Undergraduate Students' Conceptual Understanding on Abstract Algebra

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Abstract: This study aims to describe students' conceptual understanding on abstract algebra at mathematics education study program. The concept of binary and group operations is a fundamental issue in abstract algebra. This current study is a qualitative research that aims to determine level of students' understanding. Subjects of the study were students of mathematics education study program of semester 7 who were taking course of algebra structure in academic year of 2017/2018. The data were collected by using two instruments; test and interviews. After the data obtained then it was categorized into appropriate levels of understanding of students. The categories used in this study are the *low*, *medium* and *high* level of conceptual understanding. Based on the results of data analysis, the result of study indicates that more than 50% of students still have low level of understanding. It can be concluded that there are still many students who lack of conceptual understanding on binary and group operations. Students define binary and group operations based solely on their current knowledge. Therefore, there must be a special action done by lecturers to connect the concepts learned with it precondition concepts.

1 INTRODUCTION

Mathematical branches studied in mathematics education study program include algebra, analysis, applied mathematics, geometry, number theory, basic introduction to mathematics, and mathematical logic. Algebra is a course that contains basic skills expected for those who study mathematics. This means that basic knowledge of mathematics can be obtained by studying algebra (Kaput, 2000). In higher education, algebra is also called 'Abstract Algebra'. The subject of abstract algebra aims at finding the same picture of the algebraic structure, obtaining additional results based on existing results, and making the classification of operations on the structure of the material being studied. Learning a structure means studying the interrelationships among concepts involved in the structure. According to Arikan, et.all. (2015) the difficulties' experienced in a concept will cause difficulties in learning the next concept. There are several reasons for the difficulty in learning abstract algebra one of which is the lack of mastery on concepts. The less mastered concepts make students are not able to think abstractly, cannot depict verbal expressions or cannot make mathematical formulas. Therefore, in order to have good mastery in abstract algebra, a good conceptual understanding is highly required.

Procedural skill and conceptual understanding are two types of knowledge that everyone needs in learning mathematics. Procedural skill refer to the ability of a person in solving problems by using a coherent algorithm (Byrnes and Wasik, 1991; Bisson et.all, 2016). In addition, procedural skill can also be identified through the ability to change the notation used in solving a mathematical problem. On the other hand, conceptual understanding refers to an understanding of concepts, relationships, and principles in mathematics (Rittle, Siegler and Alibali, 2001; Crooks and Alibali, 2014). National Council Teaching of Mathematics (2014) states that learning mathematics should give emphasis at conceptual knowledge by reducing attention to procedural learning. Paying more attention to executional procedures and ignoring conceptual understanding will have an impact on students' success in learning the topic (Ocal, 2017). Tatar and Zengin (2016) explain ones of the causes of students having difficulty in learning mathematics especially on the topic of calculus is the lack of conceptual knowledge on the topic. A learning process that emphasizes conceptual knowledge, means providing an activity

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that makes meaningful interrelationships between concepts in mathematics, emerging understanding, and giving chance for applying concepts. Arslan (2010) states that conceptual knowledge involves a student's understanding of interpreting a concept and establishing a connection between the concepts. People who have high conceptual understanding ability can make the interrelationship between sections in mathematics better than those with low understanding of concepts. The results of Hallet, Nunes and Bryant (2010) study found that students will be more successful in mathematics learning when a conceptual approach is used more than procedural approaches. The use of a conceptual approach in a learning activity may provide students with correct procedures and better capacity in transferring knowledge (Rittle and Alibali, 1999). Joersz (2017) explain that students whom learning activities use a conceptual approach will both improve their conceptual understanding and establish correct and flexible troubleshooting procedures. The term conceptual understanding or knowledge includes what has been known about the concepts and the way in which the concept is found. Conceptual knowledge is defined as knowledge of abstract concepts and principles in general (Rittle, Siegler and Alibali, 2001). The increase of understanding about concepts in mathematics means enhancing the ability of conceptual understanding. In principle stated by the National Council of Teachers of Mathematics (2014), it is firmly stated that conceptual understanding is followed by the establishment a basic for the smooth procedure. That is why conceptual understanding becomes foundation and need for a good mastery on utilizing procedure to solve mathematical problems. The principle of NCTM shows that students must develop their basic conceptual understanding first before they have procedural knowledge. Procedural knowledge was not developed early as an action to expand the development of conceptual knowledge (Rittle, Schneider, and Star, 2015). To achieve an increased conceptual understanding of learning in classroom, a valid and reliable measuring tool is needed (Bisson, et.all., 2016) Conceptual understanding can be measured by using a variety of tasks, starting from the task of evaluating the correctness of an example or a procedure of defining and explaining the concepts learnt by students (Crooks and Alibali, 2014; Rittle, Schneider, Star, 2015). The virtues of conceptual tasks are relatively unknown or have not been encountered by the students. So in order to solve them, students must have the conceptual knowledge ability (Bisson, et.al., 2016)

Based on the above descriptions, this study aims to describe students' conceptual understanding at mathematics education program of FKIP Muhammadiyah University of Bengkulu in abstract algebra course, especially on the topic of group theory.

2 METHOD

2.1 Type of Research

This research is qualitative descriptive study using described qualitative data to produce clear and detailed description about students' conceptual understanding on topic of group theory at Mathematics Education Study Program of Muhammadiyah University of Bengkulu. Qualitative data in this study is the result of students' answers on test. After the test was conducted, then the subjects were interviewed in order to describe and dig up in-depth informations those were not obtained from student test.

2.2 Subject of the Research

Subjects of this study were 63 seventh semester students of mathematics education of FKIP of Muhammadiyah University of Bengkulu 2017/2018 academic year who were taking the course of Algebraic Structure. Based on the results of the test, 6 students had been selected to be interviewed. The students who were interviewed represented low, medium, and high conceptual ability groups.

2.3 Research Intrument

The main instrument used in this study was the researcher herself by conducting interviews to explore in-depth information on mathematics education students' understanding on the concept of binary and group operations based on test results. The instrument used in this study was a test on conceptual understanding of binary and group operation material. The questions of this test were in the form of description consisted of two questions based on the indicator. In addition, an interview guide was also used to know in-depth information about the occuring processes in term of students' algebraic thinking in solving algebra problems.

2.4 Data Collection

Data collection of this research was done by using two techniques, written test and interview. Written Test is a test of understanding on the concept of binary and group operations to obtain data on students' conceptual understanding. Meanwhile, the interview used in this study was a task-based interview so that interview guidelines only contained with outline of questions. It was conducted to obtain clearer data on conceptual understanding of binary and group operating materials.

2.5 Data Analysis Tehcnique

The data analysis on conceptual understanding of binary and group operation was done in depth after the students were divided based on their ability category. The process of data analysis began by reviewing all available data from interview and observation that has been written in field notes. Then the researcher performed data reduction, data exposure, and conclusion drawing and verifications.

3 RESULT AND DISCUSSION

3.1 Result

The problem in this research was "$Let\ \mathrm{G} = \{\mathrm{a} + \mathrm{b}\sqrt{2}, \mathrm{a}, \mathrm{b}\ \in \mathrm{Q}, \mathrm{a} \neq 0 \text{ or } \mathrm{b} \neq 0\}$ Prove that G is a Group under regular multiplication" . We wanted to analyze whether the students learnt the group principle in this problem. We marked that most of the students memorized the group principles (closure, associativity, identity element, inverse element) but they could not interpret them.

Table 1. Conceptual Understanding of the Subject in The First Problem

Number of The Problem	1
Percentage of Correct Answered in Closure Property	54
Percentage of Correct Answered in Associative	43
Percentage of Correct Answered in Identity Element	58
Percentage of Correct Answered in Inverse Element	36

In accordance with table 1, it appears that for the problem, only 54% of the subjects answered correctly in closure property. Most of the students of mathematics education in FKIP Muhammadiyah University of Bengkulu (64%) have an error in determining inverse of each element in a group. Table 1 also depicts that for the understanding on terms of a set to be a group, less than 50% (47.75%) of subjects answered correctly.

Results of this study aim to determine Mathematics Education students' conceptual understanding on binary and group operations in the subject of Structure of Algebra.

1. Result of analysis on subject 1 (represent students with high conceptual ability), based on result of the test and interview:
    a. Closure property: subject 1 could explain well the property on the multiplication of positive rational numbers derived from the closure properties of multiplication with a detail explanation at each explanatory step.
    b. Associative property: subject 1 could describe the associative property by giving explanation about the connection the multiplication operation property.
    c. Identity element: subject 1 managed to define the identity element which is also an element of a set of rational numbers with a clear and detailed explanation on each step taken.
    d. Every element has inverse: subject 1 could coherently determine the inverse element by using the identity element specified in the previous step. Subject 1 explained in sequence and detail the steps to determine the inverse element and show that the inverse is also an element of the set of rational numbers.

2. Results of analysis on subject 2 (represented students with medium conceptual ability)
    a. Closure property:Subject 2 did not mention that the set was not an empty set that has binary operations. The subject directly mentioned that the set meets the closure property and other properties. Subject 2 could show that the set of positive rational numbers has closure property to binary operation on multiplication. But in each step, subject 2 did not explain the used properties.
    b. Associative property: Subject 2 described the associative property from the left-to-right side in detail using other variables and described the inherent property of each written step. However, subject 2

was less precise in restoring the examplified variables.

c. Identity element: Subject 2 was able to determine the element of identity in the set of positive rational numbers. However, written test results indicated that the subject was not understood the property he wrote. Subject 2 also hesitated in explaining which definitions are meant in each explanatory step.

d. Identity element: Subject 2 showed that every element in the set of positive rational numbers has an inverse element by asserting the inverse with another variable. In the explanation of this property subject 2 was more flexible when it came to substituting the given variables. Eventually, subject 2 reasonably showed that the inverse is also an element of the set of positive rational numbers. The subject also explained definition of a group by repeating the explanation of each trait. Unfortunately, subject 2 was not careful enough in determining inverse.

3. Results of analysis on subject 3 (represented students with low conceptual ability)

a. Closure property: Subject 3 managed to write the definition of closure property. However, the subject failed to show closure property on the given binary operation. Subject 3 did not put meaning on the definition of binary operations properly and could not mention the notation of the set of numbers.

b. Associative property: Subject 3 could write the definition of associative properties down. However, the subject could not show that the given binary operation meets the associative property. Subject 3 copied from sample of previous problems that had different binary operations. The explanation of the left-hand side to the right-hand side did not match the definition of the binary operation assigned to the problem that was being worked on. Subject 3 has not understanding on the application of binary operations in the term of associative property.

c. Identity element: Subject 3 wrote the definition of identity element correctly. However, it could not elaborate the property using the definition of given binary operation. The subject was mimicking from binary operations has ever been undertaken without sufficient understanding on the definition itself. In addition, subject 3 assumed that the element of identity is always zero. So the operation performed was a reduction operation which is the inverse of the addition operation.

d. Inverse element: Subject 3 could denote the inverse element and wrote down the definition that characteristic of each element is having an inverse element. However, the subject did not succeed to show the inverse of the element because it wrote the wrong identity element. Furthermore, the decomposition of the binary operation definition was still incorrect. So that the inverse element obtained was also incorrect. Subject 3 was not able to link any written steps with the definitions already written at the beginning. Subject 3 mentioned that the problem that was being solved was a group but could not explain the reason correctly.

3.2 Discussion

Based on the results of the data analysis it can be concluded that subject 1 could define the binary and group operation in detail starting with a non-empty set, having binary operations, and then showing one by one closure and associative properties, having an identity element, each element has an inverse, and are distributive by using inherent properties in operation. Based on the data of Subject 2, it can be stated that subject 2 was able to identify a binary and group operation or not binary and group operations by showing one by one closure and associative properties, having an identity element, each element has an inverse and distributive. The subject did not mention that binary and group operations must be non-empty set. Subject 2 did not include attributes attached to the operation. In addition, Subject 2 was also rather careless in completing the test. Based on the data analysis on subject 3 it can be concluded that Subject 3 was able to write the definition of binary and group operation. However, it was less fluent and incomplete in showing one by one the properties. Subject 3 assumed that the identity of multiplication is always 1 and the identity of addition is always 0.

Subject 3 did not include attributes attached to the operation. In addition, Subject 3 was also less thorough in conducting the operation.

This research wanted to survey whether the students learnt the group axioms in this question. The researcher observed that most of the students memorized the group axioms (closure, associativity, neutral element, inverse element) but they could not analyze them. We categorized the mistakes into two groups on this question. In the 1st Category, students listed the group axioms but they accepted them "Correct" without sufficiently analysing it. In short, they accepted that associativity was proved without analysing. In the 2nd Category, the result we found is that the students could not comprehend the required associativity.

It could be said that the students preferred to copy rather than to think abstractly when we consider that they attended to the university as a result of test exam, i.e. the central exam system (Soylu, 2008). It sounds believable that they could have just memorized the rules of theory without internalizing the descriptions. Trying to proving the group axioms without thinking on descriptions is a sign of rote learning based education system. Whether a cognitive teaching has been done on the algebraic structures or not has not been known. Unless we internalize the meanings of the concepts covered with the different learning methods, mastering on a subject by rote will come into the question. Using computer programmes, e.g. computer algebra system CBS), will provide convenience but, are there any academic members applying to the computer programmes or how are their perspectives to these embodying processes? Doing a scientific research by academic members about this matter, their opinions and their approaches could be significantly useful (Tatar and Zengin, 2016). The questions, which measure whether definitions and features of algebraic structures are learnt, are generally measure proving, reasoning, and discernment ability. Individuals experience many problems in their daily life and they think mathematically to solve their problems. Actions like explaining a proposition, saying why it is right or wrong and choosing and using different logical thinking ways and proving types, present individual's ability on mathematical thinking. In this sense, the students of the mathematics department are supposed to use their ability of mathematical thinking and to let the operations they do make sense. Mistakes made by students, who participated in the study, came up as a result of either misunderstanding the conditions of group theory or examining these conditions wrong (Arikan, et.all., 2015). Some challenges could be experienced during the learning process but the matter is to identify them correctly and to enhance various methods to deal with them. Having difficulties at the learning abstract concepts is the most important one. Students can apply to rote learning in order to overcome this difficulty, but they can have difficulty in practice at this time. For example, student lists the axioms (closure, associativity, inverse element, neutral element) while controlling the set whether it is group or not, but he or she makes the operation supposing that the set is closed (Arikan, et.all., 2015). In other words, student cannot practice what he or she memorized or could not know what to do in other cases. We have been thinking the fact that this problem traces to the gaps of education which was received in both high school and university years. Students' infrastructure they set up with math training, which they had during their education life up to attending university, has inadequate mathematics they meet at the university. They assume that the success at this lesson to be able to perform the operations without using calculator and dealing with just practical solutions in the math exams. However, they meet theoretical mathematics after the graduation from a high school before the college and as a natural result, they are afraid of another learning difficulty, which we thought it arises from the same reasons, is the one which is dealing with proving the theories (Ocal, 2017). While it is rehearsing as if definitions and proofs have no significance at secondary education, the theoretical side of the mathematics is at the forefront at the university, especially at the Algebra Math-1 Class. Students even do not know how to study for this lesson and they are having enormous learning difficulties. Our suggestion to minimize this wavers during this gradation process is to lecture the abstract mathematics, such as logic, proving methods before Linear Algebra and Mathematics Analysis I class, in which the main subjects of the theoretical mathematics have been taken into account. Conceptual learning has much higher degree of importance in the mathematics education for the students who study at the mathematics department. Unless the students can successfully comprehend algebraic definitions, concepts and structures, they will try to memorize these phenomenon's(Soylu, 2008).

4 CONCLUSIONS

Based on the results of data analysis, it can be concluded that the conceptual understanding of students on the topic of binary operations and group theory is not satisfactory. Less than 50% of students with good conceptual knowledge. It can be concluded that there are still many students who lack of conceptual understanding on binary and group operations. Students define binary and group operations based solely on their current knowledge. This leads to the consequence that lecturers should have new strategies to improve student conceptual understanding.

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