A Guidance and Control Law for Autonomous Formation of Quadrotors

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Abstract: Formation flying means two or more flight vehicles maintain an organizational flight mode. In autonomous formation, the guidance is normally divided into formation configuration and task assignment. This paper is focused on the formation configuration of quadrotors from their initial random positions to an expected configuration in response of the leader quadrotor. Based on the position and velocity of the leader, the formation controller will generate the guidance and control instructions from the relative dynamic system. The updated positions of the followers are compared with the expected positions and furtherly processed by PI controllers to form speed instructions for the followers, and feedforward controllers are included to shape commands to provide better tracking performance. Simulations with five quadrotors result show that the designed guidance and control algorithm can help quickly achieve and keep the desired formation configuration even to follow complex motions of the leader quadrotor.

1 INTRODUCTION

The multi rotor unmanned aerial vehicle (UAV), as an important member of the UAV family, has been evolving rapidly with the development of the control theory and high technology (Giulietti F, 2005). Compared with fixed wing unmanned aerial vehicle and unmanned helicopter, the multi rotor aircraft has the advantages in terms of vertical take-off and landing, hovering, simple mechanical structures, easy maintenance, flexible operability and so on, in spite of the disadvantage of poor load capability due to limits in blade size, speed, and flapping. Therefore, the idea of multi UAV cooperative formation to accomplish complex mission has been proposed to employ its potential value while avoid the disadvantage (Sun N P, 2014; Samaneh H S, 2015). Multi UAV cooperative formation flying has the following advantages over single UAV (Escareno J, 2013; Rudio J D 2014):

(1) Redundancy is increased, because any UAV in the formation can take the place of the others;

(2) Formation flying is more adaptive to complex tasks;

(3) Information perceived in formation is more stereoscopic and more accurate

In autonomous formation, the guidance can be divided into two phases of formation configuration and task assignment, in which the formation configuration can be further divided into three sub-phases of configuration generating, configuration keeping and configuration adjustment. Current formation control methods include behaviour mode based formation, 'leader-follower' structure based formation, virtual structure based formation, artificial potential field based formation, etc (Salim N D, 2014; Karimoddini A, 2013), as compared in Table 1.

The behaviour mode based formation divides the behaviour response of each member to its input information into a number of fixed modes, and completes formation control by assigning weight. In the 'leader-follower' structure based formation, the follower sense the information of the leader and form and keep the configuration by changing its speed. The virtual structure based formation let the members in formation follow their respective trajectories by defining a virtual leader. The artificial potential field based formation keep the configuration and avoid collision by imitating the repulsion and gravitation between particles in a molecular structure.

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Table 1: Current mainstream formation algorithms

Algorithm	Advantages	Disadvantages
Behaviour mode	Formation flexibility; High reliability; Strong robustness	Poor formation keeping performance
Leader-follower' structure	Simple principle; Easy to implement	Poor environmental adaptability; Poor reliability
Virtual structure	Accurate formation keeping	Hard to expand
Artificial potential field	High reliability	Slow response

In this paper, to achieve good formation flexibility and keeping performance, while to be easy to implement, a guidance and control law for autonomous formation of quadrotors is based on combination of PI control and feedforward control, and the followers track the guidance instructions generated by the formation controller in order to keep the formation configuration.

2 MODEL DESCRIPTION

The quadrotors in this paper are small and driven by batteries. Besides the rotors, all airframe parts are taken as rigid. So, the control models include rigid body kinematics model, rigid body dynamic model, control allocation model and power system model as shown in Figure 1.

2.1 Power system model

The input and output relation of the battery, throttle, ESC and motor is described by the power system model. Among them, the motor throttle σ is input, the motor speed ϖ is output, T_m is the dynamic response constant of motor. The power system model is shown in Figure 2, and the mathematical model is described as:

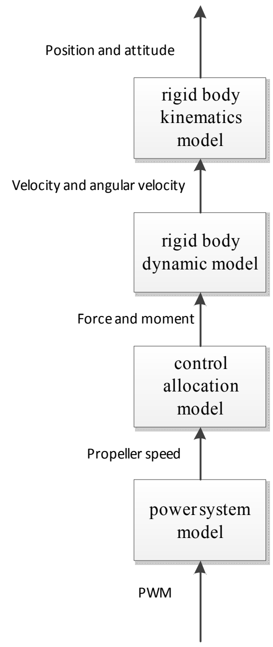

Figure 1: Quadrotor control model

$$\varpi = \frac{1}{T_m s + 1}(C_R \sigma + \varpi_b) \qquad (1)$$

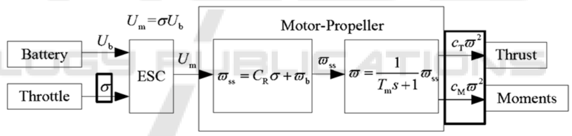

Figure 2: Signal transmission diagram of power system

2.2 Control allocation model

The input and output relation of the motor speed and force and moment is described by the control allocation model. There are two ways to control distribution of the '＋' configuration and the '×' configuration, as shown in Figure 3.

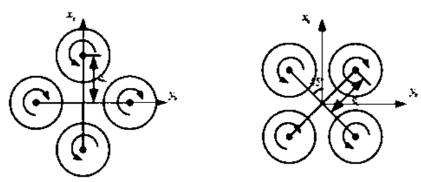

Figure 3: The '＋' configuration and '×' configuration

For the ‘ + ’ configuration quadrotor, the mathematical model is as follows:

$$\begin{bmatrix} f \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} c_T & c_T & c_T & c_T \\ 0 & -dc_T & 0 & dc_T \\ dc_T & 0 & -dc_T & 0 \\ c_M & -c_M & c_M & -c_M \end{bmatrix} \begin{bmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{bmatrix} \quad (2)$$

For the’ × ’ configuration quadrotor, the mathematical model is as follows:

$$\begin{bmatrix} f \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} c_T & c_T & c_T & c_T \\ \frac{\sqrt{2}}{2}dc_T & -\frac{\sqrt{2}}{2}dc_T & -\frac{\sqrt{2}}{2}dc_T & \frac{\sqrt{2}}{2}dc_T \\ \frac{\sqrt{2}}{2}dc_T & \frac{\sqrt{2}}{2}dc_T & -\frac{\sqrt{2}}{2}dc_T & -\frac{\sqrt{2}}{2}dc_T \\ c_M & -c_M & c_M & -c_M \end{bmatrix} \begin{bmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{bmatrix} \quad (3)$$

Where, f is the total tension acting on the body, while τ_x , τ_y , and τ_z are the moments produced by propellers along the pitching, roll and yaw axes respectively.

2.3 Control rigid body model

The control rigid body model contains dynamic and kinematic parts to determine the Euler angles θ, ϕ, ψ as in the following mathematical model:

$$\begin{aligned} {}^e\dot{p} &= {}^e v \\ \dot{\Theta} &= W\,{}^b\omega \\ {}^b\dot{v} &= -[{}^b\omega]\times {}^b v + gR^T e_3 - \frac{f}{m}e_3 - K_{drag}v \\ J\,{}^b\dot{\omega} &= -{}^b\omega \times (J\,{}^b\omega) + G_a + \tau \end{aligned} \quad (4)$$

Where, p , v , Θ , ω represent position, attitude angle and angular velocity respectively, the left tag e is expressed in the inertial system and the left tag b is expressed in the body coordinate system. The W is the coordinate transformation matrix of the body coordinate system to the inertial system. The ω and J are the mass and inertia of the quadrotor. G_a is the gyroscopic moment and it is overlooked here. All these quantities are vector forms.

3 FORMATION ALGORITHM

3.1 Formation objective

The formation configuration is defined in the leader’s speed coordinate system as x^w and y^w , it can also be converted to distance and azimuth angle. The relative kinematic model is shown in Figure 4.

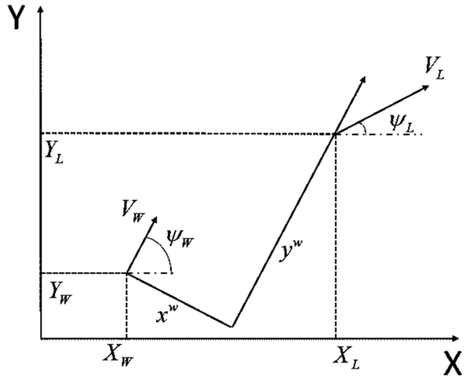

Figure 4: Geometric relation diagram of formation flying

The mathematical model is described as the following model:

$$\begin{aligned} \dot{x}^w &= V_L \cos(\psi_L - \psi_W) + \dot{\psi}_W y^w - V_W \\ \dot{y}^w &= V_L \sin(\psi_L - \psi_W) - \dot{\psi}_W x^w \end{aligned} \quad (5)$$

In the inertial coordinate system, the expected position of followers can be computed through the measured leader’s position and relative kinematic relational expression as:

$$\begin{aligned} Y_{W.Ex} &= Y_L - x^w \sin\psi_L - y^w \cos\psi_L \\ X_{W.Ex} &= X_L - x^w \cos\psi_L - y^w \sin\psi_L \\ V_{Wx.Ex} &= V_{Lx} \\ V_{Wy.Ex} &= V_{Ly} \end{aligned} \quad (6)$$

The state error of the follower can be calculated by its expected state and current state, as shown in Figure 5.

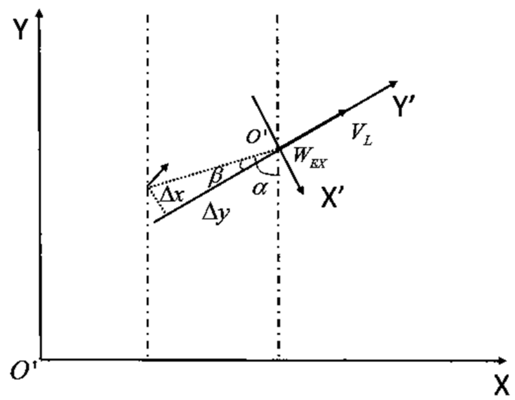

Figure 5: Expected and current states of the follower

$$
\begin{aligned}
&\alpha = \arctan \frac{Y_{W.Ex} - Y_w}{X_{W.Ex} - X_w} \\
&\beta = \alpha - \psi_L \\
&D = \sqrt{(X_{W.Ex} - X_w)^2 + (Y_{W.Ex} - Y_w)^2} \\
&\Delta x = x - x^w = D \cos \beta \\
&\Delta y = y - y^w = D \sin \beta \\
&\Delta v_x = V_{Wx.Ex} - V_{Wx} = V_{Lx} - V_{Wx} \\
&\Delta v_y = V_{Wy.Ex} - V_{Wy} = V_{Ly} - V_{Wy}
\end{aligned} \tag{7}
$$

The control objective is for the follower to fly from current state to the expected state with minimal errors:

$$(\Delta x, \Delta y, \Delta v_x, \Delta v_y) \equiv (0,0,0,0) \tag{8}$$

3.2 Formation algorithm

To achieve the above control objective, the formation controller is designed according to the forward and lateral channels.

(1) The forward channel. The control amount is Δx , and the differential effects of follower's expected position is added to the speed control as a feedforward control:

$$V_{Wxc} = k_{xp}\Delta x + k_{xI}\int_0^t \Delta x dt + \frac{dX_{W.Ex}}{dt} \tag{9}$$

(2) The lateral channel. Similar to the forward channel, the lateral control is as:

$$V_{Wyc} = k_{yp}\Delta y + k_{yI}\int_0^t \Delta y dt + \frac{dY_{W.Ex}}{dt} \tag{10}$$

4 SIMULATIONS

The control structure of the bottom adopts double loop PID, the formation controller of the top adopts the combination of PI and position instruction differential feedforward. The block diagram of the formation control structure is shown in Figure 6.

The parameters of the quadrotor used in the simulation are shown in Table 2.

Table 2: Quadrotor parameters

Parameter	Value	Parameter	Value
m / kg	1.15	c_T	$1.57*10^{-5}$
l / m	0.177	c_M	$7.5*10^{-7}$
$I_x / kg\cdot m^2$	0.0109	$I_z / kg\cdot m^2$	0.021
$I_y / kg\cdot m^2$	0.0109	K_{drag}	0.0013

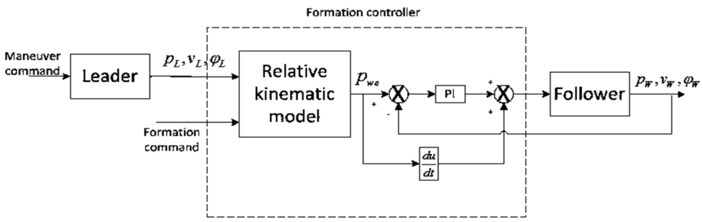

Figure 6: Block diagram of formation control

4.1 Simulations of single quadrotor control

To verify the single quadrotor control law, the initial position of quadrotor is set to $(0,0,3)$ m and the initial state of quadrotor is hover. The quadrotor starts from the initial position and tracks the given trajectory instructions. The 3D graphs of trajectory tracking are shown in Figure 7.

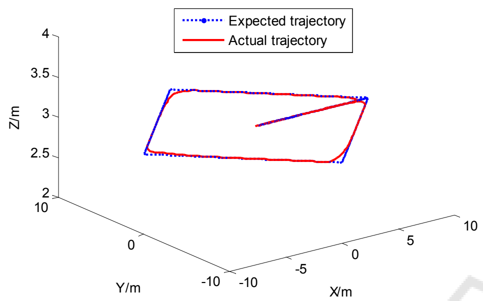

Figure 7: 3D graph of trajectory tracking

It can be seen from Figure 7, the actual tracking trajectory is approximately coincident with the desired trajectory, the height changes when the quadrotor turns, but the change is so small that can be ignored.

4.2 Formation flight simulation of five quadrotors

To verify the performance of multi aircraft formation, five quadrotors are taken as examples. Table 3 presents the initial and desired formation position of five quadrotor, followers read leader for reference to guide. The detailed formation configuration is shown in Figure 8.

Table 3: Initial and desired position for five quadrotor formation

Quadrotor	$x_0, y_0, z_0 / m$	$\Delta x, \Delta y, \Delta z / m$
Leader	0,0,3	/
Follower 1	−2,3,3	−1,−1,0
Follower 2	−1,−2,3	−2,−2,0
Follower 3	−2,−1,3	1,−1,0
Follower 4	−1,−4,3	2,−2,0

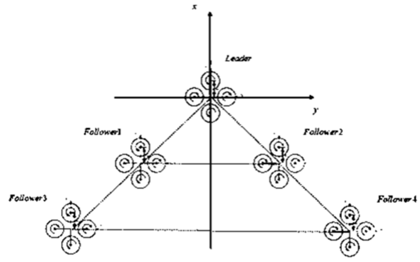

Figure 8: Five quadrotor formation configuration

In order to test the formation effect of various motion situation in the leader quadrotor, let the leader quadrotor carry out all kinds of movement including uniform motion, uniform transmission motion in the forward and lateral directions, curvilinear motion and so on. Speed command and actual speed response of the leader quadrotor in the forward and lateral directions is shown in Figure 9.

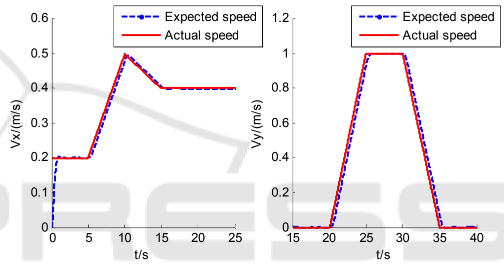

Figure 9: Speed command and actual speed response of the leader quadrotor in two direction

Figure 10 gives the result of the simulated five quadrotor formation.

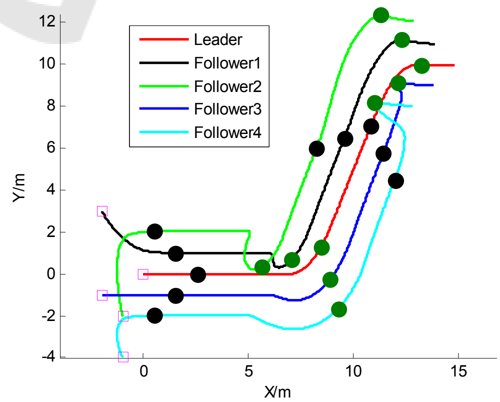

Figure 10: Five quadrotor formation configuration

As can be seen from Figure 10, in the forward and lateral directions, four following quadrotors accurately reach the desired positions in the formation.

The altitude and yaw angle channel is decoupled, and the change is so small that it is negligible.

The variation of speed in the forward and lateral directions of four follower quadrotor and the leader quadrotor in the formation is shown in Figure 11.

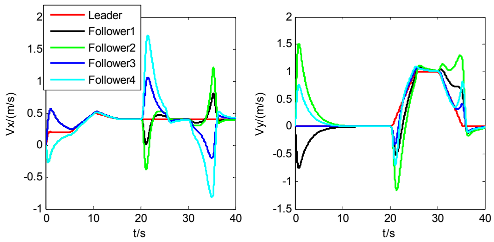

Figure 11: The speed in the forward and lateral directions of five quadrotors

As can be seen from Figure 10 and Figure 11, leader and followers form a stable formation at about 7 seconds. After that, formation configuration can be maintained nice in the case of a variety of movements in the leader quadrotor. Among them, under the condition of unidirectional linear uniform motion and uniformly variable motion of the leader, the speed in the forward and lateral directions of any follower will converge to the speed value of the leader. Under the condition of curvilinear motion of the leader, the speed of the follower in the outer circle will become larger in order to keep the formation configuration, the speed of the follower in the inner circle will become smaller and even move in reverse in order to keep the formation configuration. In this process, although the speed of any follower will not converge to the speed value of the leader, the formation configuration has been kept very well. If the leader quadrotor is allowed to continue to do circular motion, the speed in the forward and lateral directions of any follower will converge to a corresponding value.

5 CONCLUSIONS

This paper uses the combination of classical PI control and feedforward control to design the guidance law of autonomous formation of multiple quadrotors. Simulation results of single quadrotor trajectory tracking and formation of five quadrotors are given. The accuracy and response speed of trajectory tracking are verified. The formation control law as a top layer controller commands the position and controller of single quadrotor. Simulation results show that the guidance instruction generated by the presented formation control law can guide the followers to form expected formation configuration and keep the formation quite well.

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