure allows to select the best strategy. But experiments with two large real-world surveys show that there is no single best strategy for all of them. It highly depends on the structure of the surveys and sometimes even the best known strategy produces bad predictions. Dedicated research should find solutions for these cases.

1 INTRODUCTION

Web surveys present items (questions) in a paper-like format with various input fields. The survey itself interacts with a server application and transfers the inserted answers of the participants to it. In other words, web surveys are web information systems.

The interface of a web survey is intuitive and straight-forward in the common case. Besides the input fields for answers, web surveys consist usually of buttons to precede and sometimes also to recede in the survey. Headers inform the participant about the context of the survey and usually Progress Indicators (PIs) inform the participants about the state of completion. This paper focuses on the PIs as part of the interface.

Typically, the PI displays the progress in percentage between 0 and 100%. Unlike PIs of usual tasks in general software, participants of web surveys have to focus on the task (the survey), can influence the PI, and do not necessarily have an interest on the result (Villar et al., 2013). Therefore, PIs in web surveys are different from those of other software (Villar et al., 2013), e. g., for machine learning (Luo, 2017) and database queries (Li et al., 2012).

Participants in web surveys prefer to have a PI (Myers, 1983; Villar et al., 2013) to be aware of their

progress. However, the computation of the progress can be difficult in case of surveys with adaptivity (branches). (Prinz et al., 2019) propose an equation to compute the progress in adaptive surveys. The equation is based on the number of remaining items (questions) at each point of time. This number of remaining items depends on a chosen prediction strategy. Such a strategy tries to predict the number of remaining items for each page since the participants may take different paths in the survey with different numbers of remaining items. For example, two known prediction strategies are: 1) take the minimum or 2) maximum number of remaining items (Kaczmirek, 2009). However, (Prinz et al., 2019) suspects that it depends on the structure of the survey which prediction strategy is the best. Furthermore, the comparison of the quality of the strategies seems to be not trivial.

One goal of this paper is to have a measure to select that prediction strategy, which guesses the *true* number of remaining items and, therefore, the *true* progress as well as possible. We support the idea of displaying the true progress since research in Human-Computer Interaction (HCI) reveals high probable side-effects of PIs on the answer and dropout behaviour of participants (Villar et al., 2013). Especially the progress speed (the rate in which the PI increases) seems to influence the decision whether a participant

finishes a long survey (Myers, 1985; Villar et al., 2013). PIs increasing faster at the start and slower at the end seem to encourage the participant to finish the survey. On the contrary, if a PI is slow at the beginning and gets faster at the end seems to discourage and causes higher dropout rates (Conrad et al., 2003; Matzat et al., 2009; Conrad et al., 2010). A meta-analysis of PI speeds by (Villar et al., 2013) supports these observations.

The long-term effects of different PI speeds are not researched. Even though more participants complete a survey by using a fast-to-slow increase of the PI, if the participants become aware of the varying PI speed, it might decrease their willingness to participate in future surveys by the survey provider. Instead, PIs which try to display the true progress are honest to the participants and should reduce side-effects of PIs.

In this position paper, we argue that the *Root Mean Square Error (RMSE)* is a useful measure to describe the quality of prediction strategies for progress computation. This measure allows to compare uses of strategies and to choose the "best" one. Researchers conducting surveys can use the RMSE to determine the best known strategy for their surveys and can give the participant a PI, which represents the true progress as well as possible. Furthermore, this paper shows that the yet known trivial prediction strategies lead to bad predictions in some cases. Further research should find solutions for these cases.

The paper has the following structure: At first, it explains in Section 2 how we can compute the progress in adaptive surveys and how we can apply the prediction strategies. Based on this computation, the paper presents four measures for the determination of the quality of a prediction strategy in Section 3 and shows their application in Section 4. Section 4 argues further which measure is most suitable and explains some disadvantages with current prediction strategies. Section 5 ends the paper with a brief discussion and conclusion.

2 PROGRESS COMPUTATION

Although a lot of studies consider the differences in PI speeds, the computation of an accurate progress in web surveys (especially in surveys with high adaptivity) is not in the research focus. The thesis of (Kaczmirek, 2009) and the work of (Prinz et al., 2019) based on it are the only available work to the best knowledge of the authors, which try to answer how to calculate the progress in web surveys with adaptivity. The lack of knowledge about the "path" a participant takes in a survey, is the main problem in progress computations. Prinz et al. propose an algorithm, which supports

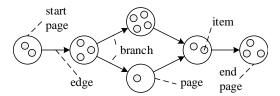


Figure 1: A simple questionnaire graph.

different prediction strategies. By applying one of several strategies it is possible to approximate the *true* progress as well as possible.

The approach with different prediction strategies is based on an abstract survey model called the *questionnaire graph* (in short Q-graph). The Q-graph describes the structure of questionnaires. It is an acyclic, connected digraph $Q = (\mathbb{P}, \mathbb{E})$ with a set of pages $\mathbb{P} = \mathbb{P}(Q)$ and a set of $edges \ \mathbb{E} = \mathbb{E}(Q)$, which connect the pages. The Q-graph Q has exactly one page without any incoming edge (the $start\ page$) and exact one page without any outgoing edge (the $end\ page$). A page $P \in \mathbb{P}(Q)$ is a finite set $\{i_1, i_2, \ldots\}$ of $items\ i_1, i_2, \ldots$ For this reason, |P| is the number of items on page P. For reasons of simplicity, each item is assumed to be unique. Figure 1 illustrates a simple Q-graph.

The edges build *paths* throughout the Q-graph. A path is a sequence $W = (P_0, \dots, P_m)$, $m \ge 0$, of pages, $P_0, \dots, P_m \in \mathbb{P}(Q)$. For this sequence, there is an edge for each two pages appearing consecutively: $\forall 0 \le i < m$: $(P_i, P_{i+1}) \in \mathbb{E}(Q)$.

(Prinz et al., 2019) propose a general equation to compute the progress for arbitrary Q-graphs. The equation is recursive and returns values between 0 and 1 (i. e., 0 and 100%):

$$\rho(P) = \rho(P_{prev}) + |P| \frac{1 - \rho(P_{prev})}{rem(P)}$$
 (1)

The equation contains the following parts: $\rho(P)$ describes the progress at the current page P. The computation of the current progress needs the progress $\rho(P_{prev})$ of the previous page P_{prev} . If the current page P is the first page of the Q-graph, then it is assumed that $\rho(P_{prev})$ is 0. The equation adds the impact on the progress of the current page, $|P|\frac{1-\rho(P_{prev})}{rem(P)}$, to the progress of the previous page. |P| is the number of items on the current page. The impact of a single item on the progress on the current page is $\frac{1-\rho(\tilde{P}_{prev})}{rem(P)}$. It contains the *remaining progress* $(1 - \rho(P_{prev}))$ and the number of remaining items (rem(P)). The usage of the remaining progress in the equation allows the progress to adopt to the number of remaining items. For example, if a participant follows a branch, which reduces the number of remaining items, then the impact of each item increases, accelerating the growth of the PI.

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Input: A Q-graph Q and a selection operator \sqcup.

Output: For each P \in \mathbb{P}(Q) the remaining items rem(P).

Set rem(P) = 0 for each P \in \mathbb{P}(Q)

worklist \leftarrow queue(\mathbb{P}(Q)), visited \leftarrow 0

while worklist \neq 0 do

P \leftarrow dequeue(worklist)

directSucc \leftarrow \{succ: (P, Succ) \in \mathbb{E}(Q)\}

if directSucc \subseteq visited then

if directSucc = 0 then

rem(P) \leftarrow |P|

else if |directSucc| = |\{Succ\}| = 1 then

rem(P) \leftarrow |P| + \bigsqcup_{Succ \in directSucc} rem(Succ)

visited \leftarrow visited \cup \{P\}

else

enqueue(worklist, P)
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Figure 2: The general algorithm for computing the number of remaining items for arbitrary prediction strategies (taken from (Prinz et al., 2019)).

Otherwise, if the number of remaining items increases, the impact with each item decreases, decelerating the growth of the PI.

At the beginning of an adaptive survey, the path, a participant takes, is unknown which makes it necessary to predict the number of remaining items rem(P). But different prediction strategies are possible making the computation of the progress a challenge. (Prinz et al., 2019) defined a general algorithm for computing the number of remaining items rem(P) for each page for arbitrary prediction strategies. A property of the algorithm is the usage of a *selection operator* \sqcup representing these strategies. The algorithm receives the selection operator as input making the algorithm independent from the operator. Figure 2 shows the algorithm. In this paper, we focus on the description of the selection operator. For a more general overview, see (Prinz et al., 2019).

There are exactly three situations during the computation of the remaining items for a page P. Either P has 1) no successor, 2) exactly one direct successor, or 3) more than one direct successor. The number of remaining items for the first both situations is the sum of the number of items on P, |P|, and the number of remaining items rem(Succ) of the direct successor Succ (or 0 in the case of situation 1)). In situation 3), different numbers of remaining items may reach P via the direct successors $Succ_1, \ldots, Succ_n, n \geq 2$. The selection operator receives all those numbers as input, $\sqcup (rem(Succ_1), \ldots, rem(Succ_n))$, and produces a single prediction for rem(P). The algorithm contains these situations in the inner if-then-else-structure.

Typical examples of selection operators are the *minimum* and *maximum* functions. Taking the minimum, the number of remaining items is the *smallest*

number of items. The progress is fast at the beginning and becomes slower if the participant takes a path containing more items than the operator has detected. For the maximum, it is vice versa. It represents the *largest* number of items.

3 MEASURES TO COMPARE PREDICTION STRATEGIES

Different prediction strategies usually result in different predicted progresses. Since a survey needs a single strategy, which provides the best prediction, we need a measure to compare the precision of such strategies.

A PI in a survey should commonly represent the *true* progress as well as possible. The computation of the true progress needs the exact number of remaining items. However, a survey engine knows this exact number only *after* the participant is finished. In other words, only after a participant completes a path $W = (P_1, \ldots, P_n), n \ge 1$, then the computation knows the exact number of remaining items on each page P_1, \ldots, P_n and can compute the true progress ρ^* .

A strategy within a set of prediction strategies $\{\sqcup_1, \sqcup_2, \ldots, \sqcup_n\}$, $n \ge 1$, is *the best one* if it minimizes the discrepancy between the predicted and the true progress best. Many measures regarding prediction accuracy are proposed in literature and a lot of recommendations explain in which situations a specific measure should be used. (Hyndman and Koehler, 2006) consider different measures of prediction accuracy in detail and provide a good overview about them. All the measures have in common that they are based on the difference between the prediction and the actual measured value (in our specific case, the true progress).

Our basic idea is that we set the true and predicted/displayed progress in relation. That means, we have a value pair $(\rho^*(P), \rho(P))$ of the true and displayed progress for each page P on a path a participant has visited. The pair $(\rho^*(P), \rho(P))$ can be read as "on page P the true progress was $\rho^*(P)$ but the progress $\rho(P)$ was displayed". If the predicted progress differs from the true progress, it results in an error $e(P) = \rho(P) - \rho^*(P)$. All the pairs $(\rho^*(P), \rho(P))$ can be computed for a prediction strategy and it results in a set \mathcal{M} , which contains all of these pairs. For the comparison of different strategies $\sqcup_1, \ldots, \sqcup_n, n \geq 2$, there is such a set for each strategy: $\mathcal{M}_1, \ldots, \mathcal{M}_n$.

Notice that ρ^* and ρ have percentage scales. That means, measures based on percentage errors are applicable. (Hyndman and Koehler, 2006) mention four typical measures of percentage errors:

1. Mean Absolute Error (MAE), |e|

- 2. Median Absolute Error (MdAE), median(|e|)
- 3. Root Mean Square Error (RMSE), $\sqrt{\overline{e^2}}$
- 4. Root Median Square Error (RMdSE), $\sqrt{median(e^2)}$.

Applying one of the measures produces a value val_i for each strategy \sqcup_i , $1 \le i \le n$. Since the true progress is always the same for each strategy and all values are on the percentage scale, statistics allows us to compare the different values. The strategy with the lowest value is the best one of the considered strategies.

A value of 0 is perfect for all measures, it means that the error between the true and predicted progress is zero. The RMSE and RMdSE have the disadvantage that they are infinite, undefined, or skewed if all *observed* values are 0 or near to 0 (Hyndman and Koehler, 2006). Since the true progress has values in the range from 0 to 100%, this disadvantage does not affect them.

The approach relies on the knowledge of the true progress and, therefore, on empirical data. Unfortunately, as with any empricial study, these data is usually not available before the survey starts. To overcome this problem, data can be generated by *pilot studies*, simulations, or path-explorations of the survey for example. Pilot studies refer to conducting the survey with a subset of the population, whereas in *simulations* virtual participants answer the questionnaire. In a pathexploration, an algorithm computes all (or most) paths of the survey and computes sample progresses for each path. But adaptive surveys may have a (exponential) large number of such paths. Furthermore, all three possibilities have in common that they should represent a "realistic" usage of the different paths. Different weights exist for the paths and influence the measure. The researcher should be aware of this.

4 EXPERIMENTS AND LESSONS LEARNED

In our department, we conduct large surveys with hundreds of variables and items and many adaptive paths. The survey engine, that we use, stores the paths on which the participants "walk" through the surveys. For each participant, it is possible to compute the true number of remaining items for each visited page. Besides the true progress, we can also compute the predicted progress for different prediction strategies in retrospect with Equation 1 and the algorithm of Figure 2. As a result, we get data sets with the true and displayed progresses for each strategy for each participant. With these it is possible to determine the most suitable measure and the best strategy.

4.1 Experimental Settings

We took two of our surveys, *survey* A and *survey* B. Table 1 describes the structure of the surveys based on empirical data. In the table, $N_{Branches}$ refers to the number of pages with branches, |Path| is the number of pages within a path, and N_{Items} refers to the number of items a participant has seen. Both surveys have similar characteristics except for $N_{Participants}$ and $N_{Branches}$. For survey A we have more available data sets, whereas survey B has much more branches.

For both surveys, we produced data sets for three different prediction strategies: minimum (min), mean, and maximum (max). If a page has more than two direct successor pages, the minimum strategy takes the smallest number of remaining items. The maximum strategy takes the largest number of remaining items in such a case, whereas the mean strategy computes the mean number. At this place, it is important that the mean represents not the empirical mean of items on all empirical paths. It represents the selection operator mean used in the general algorithm. The expected remaining items on the start page vary for both surveys (cf. Table 1, rem(start)) and are higher for survey B except for the min approach, which is very small with a value of 7. The values in parentheses represent adjustments on the surveys explained in the following.

4.2 Lessons Learned

Screening paths are paths at the start of a survey in which a participant receives a few key questions to determine if they are part of the specific target population. Depending on their answers, the survey either continues to the main part or ends quickly. Therefore, there is an exit path to the end without many items. The first lesson we learned was that the inclusion of screening paths in the progress calculation usually produces bad predictions. By taking the min strategy, the exit path has the fewest remaining items and, therefore, decreases the number of remaining items on all paths at the beginning of the survey (cf. rem(start) in Table 1). In survey B, this leads to progresses near 100% after passing the page where the screening path ends. For the strategies mean and max, the screening path does not have a great impact.

Adaptive page chains are subpaths with many adaptive pages, however, each participant only sees a small number of them. In survey *B*, there are a lot of such pages, which contain items about special topics. In general, each participant has only seen one or two of these approx. 30 pages. For *min*, such chains disappear which skews the results as most participants see at least one page. The *max* strategy includes each

	Survey A	Survey B		Survey A	Survey B
N _{Participants}	1041	193	N _{Items}		
$N_{Branches}$	11	38	min	4	6
Path			mean	246.70	290.97
min	2	2	max	339	377
mean	16.34	18.49	rem(start)		
max	25	23	$\Box = min$	46 (167)	7 (258)
Var	48.56	24.63	$\sqcup = mean$	115 (241)	254 (495)
			$\sqcup = max$	345 (288)	706 (700)

Table 1: Structure and important empirical properties of *survey A* and *survey B*.

 $N_{Participants}$ = number of participants, $N_{Branches}$ = number of branching pages in Q-graph, |Path| = empirical length of paths, N_{Items} = empirical number of items seen, rem(start) = remaining items on the start page (values in parentheses are adjustments explained in the text)

adaptive page resulting in a high number of remaining items. The *mean* strategy smooths the high number of remaining items, however, usually only by half. In Table 1, $N_{Branches}$ indicates long adaptive page chains with a value of 38 in survey B instead of 11 in A. The second lesson we learned was that such chains of adaptive pages produce bad predictions too.

As a consequence, we adjusted our experiments with the surveys *A* and *B* by removing the screening paths from the progress computation. Otherwise, it is meaningless to compare the results of the different strategies. The adjustments result in different numbers of remaining items for each strategy (see *rem(start)* in parentheses in Table 1). We left the chains of adaptive pages as they contain important items.

4.3 Experimental Results

Figure 3 shows the results of our experiments. The x axis describes the true and the y axis the displayed (predicted) progress for each strategy. The black line illustrates a perfect prediction strategy and the true progress, respectively. An observation is that the min approach results in overestimations of the progress, whereas the max approach results in underestimations. For survey A, mean has values above and below the true progress line. For survey B, the values of mean are all below the line because of the mentioned adaptive page chains.

Figure 3 contains the values for the four measures MAE, MdAE, RMSE, and RMdSE. Actually, the MdAE and RMdSE result in nearly equivalent values. The Appendix explains why.

For survey *A*, the *mean* strategy has the lowest MAE of 1.47 and RMSE of 2.13, but the *max* strategy has the lowest MdAE and RMdSE with both 1.08. The *mean* and *max* strategy seem to estimate the true progress best. The *min* approach is the worst approach. The distribution of the points supports the result.

For survey B, the strategies perform differently: the min approach is the best and the max strategy is the

worst one for all four measures. However as a whole, all strategies perform worse in survey *B*. There is no strategy which predicts the true progress well. Even though *min* is the best strategy of the three, a visual inspection of the predicted values in Figure 3 shows that for many participants the displayed progress is near 100% even though they still have around 25% of the survey to go. A reason for this behaviour are the adaptive page chains corresponding with a high number of branching pages (cf. Table 1, *N*_{Branches}). The poor fit is supported by higher values instead of those for survey *A*. The worse fitting for survey *B* could also be a result of the small number of available participants.

In our application context, high errors should be penalized more than smaller errors since higher errors have a stronger impact on the overall progress calculation and can lead to noticeable deviations from the true progress whereas small errors should be almost invisible to the participant. The RMSE is, therefore, a good choice, because it squares the error giving large errors more weight. Like Figure 3 shows, the RMSE is always the highest. Actually, the value of the RMdSE is always close to the MdAE as mentioned before. The squaring of the error in the RMdSE has not a great effect on the resulting value, as also shown in the Appendix.

For survey A, the mean (MAE and RMSE) as well as the max strategy (MdAE and RMdSE) have small values as mentioned before. Figure 3 shows that the max strategy has more outliers for survey A than the mean strategy. Following the above argumentation, the mean strategy should be used since the outliers may lead to noticeable deviations. This is supported by a higher RMSE.

Altogether, we recommend to use the RMSE for comparing different prediction strategies for PIs. It is most sensitive to high deviations.

Figure 4 shows the error distribution for all strategies in both surveys. For survey A, the *mean* and *max* strategies result in errors near zero with less variance

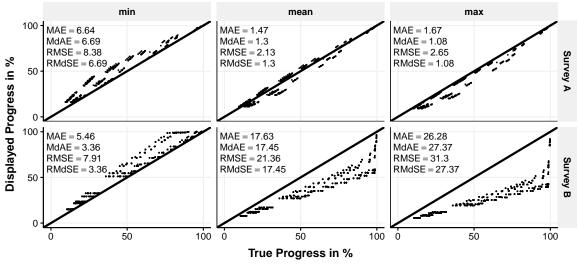


Figure 3: Charts with displayed progresses and the computed measures MAE, MdAE, RMSE, and RMSE for the three prediction strategies *min*, *mean*, and *max* for two surveys A and B. For survey A, the *mean* strategy has the lowest MAE and RMSE and the *max* strategy has the lowest MdAE and RMdSE. For survey B, the *min* approach is the best and the *max* strategy is the worst one for all four measures. But all strategies perform worse in survey B.

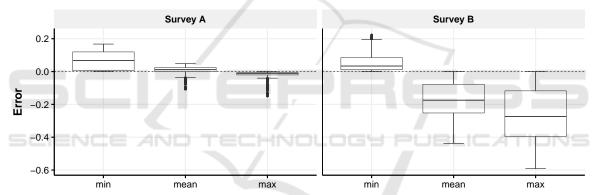


Figure 4: Error distribution of the strategies *min*, *mean*, and *max* for surveys A and B. For survey A, the *mean* and *max* strategies result in errors near zero with less variance. The *min* strategy has a higher variance. In contrast, for survey B, the *max* and *mean* strategies have a very high variance and the *min* approach has a small variance.

and only a minor number of outliers. Instead, the *min* strategy has a higher variance of the error. The *max* and *mean* strategies have a very high variance for survey *B*. The *min* approach has better results with a small variance. Altogether, Figure 4 supports the previous observations and the usage of the RMSE.

5 BRIEF DISCUSSION AND CONCLUSION

In this paper, we argued for a measure to compare prediction strategies for the computation of the progress in adaptive surveys. Experiments suggest that the RMSE is a good choice of measure. The measure is comparable between different strategies for the same survey. The strategy with the lowest RMSE is the best considered one. Our experiments with real-world surveys accentuate the hypothesis that *there is not a single best prediction strategy for all surveys*. The right strategy depends on the survey.

Using the RMSE for comparing the strategies needs empirical data. Further research has to find ways to generate this data precisely, e. g., by simulation or path-weighting.

Although a prediction strategy has the best RMSE within a set of strategies, another strategy may be better, e. g., if the simulated data does not reflect the actual population behaviour; or there is simply an yet unknown strategy, which approximates the true progress better. We presented three basic strategies, but *future research should focus on finding better strategies*, which can handle adaptive chains.

Furthermore, results of the state-of-the-art should be considered for the decision of which strategy is used. For example, if the *min* and *max* strategies perform similarly, it states to use the *min* strategy as it may reduce the dropout rate. In future work, we want to build a tool, which chooses the best of a set of strategies and includes the state-of-the-art. It should also provide a time estimate at each page of the survey. Furthermore, we also want to consider how we can inform the participants about variances in the progress and how that information influences the participant's satisfaction.

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APPENDIX

Section 4 mentioned that the MdAE and RMdSE mostly result in almost the same values. In this appendix, we are going to investigate this fact a little more closely.

Assume a vector of (real) values $v = \{v_1, \dots, v_n\}$, $n \ge 1$. The first step of computing the median \tilde{v} is to order the values of v by size. By taking the MdAE, the ordering is performed on |v|. For RMdSE, the ordering is performed on v^2 . Since for each two arbitrary (real) values v_1 and v_2 it holds true that $|v_1| \le |v_2| \iff v_1^2 \le v_2^2$, it is easy to confirm that the ordering of |v| is equal to the ordering of v^2 . Let (a_1, \dots, a_n) be the resulting order of the absolute and (s_1, \dots, s_n) be the ordering for the squared values. The following is valid:

$$\forall 1 \le i \le n \colon a_i = \sqrt{s_i} \tag{2}$$

The second step of computing the median depends on the number of dimensions n of v. There are two cases:

- 1. n is odd. The median is the value on position n/2 of the ordering. It is $a_{n/2}$ for MdAE and $s_{n/2}$ for RMdSE. For RMdSE, we have to take the root median, i. e., RMdSE is $\sqrt{s_{n/2}}$. With Equation 2 in mind, the MdAE and RMdSE are equal.
- 2. n is even. The median is half the sum of the values on positions n/2 and n/2+1. It is $1/2(a_{n/2}+a_{n/2+1})$ for MdAE and $1/2(s_{n/2}+s_{n/2+1})$ for RMdSE. For RMdSE, we take again the rooted median, $\sqrt{1/2(s_{n/2}+s_{n/2+1})}$. Actually, the MdAE and RMdSE are unequal. But on closer inspection, the value of $1/2(s_{n/2}+s_{n/2+1})$ is always between $s_{n/2}$ and $s_{n/2+1}$, and therefore, the RMdSE is always between $a_{n/2}$ and $a_{n/2+1}$ with regard on Equation 2.

Both cases result in the following facts:

- 1. MdAE and RMdSE are equal if *n* is odd.
- 2. MdAE and RMdSE are almost equal if n is even and the values on positions n/2 and n/2 + 1 are close to each other.