Hydrostatic Stiffness as Displacement Boundary Condition of Floating Cylindrical Structural Analysis in Waves

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Abstract: Motion analysis is one of the mandatory aspects to predict the performance of a floating structure, as well as how its structural strength under certain wave load. However, in majority of floating body performance prediction, the calculation of motion and strength performance is done separately. Practically, engineers calculate the motion and hydrodynamics forces that work on the structure, then do separate calculations on the structure to predict structure's strength. These separate calculations often use assumptions that tend to be unrealistic, either over-constrained or under-constrained. This paper provides an alternative to the constraint problem by introducing hydrodynamic stiffness as boundary conditions, instead of using fixed or simply supported boundary conditions, spring boundary conditions are applied with hydrodynamic stiffness of floating body properties. It is expected that this model provides a more realistic constraint to the future analyses. The results achieved are very promising, where the boundary condition resulting a close natural frequency approximation compared with the analytical calculation. This configuration is hoped to be the baseline of more complex structure to be carried out in future research, in order to represent a more realistic structural displacement boundary condition.

1 BACKGROUND

Motion analysis is one of the mandatory aspects to predict the performance of a floating structure, as well as how its structural strength under certain wave load. However, in majority of floating body performance prediction, the calculation of motion and strength performance is done separately.

Practical engineering software package tends to disintegrate calculation of motion and hydrodynamics forces that work on the structure for used to assess the strength of particular floating body.

Traditionally, engineers consider the ship structure as fixed ends beam (Okumoto, et al., 2009) or simple beam (Molland, 2008).

Several researches on analytical level proposed the methods to incorporate 'sea springiness' of floating body during strength analysis. There are researches conducted to integrate Computational Fluid Dynamics (CFD) and Finite Element Analysis (FEA) via Fluid Structure Interaction (FSI) software packages. ANSYS, for instances, is one of the established software packages that used for this intention. In maritime application, various vessel forms has been used as object. For example, composite ship structures (Ma & Mahfuz, 2012), horizontal cylinder (Raja, 2012) and ocean energy harvesting device (Agamloh, et al., 2008). Several open source software such as OpenFoam has also been used for the same intention. Wave-structure interaction method has been developed using OpenFoam (Chen, et al., 2014).

Still, the performed researches are still focused on the fluid interaction and tend to disregard the displacement boundary condition aspects. Majority of the those only consider the displacement boundary condition as buoyancy versus gravity only.

Recent studies provide the hydrostatic stiffness for linear hydroelasticity. The explicit formulation for the complete hydrostatic stiffness for flexible floating structures at rest in calm water is derived based on a consistent linearization of the external hydrostatic pressure and the internal structural stresses (Huang & Riggs, 2000). It is also found that the hydroelasticity formula deals with more terms, and, that under some assumptions, it is reduced to the known complete restoring stiffness (Senjanović, et al., 2011).

This paper introduces the practical hydrostatic stiffness to be used directly as displacement boundary condition of rigid floating body. Analytical

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calculation is introduced as an approach to assess the application of ground spring to a floating cylinder. This method is hoped to be an applied practical guide to model a better and more realistic displacement boundary conditions for those who cannot afford the luxury of FSI study.

The proposed method has been initially developed by American Bureau of Shipping, depicted at ABS Floating Production Installation (ABS, 2014), only for ship shaped structures. Authors inspired by related theory explained at ABS FPI Part 5A, Chapter 3, Appendix 4, Point 17, to be used as basis of so-called analytical-practical approach of cylindrical floating structure's displacement boundary condition.

2 UNCOUPLED HYDRODYNAMIC MOTION

Based on classical theory as commonly known, the free-floating body has six degree of freedom in hydrodynamics motion, namely (Bhattacharya, 1978):

1	Surging	=	motion backward and forward in the direction of ship travels
2	Swaying	=	athwartship motion of ship
3	Heaving	=	motion vertically up and down
4	Rolling	=	angular motion about longitudinal axis (X axis). Traditionally, the angular motion alternating from portside to starboard and vice versa
5	Pitching	=	angular motion about the transverse axis (Y axis). Traditionally, the angular motion alternating from bow to stern and vice versa
6	Yawing	=	angular motion about the vertical axis (Z axis)

Above list of motion is illustrated by Figure 1 below

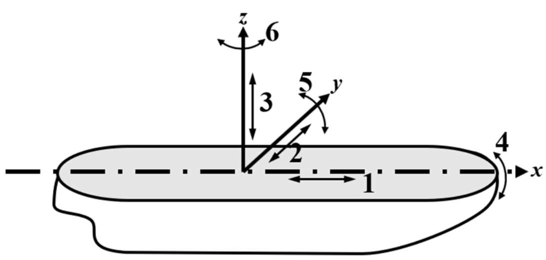

Figure 1: Six Degree of Freedom Hydrodynamic Motion.

2.1 Uncoupled Hydrodynamic Motion of Floating Cylinder

In this paper we limit the discussion only for cylindrical structure, which is a bi-symmetrical structure. Hence, the aforementioned hydrodynamic motion can be reduced due to similarities, into following motions:

1. Surging = swaying, with similar X and Y translation motion.
2. Pitching = rolling, with similar X and Y rotation motion.
3. Yawing, due to the bi-symmetrical structure, the Z rotation is considered negligible.

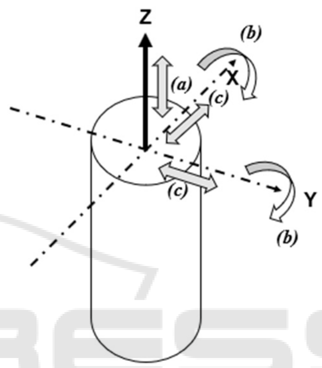

Figure 2: Floating Cylinder Motion.

Figure 2 above explains the considered motions of floating cylinder. As it can be seen, letter (a) coded the heaving motion, while (b) coded the rolling motion, and finally we have (c) coded for swaying motion. (a) and (c) are the translational motions of the cylinder, with equation described below:

$$m\ddot{u} + c\dot{u} + ku = F_0 t \quad (1)$$

Where:

$m\ddot{u}$ = translational inertial force
$c\dot{u}$ = translational damping force
ku = translational restoring force
$F_0 t$ = translational excitation force

Inertial force for translational motion, is present when the cylinder is in oscillatory motion, consist of m (cylinder mass plus hydrodynamic added mass) multiplied by $\ddot{u}$, the motion acceleration for translational.

Damping force, is the force to resist the motion. This force consists of damping coefficient c and translational velocity, $\dot{u}$.

Restoring force is the spring force that brings back the cylinder into its equilibrium position.

Restoring force is composed of k, the hydrostatic stiffness of the motion, multiplied by u for translational motion, which is the translation of cylinder's Centre of Gravity (CoG)

Furthermore, (b) is the rotational motion of the cylinder, with equation described below:

$$m\ddot{\theta} + c\dot{\theta} + k\theta = F_0 t \quad (2)$$

Where:

$m\ddot{\theta}$ = rotational inertial moment
$c\dot{\theta}$ = rotational damping moment
$k\theta$ = rotational restoring moment
$F_0 t$ = rotational excitation moment

Inertial moment for translational motion, is present when the cylinder is in oscillatory motion, consist of m (cylinder mass plus hydrodynamic added mass) multiplied by $\ddot{\theta}$, the motion angular acceleration for rotation.

Damping moment, is the moment to resist the motion. This moment consists of damping coefficient c and rotational motion angular velocity, $\dot{\theta}$.

Restoring moment is the spring moment that brings back the cylinder into its equilibrium position. Restoring moment is composed of k, the hydrostatic stiffness of the motion, multiplied by θ for translational motion, which is the rotation of cylinder's Centre of Gravity (CoG)

This stiffness properties, both for translational and rotational motion, are used for the ground spring stiffness, to represent the actual condition when we perform the structural analysis of the cylinder.

2.2 Hydrostatic Stiffness of Heaving Motion

As stated at equation 1 above, the hydrostatic stiffness of heaving is used for vertical translational spring stiffness. The heaving stiffness is the waterplane area of cylinder multiplied by the water specific weight (Patel & Witz, 1991), consequently, the hydrostatic stiffness of cylinder heaving motion is as follows:

$$k = \gamma \pi r^2 \quad (3)$$

Where:

k = heave stiffness
γ = water specific weight
πr^2 = waterplane area of cylinder

The spring stiffness is attached at the bottom of the cylinder, to represent the restoring motion of heaving.

2.3 Hydrostatic Stiffness of Rolling Motion

At the same time, for rotational motion, the hydrostatic rolling stiffness is used. Rolling stiffness is the righting moment of the cylinder. The righting moment at any particular angle of inclination is expressed as:

$$k\theta = \Delta\overline{GZ} \quad (4)$$

For small angle of inclination (in radians):

$$k\theta \cong \Delta\overline{GM_T}\theta \quad (5)$$

Hence the hydrostatic stiffness of rolling motion:

$$k = \gamma\nabla\overline{\mathrm{GM}_T} \quad (6)$$

Where:

k = rolling motion stiffness
γ = water specific gravity
∇ = water displacement
$\overline{\mathrm{GM}_T}$ = metacentre height of cylinder

3 GROUND SPRING ELEMENT

The ground spring method has very long tradition to be included in dynamic analysis of structures. It is commonly used in seismic analysis to model the damping and stiffness of soil-pile interaction (Datta, 2010). Unlike in hydrodynamic analysis, there are a lot of established coefficient that model the spring and dashpot for a variety of foundation types and soil conditions (Gazetas, 1991). Figure 3 below shows the example of ground spring applied to a building (Datta, 2010).

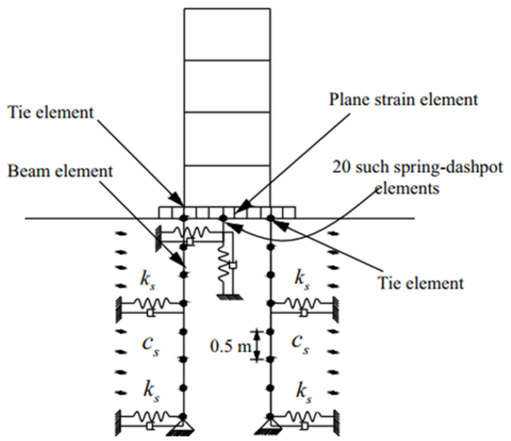

Figure 3: Example of Spring-Dashpot Equivalent Method to a Building (Datta, 2010).

The same concept applied to a floating cylinder with the spring stiffness from the hydrostatic properties of the body. In this paper, we only consider the stiffness properties of spring.

4 CASE STUDY: DISPLACEMENT BOUNDARY CONDITION OF A FLOATING CYLINDER

A simple cylindrical structure is presented to examine the usability of the ground spring applied to hydrodynamic case. Figure 4 below shows the proposed ground spring placement to create equivalent spring arrangement of hydrostatic stiffness.

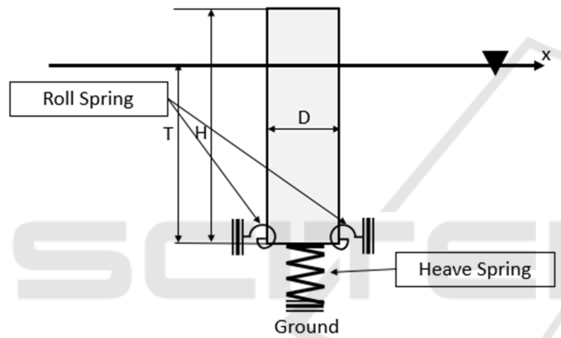

Figure 4: Equivalent Spring Arrangement.

4.1 Problem Setting

The proposed configurations above is treated as structural system with precalculated heave and roll spring as mentioned before at Chapter 2.2 and 2.3.

The heave and roll spring are placed at the bottom of the cylinder, assumed that the support is located at the bottom of the cylinder. This configuration is then compared with traditionally 'fixed' boundary condition at the bottom of the cylinder. In this paper, we only compare the 1st order natural period of heaving and rolling for:

1. Analytical hydrodynamic model
2. Rigid body-equivalent spring arrangement model
3. Rigid body-traditional fixed boundary condition arrangement model.

In analytical hydrodynamic model, the mass is calculated as addition of water displacement and added mass (Sarpkaya, 2010). Whereas the mass for rigid body arrangement, both for spring and boundary condition model, are modelled as its real mass, instead of displacement and added mass. It is important to calculate the mass with aforementioned method, to check whether the 'dry' models, which represented by rigid body model, can imitate the natural frequency of 'wet' model, which represented by hydrodynamics model.

Heave and roll spring stiffness are calculated as mentioned in Chapter 2.2 and Chapter 2.3 respectively, and then placed at subsequent arrangement as depicted in Figure 4.

4.2 Cylinder Diameter, Height and Draught

The cylinder diameter (D) and height (T) are varied with value 0.1≤D/T≤1.0. while the draught (T) is set as 0.8H.

4.3 Natural Period

The natural period of heaving and rolling will be calculated for the three configuration variations. The natural period for 'wet' arrangement is calculated as below for heaving motion:

$$T_n = 2\pi\sqrt{\frac{m + m_{AZ}}{k_z}}(s) \tag{7}$$

Where:

m = real mass
maz = added mass for heaving motion
kz = heaving motion stiffness

And for rolling motion:

$$T_n = 2\pi\sqrt{\frac{I + I_{ar}}{k_r}}(s) \tag{8}$$

Where:

I = real inertial rolling motion
Iar = added inertial rolling motion
kr = rolling motion stiffness

The natural period for heaving motion for 'dry' arrangement is calculated as below:

$$T_n = 2\pi\sqrt{\frac{m}{k_z}}(s) \tag{9}$$

Where:

m = real mass
kz = spring stiffness for heaving motion

The natural period for rolling motion for ‘dry; arrangement is calculated as below:

$$T_n = 2\pi \sqrt{\frac{I}{k_r}}\,(s) \tag{10}$$

Where:

I = real inertial rolling motion
kr = rolling motion stiffness

5 RESULT AND DISCUSSION

5.1 Hydrostatic Stiffness

Hydrostatic stiffness is calculated and used as spring stiffness and adequately inputted as spring stiffness at each motion. The natural period of each motion is then calculated and discussed as below.

5.2 Heaving Motion Natural Period

Figure 5 below shows the natural period characteristics for each boundary condition arrangement. First of all, the hydrodynamic natural period is calculated for each D/H, represented by triangle dots. The natural period increases with the increase of the D/H. Then the rigid body motion is calculated. Fixed boundary condition gives very low natural period, which is near to zero, and considered as unrealistic boundary condition due to the very wide gap between the hydrodynamic and this boundary condition.

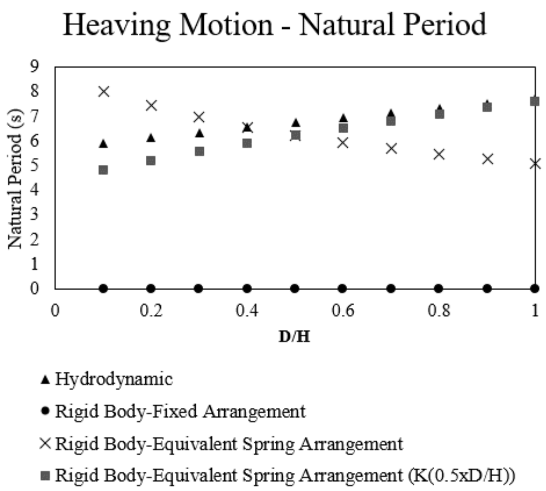

Figure 5: Heaving Motion Natural Period for Each Arrangement.

Equivalent spring motion natural period is calculated and presented at Figure 5 by the cross dots. The pattern of the equivalent spring natural period is rather different with the hydrodynamic one, but still in the same region. Maximum natural period for equivalent spring is at D/H=0.1, where the value is 8.034 s. Where the maximum natural period for hydrodynamics is D/H=1.0, the value is 7.683 s.

Adjustments is made to equivalent spring stiffness. In order to imitate the hydrodynamics properties, the spring stiffness is calculated by adding hydrostatic stiffness with the multiplication of the half ratio between diameter and height.

The value of the equivalent spring natural period is then matched with the hydrodynamics natural period, as explained by square dots at Figure 5. It turns out that by multiplying the hydrostatic stiffness with 0.5xD/T, the natural character of spring arrangement is similar to the hydrodynamic characteristic.

Similar natural period can be achieved by arranging the spring as shown at Figure 4 for the heaving motion of cylinder, by applying below equation for the spring stiffness:

$$K_{ez} = \frac{2\gamma\pi r^3}{2H} \tag{11}$$

Where:

K_{ez} = heave equivalent spring stiffness
γ = water specific weight
r = cylinder diameter
H = cylinder height

5.3 Rolling Motion Natural Period

Figure 6 below explains the natural period characteristics for every boundary condition arrangement. In the beginning, the hydrodynamic natural period is analysed for each D/H, symbolized by the triangular dots. Similar with heaving motion, the natural period increases with the increase of the D/H. The following result, which is rigid body motion, is calculated. Again, identical with heaving motion, fixed boundary condition gives very low natural period, which is near to zero. This boundary condition is considered as unrealistic due to the very wide gap between the hydrodynamic and fixed arrangement.

Equivalent spring motion natural period for rolling motion is calculated and presented at Figure 6 by the cross dots. The pattern of the equivalent spring natural period is similar with hydrodynamics motion but resulting rather higher period.

Maximum natural period for equivalent spring is at D/H=1.0, where the value is 6.344 s. Where the maximum natural period for hydrodynamics is D/H=1.0, the value is 2.006 s. It turns out that the difference between equivalent spring and hydrodynamic natural characteristic can be normalized by dividing the rolling natural period by the ratio between the diameter and the height.

The value of the equivalent spring natural period is then matched with the hydrodynamics natural period, as explained by square dots at Figure 6. It turns out that by dividing the rolling hydrostatic stiffness by D/T, the natural character of spring arrangement is similar to the hydrodynamic characteristic for rolling motion.

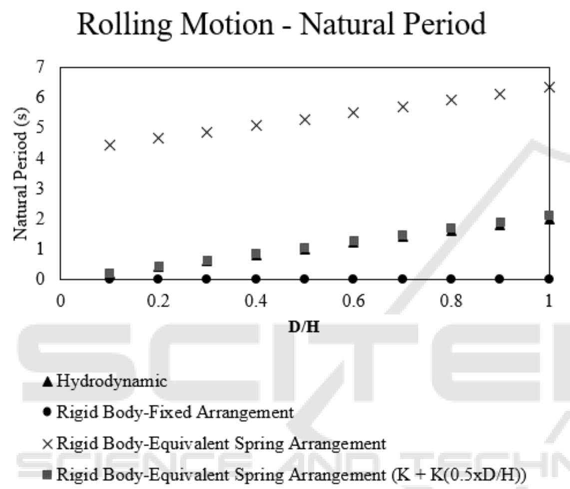

Figure 6: Rolling Motion Natural Period for Each Arrangement.

Similar natural period can be achieved by arranging the spring as shown at Figure 4 for the rolling motion of cylinder, by applying below equation for the spring stiffness:

$$K_{er} = \gamma \nabla \overline{\mathrm{GM}_T} (1 + \frac{r}{H}) \qquad (12)$$

Where:

K_{er} = roll equivalent spring stiffness
γ = water specific gravity
∇ = water displacement
$\overline{\mathrm{GM}_T}$ = metacenter height of cylinder
r = cylinder radius
H = cylinder height

6 CONCLUSION

After going with the explained procedures to create equivalent spring arrangement for heaving and rolling motion, especially to singular cylinder, we can draw conclusions as follow:

1. The value of the equivalent spring natural period matches with the hydrodynamics natural period by dividing the rolling hydrostatic stiffness by D/T the hydrodynamic characteristic for rolling motion.
2. For heaving motion, in order to imitate the hydrodynamics properties, the spring stiffness is calculated by adding hydrostatic stiffness with the multiplication of the half ratio between diameter and height.

7 FURTHER WORKS

The future works should refine the hydrostatic stiffness modelling by considering the ground spring height.

Further works also should develop more hydrostatic equivalent spring stiffness for more complex structure, e.g.: boxes, multiple cylinders, and ship shaped structures.

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REFERENCES

ABS, 2014. *Rules for Building and Classing Floating Production Installation.* Houston: American Bureau of Shipping.

Agamloh, E., Wallace, A. & Jouanne, A., 2008. Application of fluid–structure interaction simulation of an ocean wave energy extraction device. *Renewable Energy,* 33(4), pp. 748-757.

Bhattacharya, R., 1978. *Dynamics of Marine Vehicle.* New York: John Wiley & Sons.

Chen, L. et al., 2014. Numerical investigation of wave–structure interaction using OpenFOAM. *Ocean Engineering,* Volume 88, pp. 91-109.

Datta, T., 2010. *Seismic Analysis of Structures.* Singapore: John Wiley & Sons (Asia).

Gazetas, G., 1991. Foundation Vibration, Chapter 15. In: *Foundation Engineering Handbook.* New York: Van Nostrand Reinhold, pp. 553-593.

Huang, L. & Riggs, H., 2000. The hydrostatic stiffness of flexible floating structures for linear hydroelasticity. *Marine Structures,* 13(2), pp. 91-106.

Ma, S. & Mahfuz, H., 2012. Finite element simulation of composite ship structures with fluid structure interaction. *Ocean Engineering,* Volume 52, pp. 52-59.

Molland, A., ed., 2008. *The Maritime Engineering Reference Book: A Guide to Ship Design, Constructuion and Operation.* Oxford: Butterworth-Heinemann.

Okumoto, Y., Takeda, Y., Mano, M. & Okada, T., 2009. *Design of Ship Hull Structures: A Practical Guide for Engineers.* Berlin: Springer-Verlag Berlin Heidelberg.

Patel, M. & Witz, J., 1991. *Compliant Offshore Structures.* Oxford: Butterworth-Heinemann Ltd.

Raja, R., 2012. *Coupled fluid structure interaction analysis on a cylinder exposed to ocean wave loading (Master's Thesis).* Gothenburg: Chalmers University of Technology.

Sarpkaya, T., 2010. *Wave Forces on Offshore Structures.* Cambridge: Cambridge University Press.

Senjanović, I., Hadžić, N. & i Vladimir, N., 2011. Restoring Stiffness in the Hydroelastic Analysis of Marine Structures.. *Brodogradnja,* 62 (3), pp. 265-279.