A Modified iMOEA/D for Many-objective Optimization Problems with Complicated Pareto Fronts

Ghizlane Aboulbaroud[1] and Driss Mentagui[2]
[1]*Applied Mathematics Laboratory, Faculty of Sciences, Ibn Tofail University, Kenitra, Moroccoo*
[2] *Dr Professor, Applied Mathematics Laboratory, Faculty of Sciences, Ibn Tofail University, Kenitra, Moroccoo*

Keywords: Evolutionary algorithm, many-objective optimization, decomposition, Pareto front, MOEA/D, IPBI approach.

Abstract: In real life, multiobjective evolutionary algorithms have many areas of applications, such as intelligence transportations systems, management problems, data mining, data-analysis and so on. Due to the importance of these problems, researchers have investigated several approaches to deal with them. Decomposition is one of the basic strategies used in multiobjective evolutionary optimization. In this paper, a modified iMOEA/D evolutionary algorithm based decomposition is suggested. This proposition allows dealing with Many-objective optimization problems with complicated Pareto fronts. The performance of this algorithm is demonstrated using a set of benchmark problems in comparison with other recently proposed algorithms.

1 INTRODUCTION

In the majority of real life problems, many objectives (often conflicting) need to be optimized simultaneously. In that case, the output is not a single optimal solution but rather a set of possible solutions called the optimal Pareto set.

Aggregating the multiple objectives into one objective function is the simplest method to deal with an optimization problem of a multiobjective nature. The most widely used method is the weighted sum method. However, the drawback of this approach is that it is not always possible to find the appropriate weighted function.

Multiobjective Optimization deals with such simultaneous optimization of multiple, possibly conflicting, objective functions, without combining them in a weighted sum. The set of solutions of a Multiobjective Optimization Problem (MOP) is composed of the parameter vectors, which cannot be improved in any objective without degrading in at least one of the objectives, and this set called the Pareto optimal set and its image in the objective function space is usually called the Pareto front (PF).

Multiobjective Evolutionary Algorithms have been recognized as the promising techniques for solving multiobjective optimization problems. As well as the domination based and the performance indicator based algorithms, the multiobjective evolutionary algorithms based on decomposition (MOEA/D) (Zhang et al., 2014) have been widely used and investigated recently and they have shown its effectiveness. In MOEA/D, a MOP is decomposed into single objective optimization sub-problems and then solved in a single run. The objective function in each sub-problem can be a linear or nonlinear weighted aggregation function of all the objective functions in the concerned MOP. The main used approaches for converting MOP into scalar sub-problems are: Weighted Sum (WS) Approach, Tchebycheff (TCH) Approach and Boundary Intersection (PBI) Approach.

These approaches have been widely detailed in literature (Trividi et al., 2016). MOEA based decomposition have shown its effectiveness in the real-world applications.

Many-objective optimization (MaOPs) problems (four or more number of objectives) are currently a subject of great interest for the scientific research

Aboulbaroud, G. and Mentagui, D.
A Modified iMOEA/D for Many-objective Optimization Problems with Complicated Pareto Fronts.
DOI: 10.5220/0009774000940101
In *Proceedings of the 1st International Conference of Computer Science and Renewable Energies (ICCSRE 2018)*, pages 94-101
ISBN: 978-989-758-431-2

community. A great number of algorithms have been developed to solve this class of problems. In (Trividi et al., 2016), a summary of studies on MOEAs based on decomposition for many-objective optimization is given.

In this article, we propose an improved MOEA/D which deals with many-objective optimization problems with complicated Pareto fronts. The developed algorithm is based on recent research papers (Hohuu et al., 2018) and (Zhang et al., 2014).

Our main contributions include the following aspects:

Idea 1: We adopt the PBI approach in the first phase for its effectiveness in handling Maops.

Idea 2: we adopt the inverted PBI scalarizing approach to deal with problems with complicated Pareto fronts.

This paper is organized as follows: section 2 summarizes the basic concepts and defintitions related to evolutionary algorithms based decomposition framework and introduces related works. Section 3 presents our detailed algorithm. Analysis and discussion are shown in section 4 followed by conclusion.

2 PRELIMNINARIES AND RELATED WORKS

In this section, basic definitions and concepts are presented and related works are introduced

2.1 Basic Definitions

Definition 1: Many-objective Optimization Problem

A Many-objective Optimization Problem MaoP (1) can be formulated as:

$$\min F(x) = (f_1(x), \dots, f_m(x))^T$$

$$\mathit{Subject\ to}\ \ x \in \Omega \qquad (1)$$

Where $x = (x_1, \dots, x_n)$ is the decision variables vector, Ω is the search space and m is the number of objective functions.

Definition 2: Domination

We say that a solution x dominates a solution y if and only if x is better than or equal to y in all objectives and better than y in at least one.

predefined by the user. d_1 is used to measure convergence and d_2 is used to measure diversity. A solution with small d_1 and d_2 is considered as a better solution close to the Pareto front.

Another decomposition approach is introduced. This method is used in our proposed algorithm.

- The IPBI-Approach

Traditional decomposition approaches face difficulty in approximating widely spread PF in some problems like MOKPs (Sato, 2015). To deal with this problem and to conceive a decomposition method performant for many-objective optimization, inverted PBI (IPBI) decomposition method is proposed (Sato, 2015). In the conventional decomposition methods such as the TCH and the PBI, solutions are evolved towards the reference point z by minimizing the scalarizing function value. However, in the IPBI approach, solutions are evolved from the nadir point by maximizing the scalarizing function value. The experiments on MOKPs and WFG4 problem (Sato, 2015), with 2-8 objectives, demonstrated that the IPBI approach can better approximate widely spread PF in comparison to other scalarizing approaches.

The expression of the Inverted Penalty Boundary Intersection is given by:

$$\mathit{maximize}\ g_{ipbi}(x|\lambda) = d_1 - \theta d_2 \qquad (6)$$

Where

$$d_1 = \frac{\left\| \left(z^{nad} - F(x)\right)^T \lambda \right\|}{\|\lambda\|} \qquad (7)$$

$$d_2 = \left\| z^{nadir} - F(x) - d_1 \frac{\lambda}{\|\lambda\|} \right\|$$

MOEA/D has been extended to several variants using different decomposition approaches. The MOEA/D has shown its effectiveness to outperform NSGAII (Deb et al., 2002) and other existing algorithms based decomposition (Zhang et al., 2007).

In (Jiang et al., 2016), Jiang et Yang have proposed MOEA/D-TPN to solve problems with complex Pareto fronts. A two-phase strategy is adopted. The TP strategy, which conditionally divides the whole optimization process into two phases and the niche-guided strategy, which helps maintain the population diversity. The detail of this algorithm is given in (Jiang et al., 2016).

In MOEA/D-AWA (Qi et al., 2014), Qi et al. proceed in two stages strategy. In the first stage, a set of weight vectors are used until the algorithm converge to a certain extent. Then, to handle MOPs with complex Pareto fronts an adaptive weight vectors adjustment strategy is adopted.

The NSGA III algorithm proposed in (Deb et al., 2014) is an improved version of NSGA II framework. This algorithm can deal with many-objective optimization problems using reference points to implicitly decompose the objective space and a niche preservation operator to increase diversity of solutions close to every reference point. The studies prove that NSGA III performs well than MOEA/D-PBI and MOEA/D-TCH (Deb et al., 2014). More detail is presented in (Deb et al., 2014).

RVEA (Cheng et al., 2016) is another algorithm for solving many-objective optimization problems. Cheng et al. adopt a reference vectors reconstruction strategy and use a new scalarizing approach, namely angle-penalized distance (APD). RVEA can deal with MOPS with irregular Pareto Fronts and can guarantee a uniform distribution of the reference vectors. However, RVEA is unable to handle Pareto fronts with long tails or sharp peaks (Cheng et al., 2016).

Algorithm 1: iMOEA/D

Input: A MOP, subsets of odd and even weight vectors, N subproblems,
Phase (1)
Initialization
Decomposition using the Tchebycheff function with ideal point and the subset of the odd-weight vectors.
Update
Stopping criteria and output PS1 and PF1
Phase (2)
Initialization
Decomposition using the Tchebycheff function with the z nadir and the subset of the even-weight vectors.
Update
Stopping criteria and output PS2 and PF2
PS=$\{PS1, PS2\}$ Pareto set
PF=$\{PF1, PF2\}$ Pareto front

iMOEA/D Concept

Combining the ideal point $z^*$and nadir point z^{nadir} in Tchebycheff functions was reported as an effective way to get a good distribution of optimal solutions over a Pareto front. This combination allows dealing with multiobjective optimization problems characterized by complex fronts (Zhou et al., 2017). Based on this, in (Jiang et al., 2016), Jiang have designed an evolutionary algorithm proceeding in two phases. Where the Tchebycheff function with z^* is employed in the first phase and the Tchebycheff function with z^{nadir} is used in the second phase. The second phase will only be executed if a condition on the first phase is satisfied. This strategy presents limitations that have been overcome by Ho-huu in (Hohuu et al., 2018). These limitations concern the difficulty of setting a number of evaluations to pass from phase 1 to phase 2 and the computational cost of solving a multiobjective optimization problems if phase 2 is executed. In (Hohuu et al., 2018), Ho-huu proposed an improved MOEA/D (iMOEA/D) to deal with MOP with complex fronts and to overcome the limitations already mentioned. A new two phase strategy is proposed. This strategy consists of dividing the weights vector into two subsets: odd weight vectors and even weight-vectors. In the first phase, the population of the first subset is optimized using the Tchebycheff function with the ideal point z^*. The Tchebycheff function with the nadir point is applied for the second subset. The z^{nadir} is obtained from the set solutions found in the first phase. The algorithm proposed in (Hohuu et al., 2018) has shown its effectiveness and competitiveness than MOEA/D, MOEAD/TPN (Jiang et al., 2016) and NSGA II, through many test functions with complicated Pareto fronts. However, it is limited to bi-objective optimization problems and cannot deal with problems with more than 2 objective functions. The pseudo-code of iMOEA/D (Ho-huu, 2018) is given in Algorithm 1.

The iMOEA/D version includes some recent developments related to MOEA/D which are an adaptive replacement strategy (Zhang et al., 2009) and a stopping criterion introduced in (Baskar et al., 2016).

In the following section, we describe our proposed algorithm.

Algorithm 2 Main algorithm

Input:
A multiobjective optimization problem MOP,
N number of sub-problems
A StoppingCriterion
$\boldsymbol{w}^{odd} = (\boldsymbol{w}_1^i, \dots, \boldsymbol{w}_m^i)^T, \boldsymbol{i} = \mathbf{1}, \mathbf{3}, \dots, \boldsymbol{N}$: a set of odd-weight vectors;
$\boldsymbol{w}^{even} = (\boldsymbol{w}_1^i, \dots, \boldsymbol{w}_m^i)^T, \boldsymbol{i} = \mathbf{2}, \mathbf{4}, \dots, \boldsymbol{N} - \mathbf{1}$: a set of even-weight vectors;
$\boldsymbol{T_m}$: size of mating neighborhood;
$\boldsymbol{T_{rmax}}$: maximum size of replacement neighborhood;
δ: the probability for selection the mating parents from the neighborhood;
$\boldsymbol{MaxIter}$: maximum iteration;
Output: Approximation to the PF
Phase (1)
Initialization
Set $\boldsymbol{P_{01}}$ the initial population, $\boldsymbol{z}^*$the ideal point and $\boldsymbol{w}^{odd}$ and $\boldsymbol{w}^{even}$
Set $\boldsymbol{T_m}$ $\boldsymbol{and}$ $\boldsymbol{T_{rmax}}$: size of mating neighborhood and maximum size of replacement neighborhood

Set $\boldsymbol{B^i} = \{i_1, \dots, \boldsymbol{i_{Tm}}\}$ as mentioned in (Zhang et al., 2014).
Decomposition using PBI scalarizing function and $\boldsymbol{w}^{odd}$ with $\boldsymbol{z}^*$.
Solution building: a solution is generated using the 'DE/rand 1' operator
Update solution
Stopping criterion and output PS1 and PF1
Phase (2)
Initialization
Define $\boldsymbol{z}^{nad}$ with $\boldsymbol{z}_j^{nadir} = \max\{\boldsymbol{f_j}(\boldsymbol{x}) | \boldsymbol{x} \in \boldsymbol{\Omega}, \boldsymbol{j} = \mathbf{1}, \dots, \boldsymbol{m}\}$
$\boldsymbol{f_j}(\boldsymbol{x}) \in \boldsymbol{PF_1}$
Set the initial population $\boldsymbol{P_{02}} = \boldsymbol{PS_1}$
Decomposition using inverted PBI scalarizing function and $\boldsymbol{w}^{even}$ with $\boldsymbol{z}^{nad}$
Solution building
Update
Termination criterion and output PS2 and PF2
Output
PS={PS1,PS2}
PF={PF1,PF2}

3 PROPOSED ALGORITHM

This paper is a modified version of the iMOEA/D algorithm proposed by (Ho-huu et al, 2018). The said algorithm handles bio-objective optimization problems with complicated Pareto fronts. To extend the scope of this algorithm and make it suitable for Many-objective optimization problems, we propose a modified iMOEA/D which deals with many-objective optimization problems (MaOP) characterized by a complex Pareto front.

In our algorithm, we use both the PBI and the inverted PBI scalarizing approaches.

We proceed in two phases. In the first phase we run our algorithm using the PBI approach with the set of the odd-weight vectors and the ideal point z^*.

In the second phase, the Inverted PBI approach is applied with the set of the even-weight vectors and z^{nad}. The $z^{nadir}$is determined using the solutions obtained from the first phase.

Reasons behind using the PBI and the inverted PBI approaches:

The Penalty Boundary Intersection is widely applied in MOEA/D. In most cases, a uniform distribution of weight vectors in PBI approach will outcome a set of evenly distributed solutions on the Pareto-optimal front (POF). The PBI-approach has shown its performance for solving many-objective optimization problems and to handle problems with complex Pareto fronts, we apply the inverted PBI approach in the second phase.

4 EXPERIMENTS AND RESULTS

In this section, we test our algorithm along with a set of well-known algorithms including NSGA III, RVEA and MOEA/D-PBI.

4.1 Test Problems

We use DTLZ1, DTLZ2, DTLZ3 and DTLZ4 to test the ability of our algorithm to deal with more than 2 objectives. Table 1 describes the test instances, their variable domains and instance characteristics used in this work.

4.2 Parameters Setting

The population size in each algorithm is set to 800 for all test instances. The maximum number of generations was set to 400 for all test instances. For the MOEA/D-PBI and our proposed algorithm, θ is set to 5.

4.3 Results and Discussion

The inverted Generalized Distance (IGD) Indicator is used to indicate both the convergence and the diversity of our algorithm. The table 2 shows that our proposed algorithm M-iMOEA/D could perform well on all of the test instances especially on DTLZ1 and DTLZ4.

5 CONCLUSIONS

In this paper, we develop a modified version of iMOEA/D (Hohuu et al., 2018) named (M-iMOEA/D) for solving MaOPS with complicated Pareto fronts. In M-iMOEA/D, we adopt a two phase strategy. In the first strategy, the set of the odd-weight vectors is selected to be optimized using the PBI approach with the ideal point z^*. In the second phase, the Inverted-PBI approach is applied with the set of even-weight vectors and z^{nad} which is determined from the set of the obtained solutions of the first stage. Our algorithm shows its performance than other algorithms in problems with many-objectives and complicated Pareto fronts by using a set of benchmark problems.

REFERENCES

V. Ho-Huu, S. Hartjes, H. G. Visser, and R. Curran, « An improved MOEA/D algorithm for bi-objective optimization problems with complex Pareto fronts and its application to structural optimization », Expert Systems with Applications, vol. 92, p. 430‑446, feb. 2018.

Y. Zhou, Y. Xiang, Z. Chen, J. He, and J. Wang, « A Scalar Projection and Angle-Based Evolutionary Algorithm for Many-Objective Optimization Problems », IEEE Transactions on Cybernetics, p. 1-12, 2018.

S. Jiang and S. Yang, « An Improved Multiobjective Optimization Evolutionary Algorithm Based on Decomposition for Complex Pareto Fronts », IEEE Transactions on Cybernetics, vol. 46, no 2, p. 421-437, Feb. 2016.

A. Trivedi, D. Srinivasan, K. Sanyal, and A. Ghosh, « A Survey of Multiobjective Evolutionary Algorithms based on Decomposition », IEEE Transactions on Evolutionary Computation, p. 1-1, 2016.

Xinye Cai, Yexing Li, Zhun Fan, and Qingfu Zhang, « An External Archive Guided Multiobjective Evolutionary Algorithm Based on Decomposition for Combinatorial Optimization », IEEE Transactions on Evolutionary Computation, vol. 19, no 4, p. 508‑523, Aug 2015.

M. Nasir, A. K. Mondal, S. Sengupta, S. Das, and A. Abraham, « An improved Multiobjective Evolutionary Algorithm based on decomposition with fuzzy dominance », 2011, p. 765‑772.

C. A. C. Coello, « An updated survey of evolutionary multiobjective optimization techniques: state of the art and future trends », 1999, p.3‑13.

H.-L. Liu, F. Gu, and Q. Zhang, « Decomposition of a Multiobjective Optimization Problem Into a Number of Simple Multiobjective Subproblems », IEEE Transactions on Evolutionary Computation, vol. 18, no 3, p. 450‑455, June 2014.

Qingfu Zhang and Hui Li, « MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition », IEEE Transactions on Evolutionary Computation, vol. 11, no 6, p. 712‑731, dec. 2007.

H. Sato, « Analysis of inverted PBI and comparison with other scalarizing functions in decomposition based MOEAs », Journal of Heuristics, vol. 21, no 6, p. 819-849, dec. 2015.

M. Asafuddoula, T. Ray, and R. Sarker, « A Decomposition-Based Evolutionary Algorithm for Many Objective Optimization », IEEE Transactions on Evolutionary Computation, vol. 19, no 3, p. 445-460, June 2015.

H. Li, Q. Zhang, « Multiobjective Optimization Problems with Complicated Pareto Sets, MOEA/D and NSGA-II », IEEE Transactions on Evolutionary Computation, vol. 13, no 2, p. 284-302, apr. 2009.

K. M. Abdul Kadhar and S. Baskar, « A stopping criterion for decomposition-based multi-objective evolutionary algorithms », Soft Computing, vol. 22, no 1, p. 253-272, janv. 2016.

K. Deb, A. Pratap, S. Agarwal and T. Meyaruvan , «A fast and elitist multiobjective genetic algorithm: NSGA-II» Evolutionary Computation, IEEE transactions on Evolutionary Computation, vol. 6, no. 2, p. 182-197, Apr 2002.

E. Zitzler, M. Laumanns and L. Thiele, « SPEA II, Improving the strength Pareto Evolutionary algorithm for multiobjective optimization » in Evolutionary Methods dor Design, Optimization, Control. CIMNE, p. 95-100, Spain 2002.

Y. Qi, X. Ma, F. Liu, L. Jiao, « MOEA/D with adaptive weight adjusment, Evolutionary Computation. P. 231-264, 2014.

K. deb and H. Jain « An evolutionary many-objective optimization algorithm using reference point based non-dominated sorting approach, Part I: Solving problems with box constraints », IEEE Transactions on Evolutionary Computation, Vol. 18, no. 4, p. 577-601, 2014.

R. Cheng, Y. Jin, M. Olhofer and B. Sendhoff, « A reference vector guided evolutionary algorithm for many objective optimization », IEEE Transactions on Evolutionary Computation, no. 99, p1-1, 2016.

Z. Wang, Q. Zhang, M. Gong andA. Zhou, « A replacement strategy for balancing convergence and diversity in MOEA/D», Evolutionary Computation, IEEE Trans, p. 2132-2139, Jul, 2014.

K. Deb, L. Thiele, M. Laumanns, E. Zitzler, « Scalable test problems for multiobjective optimization, Springer, 2005.

Table 1: Benchmark problems: DTLZ1, DTLZ2, DTLZ3, DTLZ4

Problem	Objective function	Domain	Characteristics
DTLZ1	$f_1(x) = 0.5x_1x_2 \dots x_{m-1}(1+g(x_m))$ $f_2(x) = 0.5x_1x_2 \dots (1-x_{m-1})(1+g(x_m))$ . . $f_{m-1}(x) = 0.5x_1(1-x_2)(1+g(x_m))$ $f_m(x) = 0.5(1-x_1)(1+g(x_m))$ $where\ g(x_m) = 100[\lvert x_m\rvert + \sum_{x_i \in x_m} ((x_i-0.5)^2 + \cos(20\pi(x_i-0.5)))]$	$[0,1]^n$	Linear, multimodal
DTLZ2	$f_1(x) = \cos\left(\frac{x_1\pi}{2}\right) \dots \cos\left(\frac{x_{m-2}\pi}{2}\right)\cos\left(\frac{x_{m-1}\pi}{2}\right)(1+g(x_m))$ $f_2(x) = \cos\left(\frac{x_1\pi}{2}\right) \dots \cos\left(\frac{x_{m-2}\pi}{2}\right)\sin\left(\frac{x_{m-1}\pi}{2}\right)(1+g(x_m))$ . . . $f_{m-1}(x) = \cos\left(\frac{x_1\pi}{2}\right)\sin\left(\frac{x_2\pi}{2}\right)(1+g(x_m))$ $f_m(x) = \sin\left(\frac{x_1\pi}{2}\right)(1+g(x_m))$ $where\ g(x_m) = \sum_{x_i \in x_m} (x_i-0.5)^2$	$[0,1]^n$	Concave
DTLZ3	$f_1(x) = \cos\left(\frac{x_1\pi}{2}\right)\cos\left(\frac{x_2\pi}{2}\right) \dots \cos\left(\frac{x_{m-2}\pi}{2}\right)\cos\left(\frac{x_{m-1}\pi}{2}\right)(1+g(x_m))$ $f_2(x) = \cos\left(\frac{x_1\pi}{2}\right)\cos\left(\frac{x_2\pi}{2}\right) \dots \cos\left(\frac{x_{m-2}\pi}{2}\right)\sin\left(\frac{x_{m-1}\pi}{2}\right)(1+g(x_m))$ . . . $f_{m-1}(x) = \cos\left(\frac{x_1\pi}{2}\right)\sin\left(\frac{x_2\pi}{2}\right)(1+g(x_m))$ $f_m(x) = \sin\left(\frac{x_1\pi}{2}\right)(1+g(x_m))$ $where\ g(x_m) = 100[\lvert x_m\rvert + \sum_{x_i \in x_m} ((x_i-0.5)^2 - \cos(20(x_i-0.5)))]$	$[0,1]^n$	Concave, multimodal

DTLZ4	$f_1(x) = \cos\left(\frac{x_1^\alpha \pi}{2}\right) \cos\left(\frac{x_2^\alpha \pi}{2}\right) \dots \cos\left(\frac{x_{m-2}^\alpha \pi}{2}\right) \cos\left(\frac{x_{m-1}^\alpha \pi}{2}\right) \left(1 + g(x_m)\right)$ $f_2(x) = \cos\left(\frac{x_1^\alpha \pi}{2}\right) \cos\left(\frac{x_2^\alpha \pi}{2}\right) \dots \cos\left(\frac{x_{m-2}^\alpha \pi}{2}\right) \sin\left(\frac{x_{m-1}^\alpha \pi}{2}\right) \left(1 + g(x_m)\right)$ . . . $f_{m-1}(x) = \cos\left(\frac{x_1^\alpha \pi}{2}\right) \sin\left(\frac{x_2^\alpha \pi}{2}\right) \left(1 + g(x_m)\right)$ $f_m(x) = \sin\left(\frac{x_1^\alpha \pi}{2}\right) \left(1 + g(x_m)\right)$ $where\ g(x_m) = \sum_{x_i \in x_m} (x_i - 0.5)^2 . \alpha = 100$	$[0,1]^n$	Concave, biased

Table 2: the obtained IGD average values obtained for DTLZ1-DTLZ4 test problems

		M-iMOEA/D	NSGA-III	RVEA	MOEA/D-PBI
DTLZ1	3	**3.0001e-02**	3.0938e-01	5.0488e-01	3.4647e-02
	4	5.4290e-02	7.7805e-02	1.2131e-01	**5.4289e-02**
	5	**6.5003e-02**	5.6473e-01	3.4217e-01	6.5954e-02
DTLZ2	3	5.4769e-02	5.4920e-02	5.8780e-02	**5.4643e-02**
	4	**1.4009e-01**	1.4090e-01	1.4072e-01	1.1412e-01
	5	**2.1905e-02**	2.1626e-02	2.1396e-01	2.1318e-01
DTLZ3	3	**1.7896e-01**	1.0704e+01	8.8125e-00	1.8099e-01
	4	**1.7969e-01**	1.6565+01	1.8081e-01	1.8020e-01
	5	1.1060e-01	9.8374e-00	**1.0358e-01**	1.1361e-01
DTLZ4	3	5.3936e-02	5.7025e-02	**5.255e-02**	5.4151e-01
	4	**1.4078e-01**	1.4227e-01	1.4089e-01	7.5111e-01
	5	**2.1101e-01**	2.1833e-01	2.1487e-01	6.4572e-01

Figure 1: Parallel cordinates of PFs obtained by four algorithms :(1) RVEA, (2) NSGAIII, (3), MOEAD-PBI, (4) M-iMOEAD