Alternative Algebra for Trapezoidal Fuzzy Number and Comparison with Various Other Algebra

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Abstract: In this article will be given a new algebra for trapezoidal fuzzy number, and then the new algebra we got will be compared with other algebra by privious outhor. Finally will be given a fully fuzzy linear system which will be solved by author algebra and other algebra by privious outhor, will be shown the new algebra which the outhor suggest its better.

1 INTRODUCTION

Fuzzy logic is a branch of mathematical science first introduced by LA Zadeh, a professor from UC Berkeley's electrical engineering, computer science department in 1965. LA Zadeh thinks fuzzy logic can bridge precision machine language into human language that emphasizes meaning or significance (Zadeh, 1965).

Many methods to solve fully fuzzy linear system, but almost all methods of partitioning are in solution and in partitioning each matiks partition must be of positive or negative value (there is no mixed matrix). In this article we will give an example of solving a fully fuzzy linear system using the Cramer method, with $\tilde{x}_i = {}^{\det \tilde{A}_i}\!\big/_{\det \tilde{A}}$. In this case even though the matrix $\tilde{A}$ is a mixture of each i -th element, it can still be solved. First the fuzzy number identity value is given so that it can be used to obtain the inverse value of the fuzzy number so that $\tilde{x}_i = \det \tilde{A}_i \otimes \left(\det \tilde{A}\right)^{-1}$can be completed.

Previously (Vijayalakshmi, 2011) had defined $\tilde{u}^n = (m, n, \alpha, \beta)^n$ with n is negative is $(m^n, n^n, nm^{n-1}\beta, -nn^{n-1}\alpha)$ in this case there is a weakness in the symbol so that it can confuse the reader in interpreting n as a point on a fuzzy number or n as a rank in the fuzzy number. Furthermore (Vijayalakshmi, 2011) also gives the definition $\tilde{u}^n = (m, n, \alpha, \beta)^n$=$(m^n, n^n, nm^{n-1}\beta, -nn^{n-1}\alpha)$ with n is positive , in this case we can assume $\tilde{u}^2 = \tilde{u} \otimes \tilde{u}$ using the given multiplication formula but gives different results when the value $n = 2$ is substituted into $\tilde{u}^n$.

Furthermore (Jafarian, 2016) only defines fuzzy numbers said to be not negative if given fuzzy numbers $\tilde{u} = (m, n, \alpha, \beta)$ if $m - \alpha \geq 0$ and does not provide a definition of fuzzy numbers said to be positive or negative.

Finally author will compare the arithmetic that the writer obtained with the arithmetic given by other writers especially on the inverse value that the writer obtained and the inverse value given (Vijayalakshmi, 2011) in solving the fully fuzzy linear system.

2 PRELIMINARIES

Some basic definitions of trapezoidal fuzzy number set theory are reviewed (Jafarian, 2016), (Radhakrishnan et al., 2012), (Vijayalakshmi, 2011).

A fuzzy number $\tilde{s} = (h, k, \rho, \tau)$ is said to be a trapezoidal fuzzy numberif its membership function is given by

$$\mu_{\tilde{s}}(x) = \begin{cases} 1 - \dfrac{h - x}{\rho}, & h - \rho \leq x < h \\ 1, & h \leq x < k \\ 1 - \dfrac{x - k}{\tau}, & k \leq x \leq k + \tau \\ 0, & \text{otherwise} \end{cases}$$

The trapezoidal fuzzy number can be written in the parametric form

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$$\underline{u}(r) = h - (1-r)\rho,$$
$$\overline{u}(r) = k + (1-r)\tau.$$

Definition 2.1. Fuzzy subset $\tilde{u}$ is defined with $\tilde{s} = (x, \mu_{\tilde{s}}(x))$. In pairs $(x, \mu_{\tilde{s}}(x))$, x is a member of the set $\tilde{s}$ and $\mu_{\tilde{s}}(x)$ the value on interval [0, 1] which is called the membership function.

Definition 2.2. Fuzzy number is a fuzzy set $\tilde{s}: \mathbb{R} \to [0,1]$ which satisfies the following:
1. $\tilde{s}$ is upper semicontinuous.
2. $\tilde{s} = 0$ outside the interval $[h-\rho, k+\tau]$.
3. There exist real number $[h, k]$ in interval $[h-\rho, k+\tau]$ such that,
   i. $\tilde{s}$ monotonic increasing in $[h-\rho, h]$.
   ii. $\tilde{s}$ monotonic decreasing in $[k, k+\tau]$.
   iii. $\tilde{s} = 1$ for $h \le x \le k$.

The alternative definition of other fuzzy numbers that are often used by authors is as follows.

Definition 2.3. Fuzzy number $\tilde{u}$ in $\mathbb{R}$ is defined as a function pair $\tilde{s} = [\underline{s}(r), \overline{s}(r)]$ which satisfy the following:
1. $\underline{s}(r)$ is a bounded left continuous non decreasing function over [0,1].
2. $\overline{s}(r)$ is a bounded right continuous non increasing function over [0,1].
3. $\underline{s}(r) \le \overline{s}(r), 0 \le r \le 1$.

Definition 2.4. A trapezoidal fuzzy number $\tilde{s} = (h, k, \rho, \tau)$ is said to be zero trapezoidal fuzzy number if and only if $h = 0, k = 0, \rho = 0$ and $\tau = 0$.

Definition 2.5. Two fuzzy number $\tilde{s} = (h, k, \rho, \tau)$ and $\tilde{t} = (p, q, \sigma, \omega)$ are said to be equal if and only if $h = p, k = q$ and are said pure same if and only if $h = p, k = q, \rho = \sigma, \tau = \omega$.

Here's the algebra that other author give:

Definition 2.6. Given $\tilde{s} = (h, k, \rho, \tau) = (\underline{s}, \overline{s}) = (h - (1-r)\rho, k + (1-r)\tau)$, $\tilde{t} = (p, q, \sigma, \omega) = (\underline{t}, \overline{t}) = (p - (1-r)\sigma, q + (1-r)\omega)$ and n is real

a. Addition
$$\tilde{s} + \tilde{t} = (\underline{s} + \underline{t}, \overline{s} + \overline{t})$$
$$\tilde{s} + \tilde{t} = (h + p, k + q, \rho + \sigma, \tau + \omega)$$
b. Subtraction
$$\tilde{s} - \tilde{t} = (\underline{s} - \overline{t}, \overline{s} - \underline{t})$$
$$\tilde{s} - \tilde{t} = (h - q, k - p, \rho + \omega, \tau + \sigma)$$
c. Negative number
$$-\tilde{s} = (-k, -h, \tau, \rho)$$
d. Scalar multiplication
$$n \otimes \tilde{s} = k \otimes (h, k, \rho, \tau)$$
$$= \begin{cases} (nh, nk, n\rho, n\tau), & n \ge 0 \\ (nk, nh, -n\tau, -n\rho), & n < 0 \end{cases}$$
e. Multiplication
If $\tilde{s} \ge 0$ and $\tilde{t} \ge 0$, so
$$\tilde{s} \otimes \tilde{t} = (hp, kq, (h\sigma + p\rho), (k\omega + q\tau))$$

In this case (Jafarian, 2016) and (Vijayalakshmi, 2011) do not provide alternative algebra for other fuzzy numbers such as multiplication of positive fuzzy numbers - negative fuzzy numbers, negative fuzzy numbers - positive fuzzy numbers and negative fuzzy numbers - negative fuzzy numbers. And didnt give definition of fuzzy numbers said to be positive or fuzzy numbers said to be negative.

3 ALGEBRA OF FUZZY NUMBER

Fuzzy numbers that are said to be positive fuzzy or negative fuzzy by determining the area of the x-axis, then we will be given the identity of fuzzy numbers, inverse fuzzy numbers and alternative divisions of fuzzy numbers.

3.1 Positive and Negative Fuzzy Number

Definition 3.1. The fuzzy number $\tilde{u}$ is said to be positive (negative) fuzzy denoted $\tilde{s} \ge 0, (\tilde{s} < 0)$ by using the area rules in the x-axis, that is:
1. If the fuzzy region is exactly one of the x -axes then fuzzy $\tilde{s}$ is said to be positive (negative) if $h - \rho \ge 0 (k - \tau < 0)$.
2. If the fuzzy region is both of the x -axes so:
   a. If $h < 0$, $k \le 0$ and $k + \tau \ge 0$ $\tilde{s}$ said positive fuzzy number is $h + k + \frac{\tau}{2} - \frac{\rho}{2} + \frac{k^2}{\tau} \ge 0$, and $\tilde{s}$ said to be negative fuzzy number is $h + k + \frac{\tau}{2} - \frac{\rho}{2} + \frac{k^2}{\tau} < 0$.
   b. If $h < 0$ and $k > 0$ $\tilde{s}$ said positive fuzzy number is $k + h - \frac{\rho}{2} + \frac{\tau}{2} \ge 0$, and $\tilde{s}$ said to be negative fuzzy number is $k + h - \frac{\rho}{2} + \frac{\tau}{2} < 0$.
   c. If $h \ge 0$ and $k > 0$, $\tilde{s}$ said positive fuzzy number is $k + h + \frac{\tau}{2} - \frac{\rho}{2} - \frac{h^2}{\rho} \ge 0$, and $\tilde{u}$ said to be negative fuzzy number is $k + h + \frac{\tau}{2} - \frac{\rho}{2} - \frac{h^2}{\rho} < 0$.

3.2 Arithmetic Trapezoidal Fuzzy Number

Given $\tilde{s} = (h, k, \rho, \tau)$ with parametric function such as:
$$\tilde{s} = (\underline{s}(r), \overline{s}(r)) = (h - (1-r)\rho, k + (1-r)\tau),$$

and

$$\tilde{t} = \left(\underline{t}(r), \overline{t}(r)\right) = (p - (1-r)\sigma, q + (1-r)\omega).$$

Theorem 3.1. If $\tilde{s} = \left[\underline{s}(r), \overline{s}(r)\right]$ and $\tilde{t} = \left[\underline{t}(r), \overline{t}(r)\right]$ is two positive trapezoidal fuzzy number so $\widetilde{w} = \tilde{s} \otimes \tilde{t} = \left(\underline{w}(r), \overline{w}(r)\right)$ with

$$\underline{w}(r) = \underline{s}(r)\underline{t}(1) + \underline{s}(1)\underline{t}(r) - \underline{s}(1)\underline{t}(1),$$

For $r \in [0,1]$ is a positive trapezoidal fuzzy number.

Based on theorem 1, for two trapezoidal fuzzy number $\tilde{s} = \left[\underline{s}(r), \overline{s}(r)\right]$ and $\tilde{t} = \left[\underline{t}(r), \overline{t}(r)\right]$ following this conditions:

i. If $\tilde{s}$ positive and $\tilde{t}$ negative, so $\widetilde{w} = -\left(\tilde{s} \otimes (-\tilde{t})\right)$ negative,
ii. If $\tilde{s}$ negative and $\tilde{t}$ positive, so $\widetilde{w} = -\left((-\tilde{s}) \otimes \tilde{t}\right)$ negative,
iii. If $\tilde{s}$ negative and $\tilde{t}$ negative, so $\widetilde{w} = \left((-\tilde{s}) \otimes (-\tilde{t})\right)$ positive.

Based on theorem 1 and multiplication (i-iii), so multiplication on two trapezoidal fuzzy number $\tilde{s} = \left[\underline{s}(r), \overline{s}(r)\right]$ and $\tilde{t} = \left[\underline{t}(r), \overline{t}(r)\right]$ and $r \in [0,1]$ obtained:

i. If $\tilde{u}$ positive and $\tilde{v}$ negative, s

$$\widetilde{w} = \begin{cases} \underline{w}(r) = \overline{s}(r)\underline{t}(1) + \overline{s}(1)\underline{t}(r) - \overline{s}(1)\underline{t}(1) \\ \overline{w}(r) = \underline{s}(r)\overline{t}(1) + \underline{s}(1)\overline{t}(r) - \underline{s}(1)\overline{t}(1) \end{cases}$$

ii. If $\tilde{u}$ negative and $\tilde{v}$ positive, so

$$\widetilde{w} = \begin{cases} \underline{w}(r) = \underline{s}(r)\overline{t}(1) + \underline{s}(1)\overline{t}(r) - \underline{s}(1)\overline{t}(1) \\ \overline{w}(r) = \overline{s}(r)\underline{t}(1) + \overline{s}(1)\underline{t}(r) - \overline{s}(1)\underline{t}(1) \end{cases}$$

iii. If $\tilde{u}$ negative and $\tilde{v}$ negative, so

$$\widetilde{w} = \begin{cases} \underline{w}(r) = \overline{s}(r)\overline{t}(1) + \overline{s}(1)\overline{t}(r) - \overline{s}(1)\overline{t}(1) \\ \overline{w}(r) = \underline{s}(r)\underline{t}(1) + \underline{s}(1)\underline{t}(r) - \underline{s}(1)\underline{t}(1) \end{cases}$$

By following the description we get the fuzzy number multiplication formula as follows:

a. Multiplication of Fuzzy

I. If $\tilde{s} > 0$ and $\tilde{t} > 0$ so:

$$\widetilde{w} = \left(\underline{w}(r), \overline{w}(r)\right),$$
$$\widetilde{w} = \left(hp, kq, (h\sigma + p\rho), (k\omega + q\tau)\right)$$

II. If $\tilde{s} > 0$ and $\tilde{t} < 0$ so:

$$\widetilde{w} = \left(\underline{w}(r), \overline{w}(r)\right),$$
$$\widetilde{w} = \left(kp, hq, (k\sigma - p\tau), (h\omega - q\rho)\right)$$

III. If $\tilde{s} < 0$ and $\tilde{t} > 0$ so:

$$\widetilde{w} = \left(\underline{w}(r), \overline{w}(r)\right),$$
$$\widetilde{w} = \left(hq, kp, (q\rho - h\omega), (p\tau - k\sigma)\right)$$

IV. If $\tilde{s} < 0$ and $\tilde{t} < 0$ so:

$$\widetilde{w} = \left(\underline{w}(r), \overline{w}(r)\right),$$
$$\widetilde{w} = \left(kq, hp, -(q\tau + k\omega), -(h\sigma + p\rho)\right)$$

b. Fuzzy Number Identity

Given $\tilde{s} = (h, k, \rho, \tau)$ and $\tilde{I} = (p, q, \sigma, \omega)$, so $\tilde{I}$ said to be identity if

$$\tilde{s} \otimes \tilde{I} = \tilde{s}$$
$$(h, k, \rho, \tau) \otimes (p, q, \sigma, \omega) = (h, k, \rho, \tau)$$
$$(hp, kq, h\sigma + p\rho, k\omega + q\tau) = (h, k, \rho, \tau)$$

and

$$\begin{array}{ll} hp = h & kq = k \\ p = 1 & q = 1 \\ h\sigma + p\rho = \rho & k\omega + q\tau = \tau \\ h\sigma + \rho = \rho & k\omega + \tau = \tau \\ h\sigma = 0 & k\omega = 0 \\ \sigma = 0 \ \ if\ h \neq 0 & \omega = 0 \ \ if\ k \neq 0 \end{array}$$

So obtained $\tilde{\imath}_m = (1,1,0,0)$ is called pure identity, but it will be difficult to obtain it so given $\tilde{\imath} = (1,1,\varepsilon_1,\varepsilon_2)$ is called identity.

c. Invers of Fuzzy

Given $\tilde{s} = (h, k, \rho, \tau)$ and $\tilde{v} = (p, q, \sigma, \omega)$, so $\tilde{t}$ said to be inverse of $\tilde{s}$ if $\tilde{s} \otimes \tilde{t} = (1,1,0,0)$ with $\tilde{\imath}_m = (1,1,0,0)$.

$$\tilde{s} \otimes \tilde{t} = (1,1,0,0)$$
$$(h, k, \rho, \tau) \otimes (p, q, \sigma, \omega) = (1,1,0,0)$$
$$(hp, kq, h\sigma + p\rho, k\omega + q\tau) = (1,1,0,0)$$

So obtained

$$\begin{array}{ll} hp = 1 & kq = 1 \\ p = {}^{1}/_{h} & q = {}^{1}/_{k} \\ h\sigma + p\rho = 0 & k\omega + q\tau = 0 \\ \sigma = {}^{-\rho}/_{h^2} & \omega = {}^{-\tau}/_{k^2} \end{array}$$

$$\tilde{t} = \frac{1}{\tilde{s}} = \left(\frac{1}{h}, \frac{1}{k}, \frac{-\rho}{h^2}, \frac{-\tau}{k^2}\right)$$

d. Division of Fuzzy

Given $\tilde{s} = (h, k, \rho, \tau)$ and $\tilde{t} = (p, q, \sigma, \omega)$,so

$$\frac{\tilde{u}}{\tilde{v}} = \frac{(h, k, \rho, \tau)}{(p, q, \sigma, \omega)}$$

$$= \begin{cases} \left(\frac{h}{p}, \frac{k}{q}, \frac{-h\sigma + p\rho}{p^2}, \frac{-k\omega + q\beta}{q^2}\right), \tilde{s} > 0 \text{ and } \tilde{t} > 0 \\ \left(\frac{k}{q}, \frac{h}{p}, \frac{-k\sigma - p\tau}{q^2}, \frac{-h\omega - q\rho}{q^2}\right), \tilde{s} > 0 \text{ and } \tilde{t} < 0 \\ \left(\frac{h}{q}, \frac{k}{p}, \frac{q\rho + h\omega}{q^2}, \frac{p\tau + k\sigma}{p^2}\right), \tilde{s} < 0 \text{ and } \tilde{t} > 0 \\ \left(\frac{k}{q}, \frac{h}{p}, \frac{k\omega - q\tau}{q^2}, \frac{h\sigma - p\rho}{p^2}\right), \tilde{s} < 0 \text{ and } \tilde{t} < 0 \end{cases}$$

Furthermore, the inverse value that the writer obtained will be compared with the inverse value by Vijayalakshmi. Vijayalakshmi (2011), defines

$$\tilde{s}^n = (h, k, \rho, \tau)^n$$
$$= \begin{cases} (h^n, k^n, nh^{n-1}\tau, -nk^{n-1}\rho) & \text{for positive } n \\ (h^n, k^n, -nh^{n-1}\tau, -nk^{n-1}\rho) & \text{for negative } n \end{cases}$$

From the equation given by Vijayalakshmi an inverse value was obtained

$$\tilde{s}^{-1} = \left(\frac{1}{h}, \frac{1}{k}, \frac{\tau}{h^2}, \frac{\rho}{k^2}\right).$$

Given $\tilde{s} = (h, k, \rho, \tau)$ will be show that $\tilde{s} \otimes \tilde{s}^{-1} = \tilde{I}$.

a) Using the inverse value given by Vijayalakshmi

$$. \tilde{s} \otimes \tilde{s}^{-1} = (h, k, \rho, \tau) \otimes \left(\frac{1}{h}, \frac{1}{k}, \frac{\tau}{h^2}, \frac{\rho}{k^2}\right)$$
$$= \left(h\left(\frac{1}{h}\right), k\left(\frac{1}{k}\right), h\left(\frac{\tau}{h^2}\right) + \rho\left(\frac{1}{h}\right), k\left(\frac{\rho}{k^2}\right) + \tau\left(\frac{1}{k}\right)\right)$$
$$= \left(1, 1, \frac{\tau + \rho}{h}, \frac{\rho + \tau}{k}\right) \neq (1,1,0,0) = \tilde{\imath}_m$$

b) Using the inverse value that the author obtained

$$\tilde{s} \otimes \tilde{s}^{-1} = (h, k, \rho, \tau) \otimes \left(\frac{1}{h}, \frac{1}{k}, \frac{-\rho}{h^2}, \frac{-\tau}{k^2}\right)$$
$$= \left(h\left(\frac{1}{h}\right), k\left(\frac{1}{k}\right), h\left(\frac{-\rho}{h^2}\right) + \frac{1}{h}(\rho), k\left(\frac{-\tau}{k^2}\right) + \frac{1}{k}(\tau)\right)$$
$$= (1,1,0,0) = \tilde{\imath}_m$$

As an illustration given an example of the following fully fuzzy linear equation, then it will be solved using the Cramer method. And the end will be using two of inverse that author sugest and invers from Vijayalakshmi.

$$(1,2,3,5)\tilde{x}_1 + (2,3,4,5)\tilde{x}_2 = (7,26,37,88)$$
$$(1,3,4,5)\tilde{x}_1 + (2,4,6,7)\tilde{x}_2 = (7,36,44,115)$$

The equation can be changed in the form of a matrix as follows

$$\tilde{A} \otimes \tilde{x} = \tilde{b}$$

$$\begin{bmatrix} (1,2,3,5) & (2,3,4,5) \\ (1,3,4,5) & (2,4,6,7) \end{bmatrix} \otimes \begin{bmatrix} x_1, y_1, \rho_1, \tau_1 \\ x_2, y_2, \rho_2, \tau_2 \end{bmatrix} = \begin{bmatrix} (7,26,37,88) \\ (7,36,44,115) \end{bmatrix}$$

Obtained

$$\det \tilde{A} = (-7,6,42,46)$$

$$\tilde{A}^{(1)} = \begin{bmatrix} (7,26,37,88) & (2,3,4,5) \\ (7,36,44,115) & (2,4,6,7) \end{bmatrix}$$

$$\det \tilde{A}^{(1)} = (-94,90,641,650)$$

$$\tilde{A}^{(2)} = \begin{bmatrix} (1,2,3,5) & (7,26,37,88) \\ (1,3,4,5) & (7,36,44,115) \end{bmatrix}$$

$$\det \tilde{A}^{(2)} = (-71,65,459,475)$$

Will be solve with two of inverse value such as:

a) Using the inverse value that the author obtained

$$\tilde{x}_1 = \frac{\det \tilde{A}^{(1)}}{\det \tilde{A}} = \det \tilde{A}^{(1)} \otimes \left(\det \tilde{A}\right)^{-1}$$
$$= (-94,90,641,650) \otimes \left(\frac{-1}{7}, \frac{1}{6}, \frac{-42}{49}, \frac{-46}{36}\right)$$
$$= (13.43,15,-11,-6.67)$$
$$\tilde{x}_2 = \frac{\det \tilde{A}^{(2)}}{\det \tilde{A}} = \det \tilde{A}^{(2)} \otimes \left(\det \tilde{A}\right)^{-1}$$
$$= (-71,65,459,475) \otimes \left(\frac{-1}{7}, \frac{1}{6}, \frac{-42}{49}, \frac{-46}{36}\right)$$
$$= (10.14,10.83,-4.71,-3.89)$$

$$\tilde{x} = \begin{bmatrix} (13.43,15,-11,-6.67) \\ (10.14,10.83,-4.71,-3.89) \end{bmatrix}$$

b) Using the inverse value given by Vijayalakshmi

$$\tilde{x}_1 = \frac{\det \tilde{A}^{(1)}}{\det \tilde{A}} = \det \tilde{A}^{(1)} \otimes \left(\det \tilde{A}\right)^{-1}$$
$$= (-94,90,641,650) \otimes \left(\frac{-1}{7}, \frac{1}{6}, \frac{46}{49}, \frac{42}{36}\right)$$
$$= (13.43,15,-179.82,213.33)$$
$$\tilde{x}_2 = \frac{\det \tilde{A}^{(2)}}{\det \tilde{A}} = \det \tilde{A}^{(2)} \otimes \left(\det \tilde{A}\right)^{-1}$$
$$= (-71,65,459,475) \otimes \left(\frac{-1}{7}, \frac{1}{6}, \frac{46}{49}, \frac{42}{36}\right)$$
$$= (10.14,10.83,-132.22,155)$$

$$\tilde{x} = \begin{bmatrix} (13.43,15,-179.82,213.33) \\ (10.14,10.83,-132.22,155) \end{bmatrix}$$

From the two solutions above it can be seen that the center values of a and b are the same, but the inverse value that the author obtained provides smaller right and left area.

4 CONCLUSIONS

From the previous explanation it can be concluded that the element of identity must be distinguished between pure identity $(1,1,0,0)$ with identity $(1,1,\varepsilon_1,\varepsilon_2)$ as well as for the similarity of fuzzy numbers. Furthermore, by defining the positivity and negativity of fuzzy numbers in multiplication cases will give a better result.

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