# Learning Number in Early Childhood and Its Relationship with Mathematics Anxiety

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Abstract : Numbers and operations are basic mathematics materials that can be taught to young children. This material is very important because it is the basis for the development of the next mathematical abilities. Therefore, we need to developmental progression in this topic, so as to provide a strong mathematical foundation for children. A convenience sample of 20students was recruited from an elementary school in Malang, Indonesia. Out of 20 students, only 2 of them were chosen as the sample under the consideration that they are the most communicative.The interviews were semi-structured, covering main themes such as the number and their operations, with follow-up questions to ensure that the interviewer had interpreted the answers as intended.The tests were administered to students as individual interviews. The instrumentsused in this study weremathematics tasks and the mathematics anxiety questionnaire consistingof 20 items with five response choices in the form of face images.Students' answers to the mathematics task are analyzed based on developmental progressionin learning number and operations, that is:Number-after equals one more, Mentalcomparisons of close numbers, Number-after knowledge, Counting-based comparisons of collections larger than or equal with four, Meaningful object counting, and Subitizing (small-number recognition). The results of this study indicate that students who are incomplete in developmental progression have higher math anxiety scores while learning mathematics



## 1 INTRODUCTION

According to Regulation Minister of Education and Culture Republic of Indonesia Number 84, 2014 about early childhood education, Early Childhood Education an effort to educate newly born to six years children, is conducted by giving educational stimuli to help children grow & develop physically and spiritually so that the child has the readiness to enter further education. Therefore, mathematics learning at this stage needs special attention to prevent mathematics anxiety.It is because, anxiety might uppear during the childhood period, when change occurs rapidly when negative attitudes and anxiety occur at this age, it is usually persistent and difficult to change even when they are adult. There are some bad impacts of mathematics anxiety, that is math avoidance (Hembree, 1990), distress (Tobias, 1978; Buxton, 1981) and interference with conceptual thinking and memory processes (Skemp, 1986). Even for children, there appears to be a negative correlation between anxiety and achievement in mathematics (Hembree, 1990). Although this correlation may be indirect, it is often condisered that high levels of mathematics anxiety impair performance.

Many studies show the relationship between being good in early mathematics and achievement at the next school level (Claessens, et al, 2009; Geary et al, 2013; Sarama et al, 2012; Watts et al, 2014). In fact, early math skills became the strongest predictor of reading ability (Geary et al, 2013; Sarama et al, 2012; Watts et al, 2014) and mathematics (Clements, et al, 2014; Duncan et al 2011, Bernsteinet al, 2014).Unluckly, children with low skill in math have the tendency to be low achievers at the education level (Watts, et al, 2014; Duncan, et al, 2011; Siegler, et al, 2012)

Early mathematics refers to a variety of basic mathematical concepts for early childhood such as calculating in rounded nineteen; quantity (more, less, equal), shape (circle, square, triangle, rectangle); spatial relationships (more than, below); measurement (height, short, large, small, heavy, light); and patterns, both in the form of color patterns such as red, green, red, green, and image patterns (National Research Council, 2009;



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Sarama& Clements, 2009). In natural way such concepts of math will be explored by the children during their interaction with the environment (Sarama& Clements, 2009). For instance, when small children build beam towers, they learn mathematics by sorting the beams based on the size, color, or shape. This is very concerned about spatial relationships. In addition, these activities can also develop reasoning skills. In this case, preschoolers calculate or compare objects as they play, and find out similarities and differences in patterns and shapes (Seo and Ginsburg, 2004).

The general concern is that supporting early mathematics might mean taking away time from initial literacy. However, this does not have to be a problem. The development of initial mathematical skills and initial literacy are interrelated (IOM & NRC, 2015) and attempts to support both can occur simultaneously. In fact, when children learn mathematics along with other subjects they learn more math than if mathematics is taught in isolated way (NRC, 2009). Children learn mathematics and language in similar developments. Starting from infancy, language skills and literacy development with time when children build vocabulary, sentence length, and complexity of their sentences. Children learn how to building vocabulary, grammar mastery, and ability to produce longer and more complex sentences are the ways children use to learn how to express their ideas in words (Kipping et al, 2012).It is the same as the development phase in learning early mathematics. Firstly, they learn the basic terms in math, recognize the mathematics in tr surrounding world, and learn to use more complex mathematical concepts such as measurement, geometry, and reasoning (IOM & NRC 2015; Janzen, 2008), reading books, telling stories, and using "talking math" are an easy andeffective way to engage and improve math skills and early literacy development. Many books for children highlight mathematics in my ways.The purpose of this study was to describe the utterances of childhood when they are learning numbers and operations based on their developmental progression.

# 2 METHOD

A convenience sample of 20 children was recruited from an elementary school in Malang, Indonesia.We used participants from one school to ensure that students come from the samelearning environment. All participants returned signed consent forms and completed all interview assessment tasks.

Researchers chose to work with first graders because students ages 6–8 typically function at the uni-dimensional level (Case, 1996; Okamoto & Case, 1996). Students are considered at the uni-dimensional level because they are only able to use a single mental number line to solve numerical questions involving one dimension, such as adding ones. At the uni-dimensional level, they can determine that larger quantities, correspond to more motor movements in the tagging process (which are eventually done mentally and then not at all), to number names further up in the naming sequence, and to larger numerals, which are written further to the right on a number line. By counting up and down their mental number line, students can solve single-digit addition and subtraction problems, where addition (represented by the plus sign) results in more and subtraction (represented by the subtraction sign) results in less, and make judgments about the relative magnitudes of two single-digit numbers (Case, 1996; Griffin et al., 1995; Siegler&Ramani, 2008, 2009)

Out of 20 students, only 2 of them were chosen as the sample under the consideration that they are the most communicative. The interviews were semi-structured, covering main themes such as the number and their operations, with follow-up questions to ensure that the interviewer had interpreted the answers as intended. The interviews ranged between 20 and 30 minutes. All interviews were audio-taped and transcribed verbatim by the researcher. In this study, researchers served as interviewers.

The instrument in this study are mathematics tasks and the mathematics anxiety questionnaire contains 20 items with five response choices in the form of face images.Students' answers to the task are analyzed based on developmental progression in learning number and operations, that is: (L1) Number-after equals one more, (L2) Mental comparisons of close numbers, (L3) Number-after knowledge, (L4) Counting-based comparisons of collections larger than or equal with four, (L5) Meaningful object counting, and (L6) Subitizing (small-number recognition).

The tests were administered to students as individual interviews. Each studentwas interviewed outside of the classroom in a nearby multipurpose room using aset protocol. The student sat at a table facing the wall, and the interviewer sat nextto the student. We positioned a camera to capture the student’s gestures and work and placed an additional voice recorder near the student to pick up any audionot captured on the camera. After the student solved a problem, the interviewer asked, “How did you solve it?” to learn more about his/her strategies (Siegler, 1996). We did not give feedback on correctness but provided generic encouragement (e.g., “Great!”). For warm up, researcher ask about operations, like "*What is addition?*" If child is unsure, then ask "*What does it mean to add?*" *or "Whathappens when you add?*" Next, the researcher asked about the subtraction, "*What is subtraction?"*If the child is unsure**,** then ask**"***What does it mean to subtract?" or* "*Whathappens when you subtract?*"

**Table 1**
Interview Question

| Task | Questions |
|---|---|
| Counting backward (Griffin, 2004) | *Start at 10 and count backward as far as you can.* **If the child stops at 1:** *Can you count backward any further? Is there anything less than 1?* **If the child stops at 0:** *Can you count backward any further? Is there anything less than zero?* (Student answers) *How do you know?* **If the child stops at a positive number:** *Can you go any further?* **If the child indicates the numbers keep going:** *What would the last number be?* |

The counting task probed students' knowledge of thedecreasing verbal naming sequence. In this study, students were asked to start at "10" and count backwardas far as they could. The ordering task provided additional information on how studentsinterpret and order integers. This task also probed how students coordinated their integer value judgments with integer order because they then indicated which numbers were the least and greatest.

In this study, the math anxiety questionnaire of SEMA Questionnaire was translated into Indonesian (Wu, et al, 2012). The first tenquestions related to the mathematics curriculum and were used to measure the anxiety related to math problem solving. The other 10 questions were to assess the anxiety related to social and testing situations frequently faced the children while learning mathematics.

Two students were administered the measure individually in a one-on-one setting with an interviewer. All questions on the SEMA was presented on a piece of paper and also simultaneously read aloud by the interviewer. After each question, the children were asked to rate how anxious they felt. Ratings were made on a five-point faces response. Ratings were shown with graded anxious and non-anxious faces in order to assist the children in identifying their anxiety levels. Children answered by selecting one of the faces. The tester recorded the child's answer, making sure to ask for clarification if there was any obscurity. An individual's SEMA score was computed by summing the all items' ratings.

Figure 1.**Pictorial Rating Scale**
Source: Krinzinger et al. (2007)
Note: The type of questions are as follows: "How worried are you if you have problems with … ?" (*veryworried* to *very relaxed*). In this case, worried and relaxed faces representan anxiety.

The point of this study is an interest in how the students talk about number and operations. The students' verbally expressed experiences of number and operations. The analysis begins by examining how the students answer the questions about number and operations. Answers in which the students express their understanding were coded based on the stages of the developmental progression (Frye, 2013). Researchers explore whether six stages of specific developmental progression for number knowledge are complete or not. In this case, developmental progression is said to be incomplete when there are students who are at a stage without going through the previous stage. Furthermore, researchers found a relationship with mathematical anxiety caused by the stages that were missed in developmental progression. Due to the small-scale design, there will be no claims for generalization of the results. Data included students' written answers as well as transcripts of any verbal information they provided when solving themath problems.

# 3 RESULTS AND DISCUSSION

Initial experience with numbers and operations is very important to obtain more complex mathematical concepts and skills (NRC, 2009). This learning process depends on identifying the knowledge that children have and building that knowledge to help them take the next step. Developmental progression can help identify the next step by providing teachers with indicators that are appropriate for development to learn different skills. First, children can start a new step with a small number before moving to a larger number with the previous step (Ginsburg, et al, 1998). This can be seen from the following discussions:

*(Answer of S1)*
*Teacher : "I have an apple while Dani has two, who has more apples?"*

*S1 : "Dani"*
*Teacher : "Can you help me to buy apples so I can have more apples than Dani?"*
*S1 : "Mmmm, yes I can"*
*Teacher :" how many apples do I need to buy again?"*
*S1 : "an apple"*
*Teacher : "Oh yes, if I buy 1 apple, how many apples do I have now?"*
*S1 : "1 plus 1 to 2"*
*Teacher : "is my apple more than Dani's?"*
*S1 : "I don't think so, the apples are both two"*
*Teacher : "good, so buying 1 apple isn't enough, I need to buy 1 apple. So how many apples do I have now?"*
*S1 : "2 + 1 = 3"*
*Teacher : "Good"*

(Answer of S2)
*Teacher : "I have an apple, while Dani has two. Who's more?"*
*S2 : "Dani".*
*Teacher : "How do you know?"*
*S2 : "2 apples are the same as an apple we bought twice"*

S1 and S2 can give the correct answer, that is more Dani's Apples. In other words, S1 and S2 are able to compare which of the two numbers (1 and 2) are more. So that you can dismiss S1 and S2 in the first stage of developmental progression, namely number-after equals one more. This is not surprising because we know that children think mathematic matters in early age even before going to school.

Children show their understanding of different concepts of numeracy informally before they began to learn math at school (Song, et al, 1987; Sophian, et al, 1995). This is informal numeracy knowledge, that is, skills that children develop before starting school that does not depend on written mathematical notation (Purpura& Napoli, 2015). In this study, both S1 and S2's knowledge of numeracy indicates their counting skills development. It also proves the capacity to compare, share, order, estimate and calculate different quantities. In infancy, and innate basic skills in recognizing and responding to numerical cues and apparent (Wynn, 1995;Xu, et al, 2005).

Knowing what is 'more' and 'less' means to help children recognize the way to compare numbers. It is really important for the math developmental since children will be able to compare groups of objects and calculate them later. In this study, the important thing is that after children can mentally compare numbers and see that "2" is one more than "1" and that "3" is one more than "2." They can conclude that any number in the order of calculation is exactly one time more than the previous number. A child is ready for the next step when he recognizes, for example, that "five" is one more than "4." This can be seen from the following discussion:

*(Answer of S1)*
*Teacher :"3 and 2, which one is more?"*
*S1 : "3"*
*Teacher : How do you know?*
*S1 : "because in the number line, 2is written before number 3"*
*Teacher :" If I write number 3 before number 2, what do you think? is it still 3 greater than 2?"*
*S1 : "no, 2 is greater than 3"*

*(Answer of S2)*
*Teacher : "3 and 2, which one is more?"*
*S2 : "3"*
*Teacher : "How do you know?"*
*S2 : "Because 3 sequences are after 2"*
*Teacher : "How far is 3 to 2?"*
*S2 : "Not too far away, just one jump (laughing), I mean between 2 and 3 there are no other numbers"*

In the conversations above we can see that the S2 has passed the second stage (L2) and the third stage (L3) of the developmental progression. It is shown when S2 says numbers 2 and 3 are sequential, there are no other integers between them. Besides that, it is also indicated by the answer S2 which represents 2 apples equal to buying 1 apple 2 times.Meanwhile,S1 failed in the L2 and L3 stages. It appears that S1 is fixed on the position of numbers in the number line, and argues that the more to the right the number gets bigger, by forgetting the condition that the number must be written in sequence. Griffin (2004) has proposedcertain number sense content for the typical five-year-old child. According to her, knowing numbers in the counting sequence have a fixed position.

Once children recognize that counting can be used to compare collections and have the number-after knowledge, they can efficiently and mentally determine the larger of two adjacent or close numbers (e.g., that "9" is larger than "8"). A child has this knowledge when he or she can answer questions such as, "Which is more, seven or eight?" and can make comparisons of other close numbers.Once children can compare collections and have number-after knowledge, they can efficiently and mentally determine which is greater than 2 adjacent or near numbers (for example, that "9" is greater than "8"). A child has this knowledge when he can answer questions like, "Which is more, seven or eight?" And can make comparisons of other close numbers.

Familiarity with the order of counting allows a child to have number-after knowledge. For example, to enter a sequence at any time and specify the next number, not always counting from one. A child is ready for the next step when he can answer questions like, "What happens after 3?" By stating "3, 4" or just "4" instead of, say, counting "1, 2, 3, 4" At this stage, only S2 can enter this stage. While S1 cannot yet, so S1 still needs a better understanding of the number concept in the second stage.

After children can use small number recognition to compare small collections, they can use meaningful object counting to determine the larger of the two collections (for example, "7" items are more than "6" items because you have to count further). A child is ready for the next level when they shown two different collections. This can be seen from the following discussion.

*(Answei of S1)*
*Teacher :"9 books and 6 books, which one is more?"*
*S2 : "9 books is more"*
*Teacher : "How do you know?"*
*S2 : "Because when we mention numbers 1,2,3,4,5,6,7,8,9 we must call 6 first rather than 9 so 9 more than 6"*

*(Answer of S2)*
*Teacher :"9 books and 6 books, which one is more?"*
*S2 : "9 books is more"*
*Teacher : "How do you know?"*
*S2 : "yes because 9 means 9 steps from number 0 while 6 means 6 steps from number 0, so that 9 steps are farther than 6, so 9 is more than 6"*

The above conversation indicates that S1 and S2 are at stage (L4) Counting-based comparisons of collections larger than or equal with four and (L5) Meaningful object counting. In this case, meaningful object counting is counting in a one-to-one correspondence and recognizing that the last word used while counting is the same as the total (this is called the cardinality principle).

*(Answer of S1)*
*Teacher : (given 4 blocks) "How many?"*
*S2 : (she counts by pointing and assigning one number to each block) "1,2,3,4 …. So that the total is 4."*
*Teacher :" Next, can you count 4 + 2 = … ?"*
*S2 : "Wait, 1,2,3,4,5,6. So, the answer is 6".*

*(Answer of S2)*
*Teacher : (given 4 blocks) "How many?"*
*S2 : (she counts by pointing and assigning one number to each block) "1, 2, 3, 4…. So that the total is 4"*
*Teacher :"Next, can you count 4 + 2 = … ?"*
*S2 : "Of course, 4,5,6. So, the answer is 6".*

Aimprovement of the counting-all procedure is the counting-on procedure.In the counting-all procedure, the sum is found by counting the total number of entities which comprise the addeds. So in finding the sum of 4 and 2, S1 verbalized, "1,2,3,4,5,6". The counting-on procedure, however, is more sophisticated. Here the child begins the count with the number name whichrepresent the numerosity of one of the addeds. So in finding the sum of 4 and 2, S2 would verbalize, "4,5,6". While it is more effective to begin with the larger of the two addeds, and while indeed children can often be observed using this more effective procedure, others will begin the counting-on procedure from the first added, even if it is smaller.

Furthermore, S2 understanding of cardinality (the number of elements in a set or other grouping as property of that grouping) was found to be infrequently examined. However, with increasing age, children tend to spontaneously emphasize and repeat the last word in a count sequence (Cordes&Gelllman, 2005).

Subitization is the ability of young children to immediately recognize the total number of items in a collection. The next step is give the label them with the appropriate number words. When children are presented with many examples of different quantities (like 2 eyes, 2 socks, and 2 books) labeled with the same number word, not examples that are labeled with other numeric words (three pencils), children are in the process of making the right concept.The most frequent subitization that occurs in children is subitization of numbers 1, 5 and 10. This is because the two numbers are very familiar in the child's memory. Both, S1 and S2 represent one number by showing one finger (S1 and S2 use the index finger), and bends the other four fingers. Meanwhile, the representation of the number five is shown by S1 and S2 by opening his palm wide, where the five fingers are upright. The same thing is done by S1 and S2 when representing 10, namely by opening both of his palms and lifting all his fingers. Meanwhile the subitization process for other numbers also appears on S1 and S2 differently. This can be seen when S1 and S2 represent number 3, that is, by raising the three fingers on the right hand in the sequence, which are index finger, middle finger, and ring finger. The same thing also happens when S1 and S2 represent number 4, the fingers used are the four fingers of the right hand which is sequentially located, the little finger, ring finger, middle finger, and index finger.

In this section, we can conclude that the S1 developmental progression experienced by S1 is incomplete. Based on the results of the interview, we know that S1 only controls L1, L4, L5, L6. S1 failed in the L2 and L3 stages. It appears that the S1 is fixed on the number line, and the arguments that are the right number will be bigger, by forgetting the conditions that must be written in sequences. Meanwhile, S2 can do all the stages in developmental progression (L1-L6) completely

Based on the results of the interview, we know that developmental progression from S1 is incomplete, while S2 is complete. Furthermore, to explore their knowledge about numbers, researchers present the counting backwardtask(Griffin, 2004).

*(Answer of S1)*
*Teacher*: "*Start at 10 and count backward as far as you can!"*
*S1 : "10, 9, 8, 7, 6, 5, 4, 3, 2, 1"*
*Teacher: "Can you count backward any further? Is there anything less than 1?"*
*S1 : "I don't know"*

*(Answer of S2)*
*Teacher*: "*Start at 10 and count backward as far as you can!"*
*S2 : "10, 9, 8, 7, 6, 5, 4, 3, 2, 1"*
*Teacher: "Can you count backward any further? Is there anything less than 1?"*
*S2 : "mmm, zero maybe"*
*Teacher: "How do you know?"*
*S2 :"One means that there is something as much as one thing, but zero means that there is no object at all"*

In general, S1 and S2 can complete the backward count task properly. Even though S1 still doesn't understand numbers smaller than 1.While, S2 uses cardinality to argue about 0 which is smaller than 1. Because S2 can answer questions correctly, in this case it can be concluded that S2 has a mathematical performance in accordance with its developmental progression.

S1 has a mathematics anxiety score 60 of the maximum total score is 80 with details of 43 points coming from the first part of the questionnaire which contains questions related to mathematics anxiety related to academic. Meanwhile, the remaining 17 points are math anxiety in everyday life. So that S1 can be confirmed to have high math anxiety. Meanwhile the S2 has an anxiety score of 21 of the maximum total score of 80, with details of 11 points coming from the first part of the questionnaire, and 10 points from the second part of the questionnaire. So that it can be categorized as S2 having low mathematical anxiety. It might be that for primary school students the correlation between math anxiety and calculation performance is weak to nonexistent, for example because math anxiety may be more related to personality aspects such as general anxiety or because it might be very strongly mediated by teachers' or parents' attitudes (Stevenson et al., 2000).

# 4 CONCLUSION

Children demonstrate an interest in math well in early year of school. There are six stages in learn number and operations using developmental progression in early childhood, that is: (L1) Number-after equals one more, (L2) Mental comparisons of close or neighboring numbers, (L3) Number-after knowledge, (L4) Counting-based comparisons of collections larger than or equal with four, (L5) Meaningful object counting, L(6) Subitizing (small-number recognition). The first stage and the second stage can occur simultaneously, and allow for no sequential. Counting sequence understanding helps children to cope withmore complex problems. With good development of counting sense as a problem solving tool, children may use counting to compare size off two set count objects accurately without a need to touch the objects physically, use counting to solve problem of simple addition and subtraction and apply more complex counting strategies such as counting on from the larger set.

As a result of this study we know that S1 has incomplete developmental progression while a complete S2. But both can complete counting backward tasks well. However, math anxiety scores on S1 are much higher than S2. This means that in teaching mathematics to early childhood we cannot use orientation on academic results. But, focus on the child's process in understanding each concept in mathematics.

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