Stability Analysis of Clock Synchronization Algorithm over Lossy WSNs

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Keywords: Wireless sensor networks, Kalman filter, missing observation, estimation error covariance, stability.

Abstract: Many Wireless sensor networks(WSNs) applications are dependent on clock synchronization technology. The problem of loss of observations for clock synchronization based on Kalman filter estimation is discussed. Firstly, the clock synchronization model of incomplete observation is obtained from the sensor clock reading model. Then, according to the intermittent measurements, Kalman filter formula is deduced and the estimating error covariance recurrence equation is obtained. Considering that the observation loss is random, the statistical convergence of the error covariance is emphatically analyzed. Finally, we show the existence of the critical packet arrival rate, and prove that when the actual packet arrival rate is higher than the critical value, the mean estimation error covariance transitions from unbounded to bounded. Otherwise, we also give the bounds of the covariance of the steady-state error and the boundary of the critical packet arrival rate. Simulation results show the critical packet arrival rate determines the average error covariance transition from unbounded to bounded.

1 INTRODUCTION

Wireless sensor networks(WSNs) facilitate its deployment, low cost and high adaptability to the environment which have been widely used in medical health monitoring, smart home, and environmental monitoring(Akyildiz, 2002). These applications require a large number of synchronized nodes through the coordination of the implementation of a distributed task, so the sensor nodes have a unified time frame which is very important. However, different sensor nodes are affected by factors such as hardware timing device, ambient temperature and other factors. As time goes on, the clocks between nodes will have different deviations. Clock synchronization algorithms (Wu Y C,2011 and Tao, 2012 and Wakabayashi, 2013) are the key technology to achieve the sensor network which has the same time, its core is the estimation of clock parameters, Kalman filter algorithm is used to estimate the clock parameters. In the wireless sensor network clock synchronization technology, this paper uses two-way information exchange mechanism to obtain the observations sent by neighbor nodes. Due to the unreliability of the wireless network, the synchronization node will randomly lose the key observation, then, the stability of the Kalman filter will be greatly affected. This paper is very interested in the loss of observations of the Kalman filter estimation process.

The author have built the state transition equation with relative clock offset and fixed time delay in (Wang, 2014), and have analyzed the presence of Kalman filtering estimation packet loss, they believe that when the measured value misses, the Kalman filter is not updated, then the sampling period is random and the discussion based on random sampling convergence properties in (Micheli, 2001) and (Micheli and Jordan, 2002). With the (Wang, 2015), the Kalman filter update step and the error covariance iteration are random and all depend on the random arrival of the measured values. The authors build the Markov model with packet loss and establish the sufficient and necessary conditions for the stability based on the peak covariance stability theory in (Alexiadis, 1999). The authors of research (Moayedi, 2010) studied adaptive Kalman filtering, and took the mixed uncertainty of measured-value latency and packet loss into account. It is a novel research field to study the effect of loss of measurement on clock synchronization stability. In this paper, focus on any pair of sensor nodes which can be used in sensor networks, and establish the

Hu, Y., Duan, S., Li, K., Xu, X., Wu, Y. and Han, K.
Stability Analysis of Clock Synchronization Algorithm over Lossy WSNs.
In *3rd International Conference on Electromechanical Control Technology and Transportation (ICECTT 2018)*, pages 223-227
ISBN: 978-989-758-312-4

clock synchronization model of incomplete observation. The error covariance iterative equation of the prior form is obtained by re-deriving the Kalman filter process based on the loss of observed valued. Since the random measurement is missing, the error covariance iteration is a random process, so this paper studies the statistical properties of the estimation error covariance.

2 PROBLEM FORMULATION

We consider two sensor node $\{S_i, S_j\}$, which can communicate with each other in the sensor network. Because of the crystal oscillator and the sensor itself is different, so each sensor node has one analogy clock. The discrete clock reading model is follow as:

$$c_i(k) = k\tau_0 + \vartheta_i(k-1) + [\beta_i(k) - 1]\tau_0 \tag{1}$$

Where τ_0 is sampling period, $\vartheta_i(k)$ and $\beta_i(k)$ denote the accumulated clock offset and instantaneous clock skew of the node S_i at the k sampling, respectively.

In order to achieve sensor node clock synchronization, we assume that the clock reading of node S_j is accurate, and the node S_i is the node to be synchronized at any time, Then the goal of clock synchronization is to correct the clock read $c_i(k)$ of node S_i as node S_j clock reading. According to the discrete clock reading model, the primary task of clock synchronization is to track the clock skew and the accumulated clock offset.

We choose $x_i(k) = [\beta_i(k) \quad \vartheta_i(k)]^T$ as the state variable, can be used to obtain the S_i clock parameter evolution model:

$$x_i(k) = Ax_i(k-1) + w_i(k) \tag{2}$$

Where the state transition matrix $A = \begin{bmatrix} a & 0 \\ a\tau_0 & 1 \end{bmatrix}$, process noise is $w_i(k)$, and satisfies $E[w_i(k)] = 0$, $E[w_i(k)w_i^T(k)] = \sigma^2 \mathrm{I}_2$.

In order to establish the relationship between adjacent nodes, the timestamp exchange process can be modeled as:

$$T_2^{\{i,j\}} - \vartheta_j(t) = T_1^{\{i,j\}} - \vartheta_i(t) + d_{ij} + X_k^{\{i,j\}} \tag{3}$$

$$T_3^{\{i,j\}} - \vartheta_j(t) = T_4^{\{i,j\}} - \vartheta_i(t) - d_{ij} - Y_k^{\{i,j\}} \tag{4}$$

Where d_{ij} is the fixed time-delay part when the node S_i and the node S_j are performing bidirectional information exchange, and $X_k^{\{i,j\}}$ and $Y_k^{\{i,j\}}$ denote the variable delay part. Variable delay involves a large number of independent stochastic processes, so suppose $X_k^{\{i,j\}}$ and $Y_k^{\{i,j\}}$ are independent identically distributed Gaussian random variables with mean 0 and variance σ^2.

The actual wireless sensor network has a series of unreliable factors, often resulting in the time stamp in the transmission process of delay or loss. The binary variable γ_k is introduced to indicate whether the observed value at time k reaches the destination node, $\gamma_k = 1$ indicates that the observed value reaches the destination node successfully, and $\gamma_k = 0$ indicates that the observed value is lost, and the observed packet loss at different time is independent of each other.

Simultaneous expressions (3) and (4), the intermittent observation model is expressed as:

$$y_{i,k} = \gamma_k(Cx_i(k) + v_i(k)) \tag{5}$$

Where $y_k = T_{r,k}^{\{i,j\}} - T_{s,k}^{\{i,j\}}$, $C = [0 \quad -2]$, $v_i(k)$ is Gaussian white noise with mean zero and covariance R.

3 STATISTICAL PROPERTIES OF ITERATIVE OF ERROR COVARIANCE

In WSNs, there will inevitably be a loss of observations, and seriously affect the stability of the estimation based on Kalman filter. In this paper, we focus on the influence of missing values on the estimation stability based on Kalman filter, and then get the influence of missing values on the clock synchronization stability.

According to the intermittent observation model, the covariance of the output noise is defined as:

$$P(v_i|\gamma_k) = \begin{cases} N(0,\mathrm{R}), \gamma_k = 1 \\ N(0,\sigma^2\mathrm{I}), \gamma_k = 0 \end{cases} \tag{6}$$

When the observed value is lost, the destination node is equivalent to receiving a noise with a variance of infinity. Next, we re-derive the Kalman filtering process based on the loss of observed values, the kalman filter is as follows:

Prediction step:

$$\hat{x}_{k+1|k} = A\hat{x}_{k|k} \tag{7}$$

$$P_{k+1|k} = AP_{k|k}A^T + Q \tag{8}$$

When the observations are lost, the σ in the Kalman gain tends to infinity, then $(\sigma^2 I)^{-1} \to 0$, the update step becomes:

$$\hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + \gamma_{k+1}K_{k+1}(y_{k+1} - C\hat{x}_{k+1|k}) \tag{9}$$

$$P_{k+1|k+1} = P_{k+1|k} - \gamma_{k+1}K_{k+1}CP_{k+1|k} \tag{10}$$

The Kalman gain is $K_{k+1} = P_{k+1|k}C^T(CP_{k+1|k}C^T + R)^{-1}$.

Substituting (10) into (7) and for simplicity, let $P_{k+1} = P_{k+1|k}$, then we can get the iterative formula of P_k :

$$P_{k+1} = AP_kA^T + Q - \gamma_k AP_kC^T(CP_kC^T + R)^{-1}CP_kA^T \qquad (11)$$

For any initial value P_0 , the error covariance sequence $\{\gamma_k\}_{k=0}^{\infty}$ is also random, since the observed arrival sequence $\{P_k\}_{k=0}^{\infty}$ is random. Therefore, this paper studies the statistical properties of error covariance, focusing on the convergence of $E[P_{k+1}]$.

$E[P_{k+1} | P_k]$ is modelled as a modified Riccati differential equation (MARE):

$$g_\lambda(X) = AXA^T + Q - \lambda AXC^T(CXC^T + R)^{-1}CXA^T \qquad (12)$$

Where $\lambda = \Pr[\gamma_k = 1]$ is the statistical probability of arrival of the observed value.

Since $E[P_{k+1}] = E[E[P_{k+1} | P_k]] = E[g_\lambda(P_k)]$, the statistical convergence of $E[P_{k+1}]$ is obtained by analysing the convergence of $E[g_\lambda(P_k)]$.

4 STABILITY ANALYSIS

The estimated stability directly reflects the stability of the clock synchronization. If the clock parameters are estimated inaccurately, the logical clock of the sensor nodes will not be synchronized, and a series of tasks that rely on clock synchronization will not be completed. So this section of the clock parameter estimation for the stability analysis of the target is to ensure process stability under the premise of Kalman filter, calculate the minimum value of the arrival rate of observation that is, the critical observations of arrival rate, also calculate the convergence range of error covariance matrix.

In order to facilitate the proof of the theorem, we give an auxiliary function:

$$\Phi(K, X) = (1-\lambda)(AXA^T + Q) + \lambda(FXF^T + V) \qquad (13)$$

Where $F = A + KC$, $V = Q + KRK^T$, $X = \mathbb{R}^{n\times n} \ge 0$, $R \ge 0$ and $Q \ge 0$ 。

In this section, theorem 1 is given to prove the convergence of MARE, that is, the Riccati differential equation is bounded in the steady state, and then we prove that the steady-state mean error covariance matrix $E[P_{k+1} | P_k]$ is bounded.

Theorem 1: According to the auxiliary function $\Phi(K, X)$, suppose there exists a matrix $\hat{K}$ and a positive definite matrix $\hat{P}$, and satisfy $\hat{P} > 0$ and $\hat{P} > \Phi(\hat{K}, \hat{P})$, then:

A. For any initial value $P_0 \ge 0$, MARE converges, and the convergence value is independent of the initial value, that is $\lim_{t\to\infty} P_t = \lim_{t\to\infty} g_\lambda^t(P_0) = \bar{P}$.

B. $\bar{P}$ is the only positive definite solution of MARE.

Theorem 2 gives the conditions for the existence of the critical arrival rate λ_c . When $\lambda_k > \lambda_c$, for all initial conditions, the mean state covariance $E[P_k]$ is bounded; when $\lambda_k \le \lambda_c$, for any initial condition, the mean state covariance divergence.

Theorem 2: if $\left(A, Q^{1/2}\right)$ is controllable, (A, C) can be observed, then there is $\lambda_c \in [0,1]$, satisfying:

$\lim_{t\to\infty} E[P_t] = +\infty$, for $0 \le \lambda \le \lambda_c$ and $\exists P_0 \ge 0$;

$\lim_{t\to\infty} E[P_t] \le M_{P_0}$, for $\lambda_c \le \lambda \le 1$ and $\forall P_0 \ge 0$;

Where $M_{P_0} \ge 0$, dependent on initial conditions $P_0 \ge 0$.

Theorem 3 gives the expression of the lower bound and upper bound of the arrival rate λ_c of critical observations.

Theorem 3: If the critical observation arrival rate λ_c exists, then:

$$\underline{\lambda} = \arg\inf f_\lambda\left[\exists \hat{S} \mid \hat{S} = (1-\lambda)A\hat{S}A + Q\right] = 1 - \frac{1}{a^2} \qquad (14)$$

$$\bar{\lambda} = \arg\inf f_\lambda\left[\exists(\hat{K}, \hat{X}) \mid \hat{X} > \Phi(\hat{K}, \hat{X})\right] \qquad (15)$$

Where $a = \max_i |\sigma_i|$ and σ_i is the eigenvalue of matrix A , namely $\underline{\lambda} \le \lambda_c \le \bar{\lambda}$.

The calculation of the upper bound of the critical measurement value is equivalent to an iterative process of LMI feasibility problem. The feasibility of LMI is shown as follows.

If $\left(A, Q^{1/2}\right)$ is controllable, (A, C) is observable, assuming that K and $X > 0$ are present and that $X > \Phi(K, X)$ is satisfied. Let $F = A + KC$, then:

$$X > (1-\lambda)AXA^T + \lambda FXF^T + Q + \lambda KRK^T ,$$

Using the Shure complement decomposition, we get:

$$\Psi_\lambda(Y, Z) = \begin{bmatrix} Y & \sqrt{\lambda}(YA + ZC) & \sqrt{1-\lambda}YA \\ \sqrt{\lambda}(A^TY + C^TZ^T) & Y & 0 \\ \sqrt{1-\lambda}A^TY & 0 & Y \end{bmatrix} > 0$$

Since $\Psi(aY, aK) = a\Psi(Y, K)$, must be bound $Y \le I$.

In summary, the upper bound of the critical measurement arrival rate $\bar{\lambda}$ is the solution of the following optimal problem:

$$\bar{\lambda} = \arg\min_{\lambda} \Psi_\lambda(Y, Z) > 0 \qquad 0 \le Y \le I \qquad (16)$$

For an ideal communication network, if (A, Q) is stable and (A, C) is observable, P_k will converge to a certain value. However, for lossy communication networks, there will be no uniquely determined error covariance matrix at Kalman filter steady state, and only the boundary of the mean estimation error covariance $E[P_t]$ can be calculated. Theorem 4 gives the expression of the lower bound and upper bound of the mean error covariance $E[P_k]$ at steady state.

Theorem 4: Suppose that $(A, Q^{1/2})$ is controllable, (A, C) is observable, and $\lambda > \bar{\lambda}$ is satisfied. Then, for $E[P_0] \geq 0$, there exists $0 < \lim_{k\to\infty} S_k = \bar{S} \leq E[P_k] \leq \bar{V} = \lim_{k\to\infty} V_k$.

Where $\bar{S}$ is the solution of the equation $\bar{S} = (1-\lambda)A\bar{S}A^T + Q$, $\bar{V}$ is the solution of the equation $\bar{V} = g_\lambda(\bar{V})$.

The lower bound of average error covariance equation is $\bar{S}$, it is easy to think of the use of standard Lyapunov equation. For the upper bound of the mean error covariance, $\bar{V}$ is obtained by solving the equivalent semidefinite programming problem.

Assuming $\lambda > \bar{\lambda}$, the solution of matrix $\bar{V} = g_\lambda(\bar{V})$ is obtained by the following optimal problem:

$$\begin{cases} \arg\max_{v} \quad Trace(V) \\ s.t. \begin{bmatrix} AVA^T - V & \sqrt{\lambda}AVC^T \\ \sqrt{\lambda}CVA^T & CVC^T + R \end{bmatrix} \geq 0 \end{cases}$$

Where $\begin{bmatrix} AVA^T - V & \sqrt{\lambda}AVC^T \\ \sqrt{\lambda}CVA^T & CVC^T + R \end{bmatrix} \geq 0$ is derived from the decomposition of $\bar{V} \leq g_\lambda(V)$ using Shure.

5 NUMERICAL SIMULATION

The wireless sensor network clock synchronization model is denoted as (A, C, Q, R), where $A = \begin{bmatrix} 1.25 & 0 \\ 1 & 1 \end{bmatrix}$, $C = [0 \quad -2]$, $R = 2.5$, $Q = 100\square_{2\times 2}$. Since the observation matrix C is irreversible, then there is no $\hat{K}$, so that the $F = A + \hat{K}C$ is equal to the zero matrix. In this case, the critical measurement arrival rate can't be calculated exactly, so only the lower and upper bounds can be calculated. The green and purple solid line in Figure 1 represents the lower and upper bound of the critical measurement arrival rates, respectively. By theorem 2, the lower bound of the critical arrival rate is $\underline{\lambda} = 0.36$. In this paper, when the observed arrival rate is 0, the error covariance infinity, it is clear that, with the actual observation of the arrival rate gradually increased, when equal to 0.36, the average error covariance lower bound sharp decline, approximately converges at $\lambda = 0.6$. Similarly, the upper bound of the mean error covariance begins to decrease at $\lambda = \bar{\lambda}$, and eventually converges.

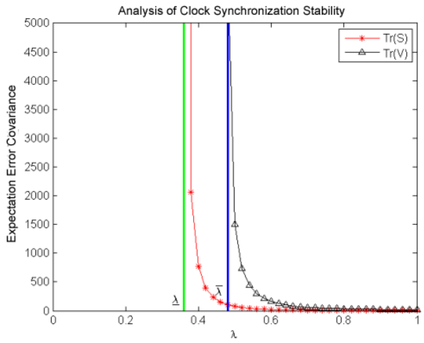

Figure.1. Upper and lower bounds transition from unbounded to bounded

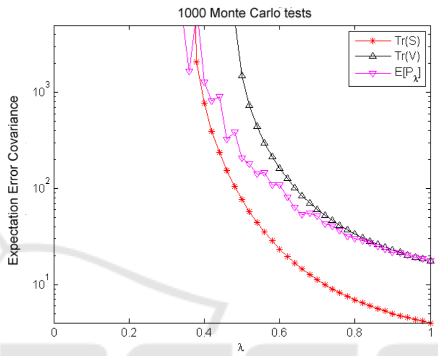

Figure.2. Monte Carlo test

The Monte Carlo simulation is used to simulate the real clock synchronization process. The inverted triangle curve in Figure. 2 represents the synchronization error covariance $E[P_k]$, which is obtained from 1000 Monte Carlo experiments. The star of red curve and the black positive triangle curve represents the lower bound and upper bound of the steady-state error covariance, respectively, calculated by the modified Riccati differential equation. In this paper, when the actual arrival rate is 0, $\lim_{k\to\infty} E[P_k]$ is equal to infinity. It is obvious that the synchronization error covariance based on Kalman filter is a monotonically decreasing function of the arrival rate. Note that when $\lambda = 0.36$, the synchronization error covariance into the lower and upper bound including area, when the measured arrival rate is larger than the critical value, the synchronization error covariance convergence, and its convergence range in the lower and upper bound, to prove the correctness of the theory.

The article propose a static state estimator for linear systems:

$$\hat{x}_{t+1}^s = A\hat{x}_t^s + \gamma_{t+1}K_s(y_t - \hat{y}_t) \qquad (17)$$

Where, K_s represents the static gain constant.

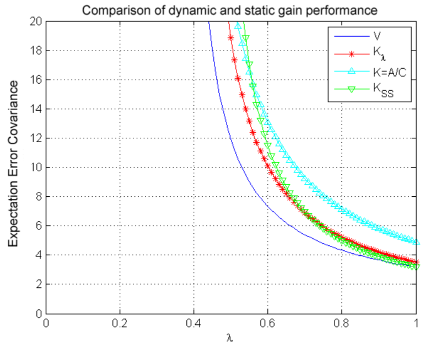

Figure.3. Comparison of dynamic and static gain performance

Three static gain methods are proposed in the reference (Sinopoli, 2004), and the Kalman filter gain in this paper belongs to the dynamic gain. By comparing the performance of these three kinds of static gain, it is shown that the Kalman filter is still the best when the measurement value is lost. Figure 3 compares the performance between the dynamic Kalman filter gain and three kinds of static gain, the star of red curve represents the average error covariance of dynamic gain, with the actual observation arrival rate increases, the most close to the upper bound of convergence theory analysis. It is shown that the steady-state error covariance is minimum and the estimation algorithm is optimal.

6 CONCLUSIONS

This paper prove that there exists the critical arrival rate of the measured value, and the average error covariance changes from unboundedness to boundedness with the arrival rate of the actual measured value increasing and exceeding the critical arrival rate. A numerical algorithm is proposed to calculate the upper and lower bound of the critical arrival rate and the boundary of the steady-state mean error covariance. The simulation results show that the average error covariance divergence and the clock parameter estimation are unstable when the actual measured value arrival rate is less than the critical value. This theory can also guide the resource allocation of wireless sensor networks. If the current synchronization accuracy does not meet the requirements, we can get better synchronization accuracy by improving the communication resources.

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