Stability Analysis of Cycle Business Investment Saving-Liquidity Money (IS-LM) Model using Runge-Kutta Fifth Order and Extended Runge-Kutta Method

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Abstract: Forecasting the future conditions of economic stability can be illustrated in business cycle model. The IS-LM business cycle model is a business cycle model which is represented in a differential equation involving investment function (I), saving function (S), and money demand function (L). In this research, stability analysis on the IS-LM business cycle model was performed. Stability analysis was performed by Routh-Hourwitz criteria. After that, numerical simulation would be performed by comparing the Runge-Kutta method of fifth order and Extended Runge-Kutta to determine the stability speed of the model. This research resulted fixed point IS-LM model that is $E = (Y^*, R^*, K^*) = (0.0979, 0.0097, 0.0351)$. Through data simulations, it was obtained numerical solution with the Extended Runge-Kutta method that has faster stable result, with $t = 25$, compared with the Runge-Kutta Fifth Order method that is stable when $t = 36$.

1 INTRODUCTION

Indonesia is one of the countries that has experienced economic crisis since the 1990s. Until now, the state of the economy has been fluctuating and tends to be unstable. One example of economic growth in Indonesia in 2018 was overshadowed by a slowdown compared to previous year. As for information, in 2017 Indonesia economic growth reached 5.07% or lower than the growth of neighboring countries, such as Malaysia which has 5.8% growth (Angriani, 2018). This situation requires Indonesia to develop economic forecasting in order to know the future condition of the economy.

Forecasting future economic conditions can be illustrated in the business cycle models, one of which is the IS-LM (Investment Saving-Liquidity Money) business cycle model. The IS-LM business cycle model is a system of differential equations involving the investment function (I), the saving function (S), and the money demand function (L) (Dwiningtias & Abadi, 2014).

The IS-LM business cycle model is a business cycle model that is represented in the form of a system of differential equations (Dwiningtias & Abadi, 2014). The method used in solving differential equation can be obtained by analytical or numerical method. However, the weakness of the analytical method is that not all mathematical equations can be solved to produce the exact value and the method takes a very long time in the process of work (Alfaruqi, 2010). Therefore, numerical methods are needed as alternative to analytical methods. The numerical methods in question include Taylor series method, Euler method, Runge-Kutta method, and Heun method. Meanwhile, methods that include many steps are Adam-Bashforth-Moulton method, Milne-Simpson method, and Hamming method.

Among these methods, researchers are more interested in using the fifth order Runge-Kutta method and the Extended Runge-Kutta Method to look at the behavior of dependent variables that affect the business cycle model. The advantage of the Runge-Kutta method is that it does not use derivatives in its process (Muhammad, 2015). In addition, the

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higher the order is used, the greater the level of accuracy produced.

This research is interesting because the IS-LM cycle model will be stability analysis of the model and model simulation is performed to show the behavior of variables affecting IS-LM business cycle model in graphic form. In addition, a comparison of the fifth order Runge-Kutta method and the Extended Runge-Kutta Method will be used to simulate the model that has been obtained with the stability speed of the model.

This research will be stability analysis of IS-LM business cycle model with variable of income rate, rate of interest, and capital stock rate. This model is a modified model of the business cycle model of Gabisch and Lorenz (1987) by substituting investment, saving, and money demand functions. Model stability analysis was performed by determining fixed point which then performed fixed point stability analysis with Routh-Hurwitz criteria followed by determining stability time through Runge-Kutta method of order five and Extended Runge-Kutta.

2 THEORITICAL FRAMEWORK

2.1 Business Cycle Model

The IS-LM model is one of the models in the field of macroeconomics. The IS-LM business cycle model involves investment (saving) function, saving function on goods market and Liquidity preference, and Money supply in money market (Rosmely, et al., 2016).

The business cycle model was first introduced by Kalecki (1935) and Kaldor (1940) in the form of a system of differential equations, ie.:

$$\frac{dY}{dt} = \alpha(I\big(Y(t), K(t)\big) - S(Y(t), K(t))$$
$$\frac{dK}{dt} = (I\big(Y(t), K(t)\big) \qquad (1)$$

with $\frac{dY}{dt}$ is the rate of income, $\frac{dK}{dt}$ is the rate of capital stock, $I\big(Y(t), K(t)\big)$ is an investment function that relies on income and capital stock, $S(Y(t), K(t))$ is a saving function that depends on income and capital stock, and $\alpha > 0$ is the acceleration due to the excess or lack of investment.

In 1977, Torre changed the model (1) by replacing the capital stock (K) variable with the interest rate variable (R). The business cycle is modelled as follows:

$$\frac{dY}{dt} = \alpha(I\big(Y(t), K(t)\big) - S(Y(t), R(t))$$
$$\frac{dR}{dt} = \beta(L(Y(t), R(t)) - M) \quad (2)$$

with $\frac{dR}{dt}$ is the interest rate, $L\big(Y(t), R(t)\big)$ is a money demand function that depends on income and interest rates, $\beta > 0$ is the acceleration caused by the lack or excess demand for money, and $M > 0$ is the constant supply of money.

Then in 1987, Gabisch and Lorenz added capital stock variables (K) into the business cycle model (2), so that it becomes a business cycle model

$$\frac{dY}{dt} = \alpha\big[I(Y(t), K(t), R(t)) - S\big(Y(t), R(t)\big)\big]$$
$$\frac{dR}{dt} = \beta\big[L\big(Y(t), R(t)\big) - M\big] \qquad (3)$$
$$\frac{dK}{dt} = I[Y(t), K(t), R(t)] - \delta K(t)$$

with $\delta > 0$ is the capital depreciation constant.

2.2 Function I,S, and L

In 2005, Cai provided assumptions on the investment function (I), saving function (S) and money demand functions (L). The amount of investment (I) is linearly dependent on the difference between income (Y) subtracted by capital stock (K) and interest rate (R). Meanwhile, the saving function (S) depends on the sum of income (Y) with interest rate (R). Money demand functions (L) depends on the difference between income (Y) and interest rate (R). All three functions can be denoted as follows:

$$I(Y, K, R) = \eta Y - \delta_1 K - \beta_1 R$$
$$S(Y, R) = l_1 Y + \beta_2 R \qquad (4)$$
$$L(Y, R) = l_2 Y - \beta_3 R$$

Annotation,
$\eta =$ growth rate of investment to income,
$\delta_1 =$ the rate of decline in investment on capital stock,
$l_1 =$ growth rate of savings to income,
$l_2 =$ the growth rate of money demand for income,
$\beta_1 =$ the rate of decline in investment on interest rates,
$\beta_2 =$ the growth rate of savings on interest rates,
$\beta_3 =$ the rate of decline in demand for money against interest rates,
where $\eta, \delta_1, l_1, l_2, \beta_1, \beta_2, \beta_3$ are positive constants in the interval [0,1].

2.3 Matrix Jacobi

In searching for stability analysis, it is necessary to have a characteristic equation on differential equations constructed from a Jacobi matrix.

Given functionality $f = f_1, f_2, f_3, \ldots, f_n$ in system $x = f(x)$ with $f_i \in C(E), i = 1,2,\ldots,n$.

$$Jf\big((x)\big) = \begin{bmatrix} \frac{df_1}{dx_1}(x) & \cdots & \frac{df_1}{dx_n}(x) \\ \vdots & \ddots & \vdots \\ \frac{df_n}{dx_1}(x) & \cdots & \frac{df_n}{dx_n}(x) \end{bmatrix} \quad (6)$$

Matrix above is called the Jacobian matrix of f at point (Hale & Kocak, 1991).

2.4 Eigen Value

If A is a matrix n x n, then a nonzero vector x in R^n called the eigenvector of A if Ax is a scalar multiple of x, i.e.

$$Ax = \lambda x \quad (7)$$

For any scalar λ, scalar λ is called the eigenvalue of A, and x is called the eigenvector of A which is related λ (Rosmely, Nugrahani, & Sianturi, 2016).

To obtain the eigenvalues of a matrix A_{nxn}, equation (7) can be rewritten as

$$Ax = \lambda I$$

or equivalent to

$$\lambda I - A\,x = 0 \quad (8)$$

So λ can be an eigenvalue. There must be a non zero solution of the equation (8). Equation (8) has a non zero solution if and only if

$$det\,(\lambda I - A) = 0 \quad (9)$$

Equation (9) is called characteristic equation of matrix A. Scalars keeping the equation (9) are eigenvalues A (Anton, 2008).

2.5 Routh-Hurwitz Criteria

One of the methods that can be used in determining fixed point stability is the Routh-Hurwitz stability criterion, which is a criterion for showing stability by not seeing the real sign of the eigen value directly but by looking at the coefficients of the characteristic equation. The Routh-Hurwitz Stability Criteria is expressed in Theorem 1 below:

Theorem 1. Example $a_1, a_2, \ldots, a_k$real numbers $a_j = 0$ if $j > k$. All values of the characteristic equation

$$P(\lambda) = \lambda^k + a_1\lambda^{k-1} + \cdots + a_{k-1}\lambda \;\; + a_k = 0$$

And Hurwitz matrix as follows:

$$H_j = \begin{bmatrix} a_1 & a_3 & a_5 & a_7 & \ldots & 0 \\ 1 & a_2 & a_4 & a_6 & \ldots & 0 \\ 0 & a_1 & a_3 & a_5 & \ldots & 0 \\ 0 & 1 & a_2 & a_4 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{2j-1} & a_{2i-2} & a_{2i-3} & a_{2i-4} & \ldots & a_j \end{bmatrix}$$

Then the eigenvalue of the equation (9) will have a negative real part if and only if the determinant of the matrix H_j is positive:

$$detH_j > 0 \text{ for all } j = 1,2,\ldots,k$$

According to Routh-Hurwitz criteria, the above theorem for value $k = 2,3,4,$, the fixed point will be stable if and only if:

$k = 2;\; a_1 > 0, a_2 > 0$
$k = 3;\; a_1 > 0, a_3 > 0\,, a_1a_2 > a_{3,}$
$k = 4;\; a_1 > 0, a_3 > 0, a_4 > 0, a_1a_2a_3 > {a_3}^2 + {a_1}^2a_4$

2.6 Runge-Kutta Method of the Fifth Order

The Runge-Kutta method is a development of the Euler method with the completion calculation performed step by step (Alfaruqi, 2010). This method is an alternative to the Taylor series method that does not require derivative calculations (Iffatul, 2016). However, not all functions can be easily counted. The higher the order in the Taylor series is, the higher the derivative should be calculated and it makes the Taylor series rarely used in the solution of ordinary high-order differential problems (Alfaruqi, 2010). The fifth order Runge-Kutta method can be written as follows:

$$y_{n+1} = y_n + \tfrac{1}{6}(k_1 + 4k_4 + k_5) \quad (10)$$

with:

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf\big(x_n + {}^1\!/_3\, h, y_n + {}^1\!/_3\, hk_1\big)$$
$$k_3 = hf\big(x_n + {}^1\!/_3\, h, y_n + h({}^1\!/_6\, k_1 + {}^1\!/_6\, k_2)\big)$$
$$k_4 = hf(x_n + {}^1\!/_2\, h, y_n + h({}^1\!/_8\, k_1 + {}^3\!/_8\, k_3))$$
$$k_5 = hf(x_n + h, y_n + h({}^1\!/_2\, k_1 - {}^3\!/_2\, k_3 + 2k_4))$$

2.7 Extended Runge-Kutta Method

The Extended Runge-Kutta method is an extension of the Runge-Kutta method on the main function and its evaluation function (Muhammad, 2015). In general, the Extended Runge-Kutta equation model can be written as follows:

$$y_{n+1} = y_n + \textstyle\sum_{i=1}^{m}(hb_ik_{i1} + h^2c_ik_{i2}) \quad (11)$$

with,

$$k_{i1} = f\left(x_n + c_ih, y_n + h\sum_{s=1}^{i-1} a_{is}k_{s1}\right)$$

$$k_{i2} = f'\left(x_n + c_ih, y_n + h\sum_{s=1}^{i-1} a_{is}k_{s1}\right)$$

Equation (11) is a major function of the general equation of *Extended* Runge-Kutta model. So that equation of *Extended* Runge-Kutta fourth order is can be obtained as follows:

$$y_{n+1} = y_n + h(b_1k_{11} + b_2k_{21} + b_3k_{31} + b_4k_{41}) + h^2(c_1k_{12} + c_2k_{22} + c_3k_{32} + c_4k_{42}) \quad (12)$$

with,

$$k_{11} = f(x_n, y_n)$$
$$k_{21} = f(x_n + c_2h, y_n + ha_{21}k_{11})$$
$$k_{31} = f(x_n + c_3h, y_n + ha_{31}k_{11} + ha_{32}k_{21})$$
$$k_{41} = f(x_n + c_4h, y_n + ha_{41}k_{11} + ha_{42}k_{21} + ha_{43}k_{31})$$
$$k_{12} = f'(x_n, y_n)$$
$$k_{22} = f'(x_n + c_2h, y_n + ha_{21}k_{11})$$
$$k_{32} = f'(x_n + c_3h, y_n + ha_{31}k_{11} + ha_{32}k_{21})$$
$$k_{42} = f'(x_n + c_4h, y_n + ha_{41}k_{11} + ha_{42}k_{21} + ha_{43}k_{31})$$

3 METHODOLOGY

3.1 Type of Research

This research is a quantitative research because this research used quantitative data so that data analysis used quantitative analysis.

3.2 Variable of Research

In this research, variables consist of independent variables and dependent variables. The independent variables used are the acceleration due to excess/shortage of investment stock, the constant of money supply, the acceleration due to excess/shortage of money demand, the constant of capital depreciation, the rate of investment growth on capital stock, the level of investment to income, the growth rate of savings to income, money to earnings, decreased demand for money on interest rates, reduced rates of investment on interest rates, and growth rates on savings on interest rates.

Meanwhile, the dependent variables used are the national income rate, the rate of interest rate, and the rate of capital stock. The variables used in this research are secondary data obtained from Bank Indonesia in 2016.

3.3 Method of Research Analyze

The process of analysis in this research is divided into two, namely:

a. Analysis of fixed point stability of IS-LM model

The algorithm for performing fixed point stability analysis of IS-LM model as follows:

1. Determining the IS-LM model used by substituting the Gabisch-Lorenz IS-LM model into the investment, saving, and money demand functions of Cai. (Equation (4)).
2. Determining the fixed point of the IS-LM model using the method of elimination and substitution to obtain the point Y^*, R^*, and K^*.
3. Determining Jacobi's matrix in Equation (6).
4. Determining characteristic equation that fullfills $\det(J - \lambda I) = 0$.
5. Determining the stability of fixed points through Routh-Hurwitz criteria

b. Numerical simulation of the IS-LM model

1. Variable declarations used in the IS-LM business cycle model $(\eta, \delta_1, l_1, l_2, \beta_1, \beta_2, \beta_3)$.
2. Determining a numerical solution using the fifth-order Runge-Kutta method
3. Determining the stability time of each method
4. Determining the best method of both methods according to the fixed point stability time velocity.

4 ANALYSIS AND DISUSSION

4.1 Model Stability Analysis

The business cycle model used in this research was a model introduced by Gabisch and Lorenz (1987) by substituting the investment, saving, and capital stock functions of equation (4) into equation (3) with the parameters given by Cai (2005), namely:

$$Y(t) = \alpha[(\eta - l_1)Y - (\beta_1 + \beta_2)R - \delta_1K]$$
$$R(t) = \beta[l_2Y - \beta_3R - M] \quad (13)$$
$$K(t) = \eta Y - \beta_1R - (\delta_1 + \delta_2)K$$

with $\eta, \delta_1, l_1, l_2, \beta_1, \beta_2, \beta_3$ are positive constants in the interval $[0,1]$.

The fixed point in the equation (13) can be obtained if it fulfills

$$Y(t) = R(t) = K(t) = 0.$$

so it obtains

$$\alpha[(\eta - l_1)Y - (\beta_1 + \beta_2)R - \delta_1K] = 0 \quad (14)$$
$$\beta[l_2Y - \beta_3R - M] = 0 \quad (15)$$
$$\eta Y - \beta_1R - (\delta_1 + \delta_2)K = 0 \quad (16)$$

Next is to find fixed point (Y^*, R^*, K^*) by using elimination and substitution on each equation.

Thus, fixed point model, that is $E = (Y^*, R^*, K^*)$, was obtained where:

$$
\begin{aligned}
Y^* &= \frac{M(-\beta_2\delta_1-\beta_1\delta_2-\beta_2\delta_2)}{-\delta_1(\beta_2 l_2+\beta_3 l_1)+\delta_2(\eta\beta_3-l_1\beta_3-\beta_1 l_2-\beta_2 l_2)}\\
R^* &= \frac{M(-\eta\delta_2+l_1\delta_1+l_1\delta_2)}{-\beta_2 l_2\delta_1+\eta\beta_3\delta_2-l_1\beta_3\delta_2-\beta_1 l_2\delta_2-\beta_2 l_2\delta_2-l_1\beta_3\delta_1}\\
K^* &= \frac{-M(\beta_1 l_1+\beta_2\eta)}{\delta_2(\eta\beta_3-\beta_1 l_2)-(\delta_1+\delta_2)(\beta_3 l_1+\beta_2 l_2)}
\end{aligned}
\qquad (17)
$$

The stability of the fixed point is obtained by looking at the eigenvalues of the characteristic equations derived from the model by searching $det(J - \lambda I) = 0$, so we can get the characteristic equation

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0 \qquad (18)$$

when

$$
\begin{aligned}
P &= \beta\beta_3 - \alpha\eta + \delta_1 + \delta_2 + \alpha l_1\\
Q &= \alpha\delta_1\eta + \alpha\beta\beta_2 l_2 + \alpha\beta\beta_1 l_2 + \beta\beta_3\delta_2 + \beta\beta_3\delta_1\\
&\quad + \alpha l_1\delta_2 + \alpha l_1\delta_1 + \alpha l_1\beta\beta_3\\
&\quad - \alpha\eta\delta_2 - \alpha\eta\delta_1 - \alpha\eta\beta\beta_3\\
R &= -\alpha\eta\beta\beta_3\delta_1 - \alpha\eta\beta\beta_3\delta_2 + \alpha l_1\beta\beta_3\delta_2\\
&\quad + \alpha l_1\beta\beta_3\delta_1 - \alpha\beta\beta_1 l_2\delta_1\\
&\quad + \alpha\beta\beta_1 l_2\delta_1 + \alpha\beta\beta_1 l_2\delta_2\\
&\quad + \alpha\beta\beta_2 l_2\delta_1 + \alpha\beta\beta_2 l_2\delta_2\\
&\quad + \alpha\eta\beta\beta_3\delta_1
\end{aligned}
$$

Because the eigenvalues of the equations (18) was difficult to determine, the stability of the fixed point can be investigated using the Routh-Hurwitz criterion. According to Routh-Hurwitz criteria, the eigenvalues of the equation (18) will make a fixed point $E = (Y^*, R^*, K^*)$ (4.12) stable if and only if $P > 0, R > 0$, and $PQ - R > 0$.

4.2 Numerical Simulation using Runge-Kutta Method of the Fifth Order

In this section, a numerical simulation of the IS-LM business cycle model will be conducted to find the stability time of the model using the Runge-Kutta method of order five.

The initial value used in the simulation was data from Bank Indonesia in 2016 in the form of income rate $Y(t) = 5$, rate of interest $R(t) = 9.18$, and capital stock rate $K(t) = 4.47$ [9]. The parameters used in the simulation of this business cycle model are presented in Table 1.

Based on these parameters and by taking into account the data in Table 1, it was obtained a fixed point $E = (Y^*, R^*, K^*) = (0.0979, 0.0097, 0.0351)$. Before performing numerical simulations, a fixed point stability simulation was performed by substituting the parameters in Table 1 into equations (18). It will be shown that the parameter meets Routh-Hurwitz criteria such that the point remains stable.

Table 1: Value of business cycle parameters.

Symbol	Parameter Definition	Value
η	The growth rate of investment to income	0.5
δ_1	The rate of decline in investment on capital stock	0.7
δ_2	Constant depreciation of capital	0.5
l_1	Saving growth rate against income	0.1
l_2	The growth rate of money demand for income	0.6
b_1	The rate of decline in investment on interest rates	0.7
b_2	The rate of saving growth against interest rates	0.8
b_3	The rate of decline in money demand on interest rates	0.9
M	Constant money supply	0.05
α	Acceleration due to the excess or lack of investment	1
β	Acceleration due to the excess or lack of money demand	4

According to Routh-Hurwitz criteria, the point remains stable if and only if $P > 0, R > 0$, and $PQ - R > 0$. The simulation results based on these criteria can be seen in Table 2.

Table 2: Eigen Value of Characteristic Equations.

P	4.4000
Q	6.3500
R	2.6760
$PQ - R$	25.2640

Based on the table above, it can be seen that the eigenvalue of the characteristic equation in equation (18) satisfied the Routh-Hurwitz criterion $P = 4.4000 > 0, R = 2.6760 > 0$, and $PQ - R = 25.2640 > 0$ so the fixed point of the model was stable.

The next was to develop numerical simulation with the Runge-Kutta method of order five with the help of MATLAB R2013 software by substituting the parameters in Table 1 into Eq. (13) to obtain the graph in Figure.1.

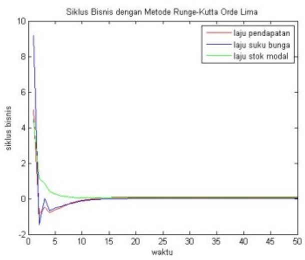

Figure 1: Numerical simulation by Runge-Kutta method of the fifth order.

The business cycle will experience different fluctuations and then move constantly to a fixed point so as to obtain a stable point. Figure 1 shows business cycle stability around the point. Figure 1 shows business cycle stability around the point $t = 36$ with value $Y = 0.0979, R = 0.0097,$ and $K = 0.0351$. Therefore, according to Routh-Hurwitz criteria, this business cycle is stable.

4.3 Numerical Simulation with Extended Runge-Kutta Method

The initial value used in this simulation was the same as the previous simulation of the income rate $Y(t) = 5$, rate of interest $R(t) = 9.18$, and capital stock rate $K(t) = 4.47$. The resulting numerical simulation is as follows.

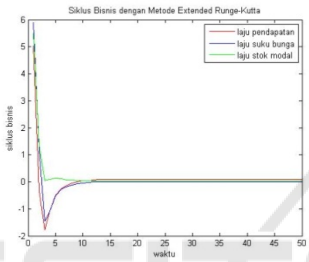

Figure 2: Numerical simulation with Extended Runge-Kutta method.

Based on Figure 2,. the business cycle model with the Extended Runge-Kutta method has similar pattern and behavior that decrease (recess) and then move constantly to a stable point. In Figure 2, the business cycle has stability around the point $t = 25$ with value $Y = 0.0979, R = 0.0097,$ and $K = 0.0351$. Therefore, according to Routh-Hurwitz criteria, this business cycle is stable.

4.4 Comparison Method

In this research, the comparison of the two methods was developed by comparing the stability time of each method and computation time to obtain a stable point.

Table 3: Comparison Method.

	Income Rate ($Y = 0.0979$)	**Rate of Interest** ($R = 0.0097$)	**Capital Stock Rate** ($K = 0.0351$)
RK5	$t = 36$	$t = 35$	$t = 32$
ERK	$t = 25$	$t = 25$	$t = 24$

Based on Table 3, we can shows the stability time of each method. The stability time resulting from the Runge-Kutta method of order five was the rate of income that was stable at $t = 36$, rate of interest that had stable moment $t = 35$, and the capital stock rate that was stable at the moment $t = 32$. Meanwhile, the stability time generated by the Extended Runge-Kutta method was the income rate that was stable at the moment $t = 25$, rate of interest that was stable at the moment $t = 25$, and the capital stock rate that was stable at the moment $t = 24$.

Based on the Runge-Kutta method, the fifth order business cycle model will experience stability in the next 36 years, or in 2052, and with the Extended Runge-Kutta method, stability of the next 25 years will be in the year 2041. Both will be stable at the point $E = (Y^*, R^*, K^*) = (0.0979, 0.0097, 0.0351)$.

Therefore, the Extended Runge-Kutta method has a faster stability time than the Runge-Kutta method. This is because the main function of the Extended Runge-Kutta is added with the derivative function. So, the results obtained by the Extended Runge-Kutta method are faster to be stable than the fifth order Runge-Kutta method.

5 CONCLUSIONS

Based on the formulation of the problem and the results of the discussion, the following conclusions can be obtained:

1. In the IS-LM business cycle model, we get a stability model of equilibrium point $E = (Y^*, R^*, K^*) = (0.0979, 0.0097, 0.0351)$. Model of business cycle stability can be shown by the value of the equation of characteristics obtained from the model, that is
$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0$$
with
$$P = \beta\beta_3 - \alpha\eta + \delta_1 + \delta_2 + \alpha l_1$$
$$Q = \alpha\delta_1\eta + \alpha\beta\beta_2 l_2 + \alpha\beta\beta_1 l_2 + \beta\beta_3\delta_2 + \beta\beta_3\delta_1 + \alpha l_1\delta_2 + \alpha l_1\delta_1 + \alpha l_1\beta\beta_3 - \alpha\eta\delta_2 - \alpha\eta\delta_1 - \alpha\eta\beta\beta_3$$
$$R = -\alpha\eta\beta\beta_3\delta_1 - \alpha\eta\beta\beta_3\delta_2 + \alpha l_1\beta\beta_3\delta_2 + \alpha l_1\beta\beta_3\delta_1 - \alpha\beta\beta_1 l_2\delta_1 + \alpha\beta\beta_1 l_2\delta_1 + \alpha\beta\beta_1 l_2\delta_2 + \alpha\beta\beta_2 l_2\delta_1 + \alpha\beta\beta_2 l_2\delta_2 + \alpha\eta\beta\beta_3\delta_1$$
By substituting the parameter values, it was obtained $P = 4.4000 > 0, R = 2.6760 > 0$, and $PQ - R = 25.2640 > 0$ which meets the Routh-

Hurwitz criteria so that the IS-LM business cycle model is stable.

2. The numerical solution using the Runge-Kutta method of order five obtained the time stability of the IS-LM business cycle model by substituting the parameters when $t = 36$.
3. Numerical solution using Extended Runge-Kutta method obtained the time stability of the IS-LM business cycle model by substituting the parameters when $t = 25$.
4. In determining the stability of the IS-LM model, the Extended Runge-Kutta method has a faster stability time with $t = 36$ than the current fifth order Runge-Kutta method with $t = 25$. This happens because the Extended Runge-Kutta method adds the derivation function to the main function so that the stability point gets faster.

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