Ordinary Kriging Method using Isotropic Semivariogram Model for Estimating the Earthquake Strength in Bengkulu Province

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Abstract: Ordinary kriging method is a method that gives the best unbiased estimator. Kriging method has been applied to several earthquake studies. Faisal (2014) uses isotropic semivariogram models using the linear program method in the earthquake event data sample in Bengkulu. However, the kriging method has not been used to estimate the earthquake strength. This study aims to apply the ordinary kriging method to estimate the strengths of the earthquake in Bengkulu Province and its surrounding areas. The semivariogram model used in the parameter estimation is an isotropic semivariogram model. We use data earthquake events in the range from 99.00°E to 104.00°E on the latitude position and from 6.00°S to 2.00°S on the longitude position. The result shows that the semivariogram model that can be used is the spherical model with Sill = 0.1907015 and Range = 0.07929616 and the exponential model with Sill = 0.1898991 and Range = 0.01622501. The maximum earthquake strength estimation on the spherical model is Mw 7.5249 and the maximum on the exponential model is Mw 7.5254. These models show that the epicenter of the greatest earthquake estimation is on 102.0833°E, 4.7045°S.

1 INTRODUCTION

Spatial data is data that contain information about location attributes and information. Spatial data are presented in the geographical position of objects, locations, shapes, and relationships with other objects. Analysis of spatial data requires more attention than analysis of non-spatial data. Spatial analysis is an analysis that considers location attributes, as well as absolute locations (coordinates). One of the sciences that use spatial analysis is geostatistics.

Geostatistics emerged in the early 1980s as a combination of mining science, geology, mathematics, and statistics. Geostatistics was originally developed in the mineral industry to assess the mineral reserves in the earth. Geostatistics recognizes spatial variations on the large and small scale, or the spatial trends and the spatial correlation. Geostatistics is used to predict data at locations that have not been measured (Fischer & Getis, 2010)

One of the basic tools in geostatistics is a semivariogram that explains the variability of data at certain distances and directions. Semivariogram is a real function to show the spatial correlation measured at the sample location. Semivariogram is presented as a graph that shows the variance in measuring the distance between all pairs of sample locations. There are two types of semivariogram namely isotropic semivariogram and anisotropic semivariogram. The semivariogram used in the kriging procedure to interpolate unobserved locations.

Kriging is a geostatistics method that utilizes a semivariogram model to estimate values in other regions that have not been sampled (Rubeis et al., 2005). There are different types of Kriging techniques, such as Ordinary Kriging, Universal Kriging, Indicator Kriging, Co-kriging, and others. The choice of which kriging to use depends on the characteristics of the data (Mesić Kiš, 2015). Ordinary kriging is the most widely used kriging method. It serves to estimate a value at a point of a region for which a variogram is known, using data in the neighborhood of the estimation location (Wackernagel, 2003).

Along with the development of science and technology, geostatistics continues to grow, and its application extends to various fields, such as weather, climate, disaster, and population. In the field of disaster, several earthquake studies using the Kriging

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method have been carried out before. Research conducted by Carr et.al (1986) applied disjunctive kriging to estimate ground movements in the event of an earthquake. Furthermore, Carr et al. (1989) continued his research by comparing universal kriging and ordinary kriging method to estimate the ground motion in the event of an earthquake. Sugai et al. (2015) introduced a practical method to estimate the specific distribution of ground motion earthquake events based on ordinary kriging analysis. Recent research on the application of ordinary kriging was carried out by Bekti et al. (2017) on the peak ground acceleration (PGA) data in Banda Aceh in 2006 and the results are Ordinary Kriging can be applied to predict PGA.

The kriging method is considered as the best estimation method in terms of the accuracy of its assessment. The ordinary kriging method is a method of estimating a random variable at a certain point (location) by observing similar data in another location with the mean data assumed to be constant but unknown in value. This method is a method that provides the best linear unbiased estimator (BLUE). Hence, the ordinary kriging method can be used in earthquake events. The other hand, in reality, the population average of earthquake events is difficult to be known.

Bengkulu Province is one of the provinces located at the meeting of the Indo-Australia and Eurasia tectonic plates which are the main generators of high earthquake activity. The movements caused by these two plates can cause active faults which are generators of seismicity in parts of Sumatra. Bengkulu is also among two active faults, the Semangko and Mentawai faults. This condition makes Bengkulu Province the most vulnerable to earthquake disasters (Hadi et al., 2010).

Faisal (2014) compared Matheron (classic) and Cressie-Hawkins (robust) isotropic semivariogram models using the linear program method in the earthquake event data sample in the coastal area of Bengkulu. From the results of the case study, the Spherical theoretical semivariogram model was the best semivariogram model obtained from the results of the Matheron isotropic experimental semivariogram. However, in this study, the kriging method has not been used to estimate earthquake strength in Bengkulu area

Based on the previous research, we interested in conducting further research. The purpose of this study was to predict the strength of earthquakes in Bengkulu Province and its surrounding areas based on the epicenter of an earthquake that occurred in some years. We use the ordinary kriging with the isotropic semivariogram model. The final objective is a mapping of the predicted strength of the earthquake from the selected model. To facilitate the calculation of the kriging and mapping, we use R software.

2 LITERATURE REVIEW

2.1 Semivariogram Analysis

Variogram is a tool in geostatistics that is used to describe spatial correlations of data. In Plotting of variogram, the low value will be close to other low costs and large value will be near other large values. The values can be visualized in a variogram graph as a function of distance. Variogram determines the distance where the values of the observation data are independent or have no correlation. The symbol of the variogram is 2γ, while semivariogram is half of the variogram with the symbol γ. This semivariogram is used to measure spatial correlation (Wackernagel, 2003).

The experimental variogram is a variogram calculated from the observed data. Variogram is formulated as follows:

$$2\gamma(h) = Var[Z(s+h) - Z(s)], \quad (1)$$

assume that the data are second order stationary, equation (1) can be written as follows:

$$2\gamma(h) = E[Z(s+h) - Z(s)]^2. \quad (2)$$

Since semivariogram is half of the variogram, then from equation (2) can be written:

$$\gamma(h) = \frac{1}{2}E[Z(s+h) - Z(s)]^2, \quad (3)$$

where $\gamma(h)$ is a semivariogram. The above semivariogram is also called theoretical semivariogram. There are two types of semivariogram: isotropic semivariogram ($\gamma(h)$ depends only on distance h) and anisotropic semivariogram (γ (h) depends on distance h and direction).

A semivariogram is a statistical tool for describing, modeling and explaining spatial correlations between observations. An experimental semivariogram is a semivariogram obtained from known data. The estimator of semivariogram proposed by Matheron (1962) in Cressie (1993) is:

$$\hat{\gamma}(h) = \frac{1}{2|N(h)|}\sum_{i=1}^{N(h)}\left[z(s_i + h) - z(s_i)\right]^2 \qquad (4)$$

where s_i is location of sample (coordinate), Z(S_i) data value in location S_i and |N(h)|: #pairs (S_i , S_i +h) with distance h .

Semivariogram has properties s follows:

1. Semivariogram value at the origin is zero: $\gamma(0) = 0$.
2. Semivariogram values are always non-negative: $\gamma(h) \geq 0$
3. Semivariogram is an even function: $\gamma(h) = \gamma(-h)$.

The semivariogram plot of distance h gives an experimental semivariogram plot. Experimental semivariograms obtained from data usually have irregular shapes that are difficult to interpret and cannot be directly used in assessments. These parameters are:

a. Range is the maximum distance where there is still a correlation between data.
b. Sill is a semivariogram value that does not change for unlimited h. The sill value generally approaches the data variance.
c. The Nugget effect is a discontinuous phenomenon around the starting point.

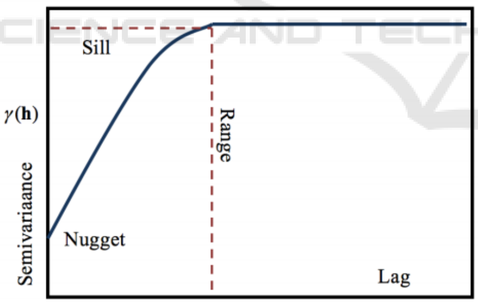

Figure 1: Component of semivariogram graph.

In the semivariogram prediction, the theoretical semivariogram model is fitted in the experimental semivariogram $\hat{\gamma}(h)$. There are four theoretical semivariogram models are (Cressie, 1993; Amstrong, 1998):

1. Spherical model:

$$\gamma(h) = \begin{cases} C_0 + C\left[\frac{3h}{2a} - \frac{1}{2}\left(\frac{h}{a}\right)^3\right] & ,0 < h \leq a \\ C_0 + C & ,h > a \end{cases} \qquad (5)$$

2. Exponential model:

$$\gamma(h) = \begin{cases} C_0 + C\left[1 - exp\left(-\frac{h}{a}\right)\right]; & h > 0, \\ 0; & h = 0. \end{cases} \qquad (6)$$

3. Gaussian model:

$$\gamma(h) = \begin{cases} C_0 + C\left[1 - exp\left(-\frac{h^2}{a^2}\right)\right]; & h > 0, \\ 0; & h = 0, \end{cases} \qquad (7)$$

where C_0: nugget variance, $C_0 + C$: sill, and a: range

4. Linear Model

$$\gamma(h) = \alpha h, \qquad (8)$$

where α is a gradient.

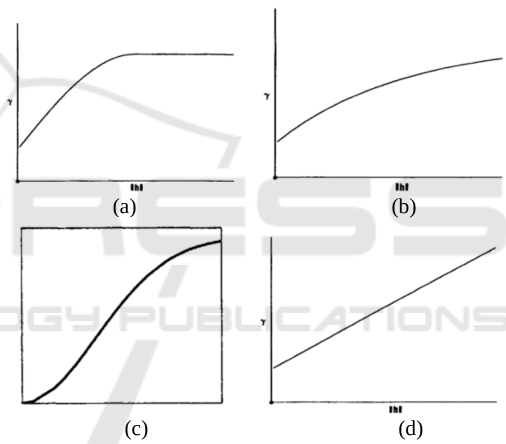

Figure 2: The theoretical semivariogram models: (a) Spherical Model, (b) Exponential Model, (c) Gaussian Model and (d) Linear Model.

2.2 Ordinary Kriging

The ordinary kriging method is a method of estimating a random variable at a certain point (location) by observing similar data in another location with the mean data assumed to be constant but unknown in value. In the ordinary kriging method, the known sample are used as linear combinations to estimate the points around the sample area (location). In other words, to estimate any non-sampled point (s_0) can use a linear combination of random variables $Z(s_i)$ and kriging weight values respectively, mathematically can be written with:

$$\hat{Z}(s_0) = \sum_{i=1}^{n} \lambda_i Z(s_i), \qquad (8)$$

Where $\hat{Z}(s_0)$ is the estimated value of the random variable at points s_0, and $Z(s_i)$ is the value of the random variable $Z(s)$ at the i-point, and λ_i is the kriging weight at the i-point (Pfeiffer and Robinson, 2008). Whereas the variance of the estimated error (kriging variance) can be expressed by:

$$\sigma_{OK}^2(s_0) = E\left(\sum_{i=1}^{n} \lambda_i Z(s_i) - Z(s_0)\right)^2 = \sum_{i=1}^{n} \lambda_i \gamma(s_0 - s_i) + m, \qquad (7)$$

where parameter *m* is the Lagrange multiplier.

3 METHODS

We use data of the earthquake events with magnitudes above Mw 5.0 that occurred in Bengkulu Province from 2000 to 2016. The 365 data obtained from the website www.usgs.com. The variables of data are the coordinate position of the center of the earthquake, latitude, longitude, and magnitude. Based on the longitude position, earthquake events in the data range from 100.00°E to 105.00°E, while based on the latitude position, the minimum data is at 6.00°S, and the maximum is at 2.00°S.

The steps of analysis in this study are:

1. Collect data of earthquake occurred in the Bengkulu Province and its surrounding areas.
2. Perform descriptive statistics for magnitude variable.
3. Calculate Experimental Semivariograms using equation (4) and display a variogram graph.
4. They fit theoretical Semivariogram from variogram graph, Spherical, Exponential, Gaussian and Linear models.
5. Perform a model validation test to determine whether the theoretical semivariogram model that will be used in the kriging method is the best model with the smallest remaining sum squares (RSS) value compared to the other models.
6. Map a contour of earthquake strength estimation in Bengkulu Province and the surrounding areas.

4 RESULTS AND DISCUSSION

The descriptive statistics of the data are shown in table 1. shows that the minimum earthquake is Mw 5 and the maximum earthquake occurred with a strength of Mw 8.4. This Mw 8.4 earthquake occurred on December 9, 2007, with the epicenter of the earthquake at 101.37°N and 4.44°S. The average earthquake data used is Mw 5.33 with a standard deviation of 0.417. The median earthquake strength is Mw 5.2 and the range is Mw 3.4.

Table 1: Statistics of the earthquake strength in Bengkulu Province.

Statistic	Magnitude
Mean	5.333
Standard Deviation	0.417
Median	5.200
Range	3.400
Maximum	8.400
Minimum	5.000

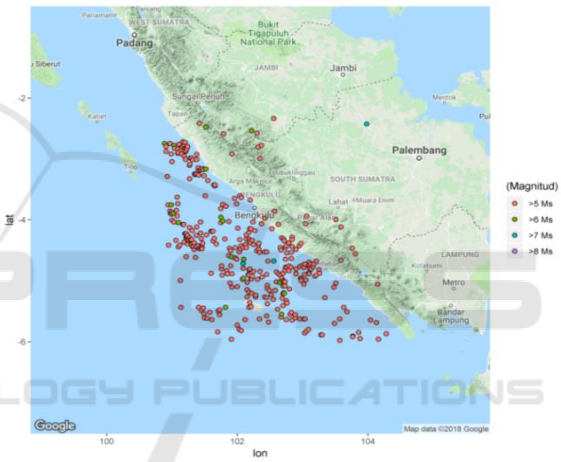

Figure 3: The distribution of earthquake events with magnitudes above Mw 5 that occurred in Bengkulu Province from 2000 to 2016.

Figure 3 describes that the central point of the earthquake event in Bengkulu Province and its partners occurred a lot in the sea and most of the earthquakes were between Mw 5-6.

The results of the calculation and experimental semivariogram images can be seen in Table 2 and Figure 4. The experimental semivariogram plot in figure 4 shows that the first class has a semivariogram value of 0.1875, then slightly decreases and rises back to the maximum semivariogram of 0.2208 at a distance of 0.2773. The semivariogram value fluctuates around 0.1800 and slightly decreases towards a stable value of 1.6 with a value of 0.1639.

Table 2: Experimental semivariogram.

Number of pairs	Distance	Semivariogram
789	0.0709	0.1875
1564	0.1682	0.1786
1935	0.2773	0.2208
2280	0.3889	0.1903
2699	0.4990	0.1871
2966	0.6082	0.2061
3295	0.7201	0.1987
3676	0.8294	0.1685
3560	0.9401	0.1903
3656	1.0504	0.1857
3547	1.1595	0.1691
3473	1.2703	0.1644
3253	1.3815	0.1783
3251	1.4923	0.1699
3205	1.6018	0.1639

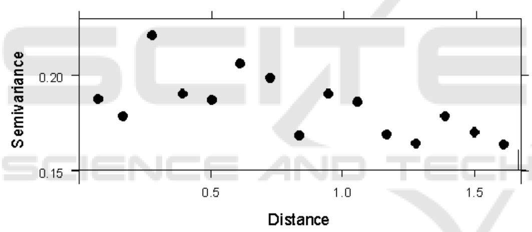

Figure 4: Experimental semivariogram graph.

Furthermore, the fitting of the theoretical semivariogram model is conducted to determine the value of the parameters and the model validation test to determine the theoretical semivariogram model to be used in the kriging method. The best model is chosen based on the smallest value of Residual Sum Squares (RSS). Parameters in the spherical, exponential, Gaussian and linear models are shown in Table 3 while fitting the experimental semivariogram model with the theoretical model is shown in Figure 5.

The best semivariogram model is chosen based on the sum square error value, where the smallest sum square error value shows the best model. Based on the sum square value error the spherical semivariogram model shows the smallest number with a value of 39.68526. This number is slightly smaller than the sum square error in the exponential model which is worth 40.41875. Hence, this kriging estimation will be used two models, namely spherical model with Sill parameter = 0.1907015, Range = 0.07929616, and Nugget = 0 and exponential model with Sill = 0.1898991, Range = 0.01622501, and Nugget = 0. Following are the theoretical semivariogram models used:

Spherical model:

$$\gamma(h) = \begin{cases} 0.1907\left(\frac{|h|}{2(0.0793)} - \frac{|h|^3}{2(0.0793)^3}\right), & |h| < 0.0793, \\ 0.1907, & |h| \geq 0.0793. \end{cases}$$

Exponential model:

$$\gamma(h) = \begin{cases} 0.1898991\left[1 - exp\left(-\frac{h}{0.01622501}\right)\right]; & h > 0, \\ 0, & h = 0. \end{cases}$$

Table 3: Parameter of theoretical semivariogram model.

Variogram Model	Sill (Co+C)	Range (A)	Residual SS
Spherical	0.1907015	0.07929616	39.68526
Exponential	0.1898991	0.01622501	40.41875
Gaussian	0.1968073	0.01198644	60.852
Linier	0.3135558	0	6463.833

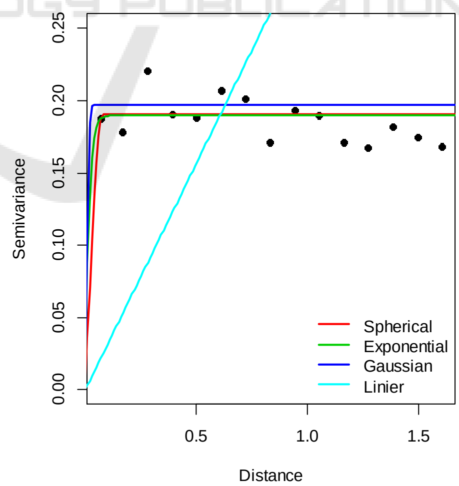

Figure 5: Theoretical semivariogram model.

The central points that are estimated are in the range from 100.00°E to 105.00°E, while based on the latitude position, the minimum data is on 6.00°S, and the maximum is on 2.00°S. The number of points to be estimated is 10,000 points. Estimates are carried out using the ordinary kriging at the determined location to estimate the *Z* value. Here are the results of estimating earthquake strength at points that are not sampled using the spherical model:

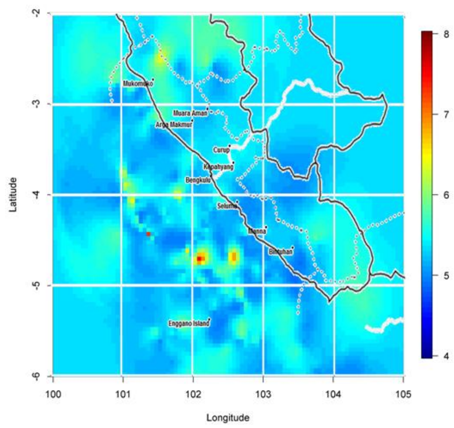

Figure 6: Contour map of the earthquake strength estimation based on the spherical model.

Based on the earthquake strength contour map above, the location of the dark blue is the location of the epicenter of the earthquake with a power of Mw 4 – 5, while the light blue indicates the strength of Mw 5 – 6. From the color on the contour map on the coordinates 100.00°E to 105.00°E and 6.00°S to 2.00°S, the potential for earthquakes with a strength of Mw 4 – 6 is wider than that of an earthquake with a magnitude of more than Mw 6. Besides that, there are also areas with yellow and red colors. The yellow color shows the area with an earthquake strength of Mw 6 – 7 and the red color of the area with an earthquake strength of Mw 7 – 8. The maximum earthquake strength estimation on this contour map is Mw 7.5249. The red areas are concentrated at several points, which are around the points 101.3611°E, 4.4318°S and 102.0833°E, 4.7045°S, so that this area is estimated to have earthquake strength above Mw 7. In the area around 102.5833°E, 4.6590°S there is an area with an orange color which shows the estimated strength of the earthquake in the area around that point is close to Mw 7.

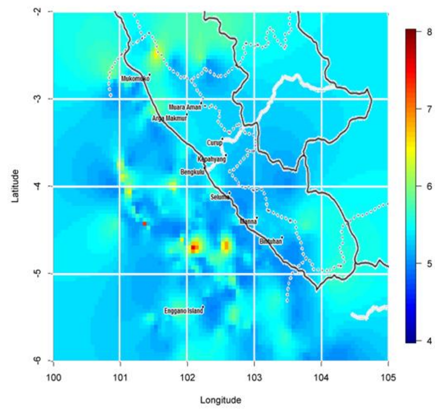

Figure 7: Contour map of the earthquake strength estimation based on the exponential model.

Figure 7 shows the estimated contour of earthquake strength using the exponential model. This contour shows the same pattern as the contour in the spherical model, where light blue and dark blue dominate the estimation area. The dark blue area is the location of the epicenter of the earthquake with a power of Mw 4-5, while the light blue indicates the strength of Mw 5-6. The yellow color shows the area with the earthquake strength of Mw 6 – 7 have lied in the same area in the spherical model. The red color is the area with the earthquake strength of Mw 7 – 8 are around the points 101.3611°E, 4.4318°S, and 102.0833°E, 4.7045°S. Maximum earthquake on the exponential model predicted in the red area is Mw 7.5254.

5 CONCLUSIONS

From the results and discussion concluded that the semivariogram model that can be used is a spherical model with sill = 0.1907015 and range = 0.07929616 and an exponential model with sill = 0.1898991 and range = 0.01622501. Based on the spherical model and exponential model, the potential for earthquakes with a strength of Mw 4 – 6 is wider than that of an earthquake with a magnitude of more than Mw 6. The maximum earthquake strength estimation on the spherical model is Mw 7.5249 and maximum on the exponential model is Mw 7.5254. Using both of models, spherical and exponential model, it's known that the epicentre of the greatest earthquake estimation is on 102.0833°E, 4.7045°S.

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