Gain-Scheduling Position Control Approaches for Electromagnetic Actuated Clutch Systems

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Abstract: The paper proposes three Gain-Scheduling (GS) control design approaches dedicated to the position control of electromagnetic actuated clutch systems. The initial nonlinear mathematical model of the plant is simplified and next linearized at six operating points to use it in the design approaches. Starting with classical Proportional-Integral (PI) controllers, three GS control versions, namely Lagrange, Cauchy and Switching GS, are next designed to ensure zero steady-state control error and the switching between PI controllers. All control solutions are tested and validated on the nonlinear model of the plant and a comparative analysis is included.

1 INTRODUCTION

The paper is focused on the development of three gain-scheduling control solutions (CSs) for electromagnetic actuated clutch systems (the plant) in the framework of electrically driven clutches which belong to vehicular power train system.

Several classical and modern control solutions for electromagnetic actuated clutch systems have been proposed recently including the following ones: two electromagnetic clutch water pumps that can control the coolant in terms of a nonlinear servo are designed in (Shin et al., 2013) using a model-free approach based on an online self-organizing adaptive fuzzy controller. A nonlinear feedforward–feedback control scheme is proposed in (Gao et al., 2014) to improve the performance of the position tracking control that consist of steady-state-like control, feedforward control based on reference dynamics, and state dependent feedback control. The design of an estimator for each clutch of the dual clutch transmission is carried out in (Oh et al., 2014) using shaft model-based observer, unknown input observers, and adaptive output torque observer. A position controller for a clutch actuator is suggested in (Losero et al., 2016) using a quasi-Linear Parameter Varying (LPV) Takagi-Sugeno representation and, in order to use unmeasured values in the controller, a Takagi-Sugeno switching observer. A parallel adaptive feedforward and bang-bang controller is proposed in (Temporelli et al., 2017) to control the clutch pressure with an electromechanical clutch actuator. A controller for an electromagnetic linear clutch actuator is given in (Ranjan et al., 2017).

Since linear controllers can usually ensure the CS performance specifications only in some neighbourhood of a single operating point, the Gain-Scheduling (GS) technique is popular as it generalizes the performance specifications over various operating points. An analysis of GS controllers which can vary slowly and can capture the plant’s nonlinearities and the conditions which guarantee the stability using Lyapunov’s stability theory, robustness and performance of the overall gain-scheduled design are given in (Shamma and Athans, 1990; Veselý and Ilka, 2013), with recent results outlined as follows: a Proportional-Integral (PI) GS CS for second-order LPV systems, which excludes time varying delay and uses a Smith predictor, is given in (Puig et al., 2012). GS deals in (Andonovski et al., 2015) with the adaptation of gains of a robust evolving cloud-based controller.
(RECCo) designed for a class of nonlinear processes; the robust modification of the adaptive laws and the performance analysis are introduced. A practical implementation of RECCo with normalized data space for a heat-exchanger plant is shown in (Andonovski et al., 2016). Other interesting GS (adaptive) control techniques for real practical applications are discussed in (Haidegger et al., 2012; Costa et al., 2015; Precup et al., 2015; Yang and Yan, 2016; Precup et al., 2017).

This paper continues with the modelling of a nonlinear electromagnetic actuator system in Section 2. Six linear PI controllers are designed in Section 3 based on the linearized mathematical models (MMs) of the plant. Three GS controllers are next suggested to ensure the switching between these linear PI controllers and to improve the performance indices. The parameters of the suggested controllers can be relatively easily adapted to the modifications of the operating points. The simulation results and comparisons between the suggested GS controllers are given in Section 4. Section 5 presents the conclusions.

2 MATHEMATICAL MODELING OF CONTROLLED PLANT

In this paper, the controlled plant is an electromagnetic actuator as part of a clutch system. The state-space MM of the electromagnetic actuated clutch is built around a magnetically actuated mass spring damper system, Figure 1 (Di Cairano et al., 2007). The mass moves linearly under the effect of the magnetic force \( F \), which is generated by the coil. Additional forces which act on the mass are generated by the spring and the damper.

\[ \begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= -(k / m)x_1 - (c / m)x_2 + (k_x x_2^2) / [m (k_x + d - x_1)^2], \\
\dot{x}_3 &= -(R (k_x + d - x_1) / (2k_x))x_1 - [1 / (k_x + d - x_1)]x_1 x_3 + [(k_x + d - x_1) / (2k_x)]u, \\
y &= 1000x_1,
\end{align*} \]

with the following characteristic variables: \( u \in [0, 12] \) [V] is the control input, \( x_1 \in [0, 0.004] \) [m] is the mass position, \( x_2 \) [m/s] is mechanical subsystem’s speed and \( x_3 = \lambda \) [V ⋅ s] is the magnetic flux; \( y \) [m] is the output variable, i.e., the measured mass position, \( m = 1 \) [kg] is the mass, \( d = 0.004 \) [m] is the distance between contact position and spring neutral position, \( R = 1.2 \) [Ω] is the resistance, \( c = 700 \) [N ⋅ s/m] coefficient of the damper, \( k = 37500 \) [N/m] is stiffness of the spring, \( k_x = 0.5 \) is a constant \( k_0 = 0.375 \) is a constant, \( i \in [0, 10] \) [A] is the current, and \( F \in [0, 150] \) [N] is the external magnetic force.

To design the proposed CS, the reduced model (2) is linearized at six operating points (o.p.s) with the following coordinates \( P^{(j)}(x_1^{(j)}, x_2^{(j)}, u^{(j)}) \), where \( j \) is the index of the current operating point, \( j = 1, 6 \):

\[ \begin{align*}
P^{(1)}(0.002, 5, 6), P^{(2)}(0.0021, 6, 7.2), \\
P^{(3)}(0.0023, 7, 8.4), P^{(4)}(0.0027, 8, 9.6), \\
P^{(5)}(0.0033, 9, 10.8), P^{(6)}(0.0038, 9.8, 11.76).
\end{align*} \]

The simplified linearized state-space models (Ln-Ms) are
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\[
\Delta \mathbf{x}^{(j)} = A^{(j)} \Delta \mathbf{x}^{(j)} + b^{(j)} \Delta u^{(j)},
\]
\[
\Delta y^{(j)} = c^{T(j)} \Delta \mathbf{x}^{(j)},
\]
\[
\Delta \mathbf{x}^{(j)} = [\Delta \mathbf{x}_1^{(j)}, \Delta \mathbf{x}_2^{(j)}, \Delta \mathbf{x}_3^{(j)}]^T,
\]
\[
\Delta \mathbf{x}^{(j)} \in \mathbb{R}^{3\times1}, \Delta u^{(j)} \in \mathbb{R},
\]
\[
A^{(j)} = \begin{bmatrix} a_{11}^{(j)} & a_{12}^{(j)} & a_{13}^{(j)} \\ a_{21}^{(j)} & a_{22}^{(j)} & a_{23}^{(j)} \\ a_{31}^{(j)} & a_{32}^{(j)} & a_{33}^{(j)} \end{bmatrix},
\]
\[
b^{(j)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},
\]
\[
e^{T(j)} = [1000 \ 0 \ 0],
\]
\[
A^{(j)} \in \mathbb{R}^{3\times3}, b^{(j)} \in \mathbb{R}^{3\times1}, e^{T(j)} \in \mathbb{R}^{1\times3},
\]
with the matrix parameters
\[
a_{11}^{(j)} = -k/m, a_{12}^{(j)} = -c/m, a_{13}^{(j)} = 2k_r - s_{2\alpha} / (m \cdot k_c^2),
\]
\[
a_{21}^{(j)} = s_{2\alpha} / k_s, a_{22}^{(j)} = -R / (2 \cdot k_r), a_{23}^{(j)} = k_s / (2 \cdot k_r).
\]

The variables in (4) are: \(\Delta x_j^{(j)} = x_j^{(j)} - x_j^{(j-1)}\), \(\Delta y_j^{(j)} = y_j^{(j)} - y_j^{(j-1)}\), \(j = 1, 6\), \(\gamma = \sqrt{3}\), representing the differences of the variables \(x_j^{(j)}\) and \(y_j^{(j)}\) with respect to their values at the current operating point \(P^{(j)}\), and referred to as \(x_j^{(j)}\) and \(y_j^{(j)}\), respectively.

The transfer function (t.f) corresponding to LmNs (4) has the general expression
\[
H_j^{(j)}(s) = c^{T(j)} (sI - A^{(j)})^{-1} b^{(j)},
\]
\[
k_j^{(j)} = \frac{k_j^{(j)}}{\prod_{\eta=1}^{3} p_\eta^{(j)}},
\]
\[
H_j^{(j)}(s) = \frac{k_j^{(j)}}{\prod_{\eta=1}^{3} (s - p_\eta^{(j)})} = \prod_{\eta=1}^{3} (1 + T_\eta^{(j)}),
\]
where \(k_j^{(j)} = k_j^{(j)} / \prod_{\eta=1}^{3} p_\eta^{(j)}\), \(I\) is the third-order identity matrix and the time constants of the plant are \(T_\eta^{(j)} = -1/p_\eta^{(j)}\), \(\eta = \sqrt{3}, j = 1, 6\). The numerical values of the t.f.s. \(H_j^{(j)}(s)\) at six operating points are synthesized in Table 1 (Dragos et al., 2012b).

### 3 DESIGN OF POSITION CONTROL SOLUTIONS

Four CSs are developed and analyzed as follows to obtain good performance of electromagnetic actuated clutch systems: PI controller and three PI gain-scheduling controllers.

![Table 1: The numerical values of \(H_j^{(j)}(s)\).](image)

<table>
<thead>
<tr>
<th>(P_j^{(j)})</th>
<th>(H_j^{(j)}(s), j = 1, 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_0)</td>
<td>(\frac{1}{(1 + 0.064s)(1 + 0.016s)(1 + 0.0016s)})</td>
</tr>
<tr>
<td>(P_2)</td>
<td>(0.38)</td>
</tr>
<tr>
<td>(P_3)</td>
<td>(0.47)</td>
</tr>
<tr>
<td>(P_4)</td>
<td>(0.58)</td>
</tr>
<tr>
<td>(P_5)</td>
<td>(0.745)</td>
</tr>
<tr>
<td>(P_6)</td>
<td>(0.9)</td>
</tr>
</tbody>
</table>

### 3.1 Design of PI Controllers

Depending on the operating points, six control solutions with PI controllers have been designed to ensure a small overshoot, offering a adequate phase margin (\(\varphi_m = 60^\circ\)) and a relatively small settling time.

The Modulus Optimum method is applied to initially tune the parameters of PI controllers (Åström and Hägglund, 1995):
\[
H_j^{(j)}(s) = k_j^{(j)} (1 + s T_j^{(j)}) / s,
\]
where \(k_j^{(j)} = 1/(2 \cdot k_c^{(j)} \cdot T_c^{(j)})\) is the controller gain, \(T_r^{(j)} = T_i^{(j)}\) is the integral time constant. The numerical values of tuning parameters are: \(k_0^{(j)} = 89\), \(T_0^{(j)} = 0.064\), \(k_1^{(j)} = 72.95\), \(T_1^{(j)} = 0.066\), \(k_2^{(j)} = 59.88\), \(T_2^{(j)} = 0.07\), \(k_3^{(j)} = 48.79\), \(T_3^{(j)} = 0.077\), \(k_4^{(j)} = 38.92\), \(T_4^{(j)} = 0.087\), \(k_5^{(j)} = 31.92\), \(T_5^{(j)} = 0.098\).

The continuous PI controller (7) is discretized using Tustin’s method with the sampling period \(T_s = 0.003\) s. Six discrete-time PI controllers with the following t.f.s are obtained:
\[
H_j^{(j)}(z^{-1}) = (q_0 + q_1 z^{-1}) / (1 - z^{-1}),
\]
where \(z^{-1}\) is the backward shift operator. The numerical values of tuning parameters are:
3.2 Gain-scheduling Control Solutions 
Design

In order to ensure the switching between these six discrete PI controllers, three GS control solutions, namely Lagrange, Cauchy and Switching GS, Figure 2, are developed:

\[ q_{i,LGS} = \sum_{j=0}^{n} \alpha_{j,LGS}^{(j)} q_{i}^{(j)}, \quad i \in \{0,1\}, \tag{11} \]

where the superscripts \( j \) denote different operating points, \( n = 7 \), \( LGS \) is Lagrange GS version, \( \| P - P^{(j)} \| \) is the Euclidean distance between the current operating point in the form of \( P = (x_1, y, \hat{s}, \hat{u}, \hat{d})^T \) and the nearest operating point \( P^{(0)} \). All coefficients \( \alpha_{j,LGS}^{(j)} \) in the first summation in (11) are normalized to add up to 1.

The Cauchy GS control solution is the second GS version which is based on a Cauchy kernel distance metric (Andonovski et al., 2016). This approach directly takes into account all previous data samples using:

\[ \alpha_{j,CGS}^{(j)} = \prod_{i=0}^{q-1} \frac{\| P - P^{(j)} \|^2}{\| P - P^{(i)} \|^2}, \tag{12} \]

The proposed Lagrange GS (LGS) control solution is the first GS version, and it is based on a generalization to the multivariable case of the Lagrange interpolating parameter value method:

\[ q_{i} = \beta \cdot q_{i-1} + q_{i,GS}(k), \quad \tag{10} \]

extended or not with a first-order lag filter, where the parameter \( \beta \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\} \) controls the transition speed between different controller parameters, and \( q_{i,GS}(k) \) are regarded as reference inputs calculated as follows.

The proposed Lagrange GS (LGS) control solution is the first GS version, and it is based on a generalization to the multivariable case of the Lagrange interpolating parameter value method:

\[ q_{0}^{(1)} = 5.96, \quad q_{1}^{(1)} = -5.70, \quad q_{2}^{(1)} = 5.06, \]
\[ q_{0}^{(2)} = -4.82, \quad q_{1}^{(2)} = 4.38, \quad q_{2}^{(2)} = -4.20, \]
\[ q_{0}^{(4)} = 3.92, \quad q_{1}^{(4)} = -3.77, \quad q_{2}^{(4)} = 3.51, \]
\[ q_{0}^{(5)} = -3.40, \quad q_{1}^{(5)} = 3.33, \quad q_{2}^{(5)} = -3.23. \]
and CGS is Cauchy GS version, \( \| P - P^{(i)} \| \) is the Euclidean distance between the current operating point \( P \) and the nearest operating point \( P^{(i)} \).

The Switching GS (SGS) control solution is the third version. It is based on the switching between PI controllers and the PI controller parameters correspond to the nearest operating point during the real-time experiments. The selection is supported by the Euclidean distance metric resulting in

\[
q_{i,SGS} = \sum_{j=1}^{n} \frac{\alpha_{i,SGS}^{(j)} \cdot q_{j}^{(i)}}{\sum_{j=1}^{n} \alpha_{i,SGS}^{(j)}}, \quad i \in \{0,1\},
\]

where

\[
j^* = \arg \min_{j \in \{0,1\}} \| P - P^{(j)} \|^2, \quad i \in \{0,1\},
\]

and SGS is Switching GS version.

4 SIMULATION RESULTS

The proposed adaptive control structures presented above are tested and validated by six simulation results. A staircase change of the reference input signal was employed and the control structures responses were tested on the time frame of 10 s. The illustrated results include the evolutions of mass position \( x_1(t) \) versus time \( t \) for Lagrange, Cauchy and Switching GS control solutions designed for the electromagnetic actuator as part of clutches system for \( \beta \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\} \). Due to the lack of space, in this paper, only the results corresponding to \( \beta = 0, \beta = 0.3 \) and \( \beta = 0.5 \) are illustrated in Figures 3, 4 and 5.

The mean square error \( J_{MSE,GS} \) is computed for all three GS versions as:

\[
J_{MSE,GS} = \frac{1}{N} \sum_{t_j=1}^{N} (r(t_j) - y(t_j))^2,
\]

where \( \beta \in \{\text{LGS, CGS, SGS}\} \) is the designed control solution, \( r(t_j) \) is the reference input at time.

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>0</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>LGS</td>
<td>8.52 \times 10^{-2}</td>
<td>8.12 \times 10^{-2}</td>
<td>7.72 \times 10^{-2}</td>
<td>7.33 \times 10^{-2}</td>
<td>6.94 \times 10^{-2}</td>
<td>6.57 \times 10^{-2}</td>
</tr>
<tr>
<td>CGS</td>
<td>8.31 \times 10^{-2}</td>
<td>7.93 \times 10^{-2}</td>
<td>7.55 \times 10^{-2}</td>
<td>7.17 \times 10^{-2}</td>
<td>6.80 \times 10^{-2}</td>
<td>6.44 \times 10^{-2}</td>
</tr>
<tr>
<td>SGS</td>
<td>8.68 \times 10^{-2}</td>
<td>8.28 \times 10^{-2}</td>
<td>7.89 \times 10^{-2}</td>
<td>7.51 \times 10^{-2}</td>
<td>7.14 \times 10^{-2}</td>
<td>6.77 \times 10^{-2}</td>
</tr>
</tbody>
</table>

Figure 3: Mass position \( x_1 \) versus time \( t \) in all the three GS versions (namely CGS, LGS, SGS) for \( \beta = 0 \).
Figure 4: Mass position $x_1$ versus time ($t$) in all the three GS versions (namely CGS, LGS, SGS) for $\beta = 0.3$.

Figure 5: Mass position $x_1$ versus time ($t$) in all the three GS versions (namely CGS, LGS, SGS) for $\beta = 0.5$.

moment $t_j = 1...N$, $N=3333$ is the number of samples and $y(t_j)$ is the measured mass position at time moment $t_j=1...N$. The values are presented in Table 2.

The conclusion drawn by analyzing the plots given in Figures 3, 4 and 5, and after comparing the results presented in Table 2 is that the zero steady-state control error is ensured in all versions and the reference input is well tracked.

Analyzing in terms of GS versions the smallest mean square error is obtained in CGS version and the biggest mean square error is obtained in SGS version, and analyzing in terms of $\beta$ the smallest mean square error is obtained for $\beta = 0.5$ and the biggest mean square error is obtained for $\beta = 0$ in all GS versions.

The above analysis of control system performance can lead to different results for other controlled plants. Such suggestive examples of plants include motion control (Korondi et al., 1996), chaotic systems (Precup et al., 2007), large-scale complex systems (Filip, 2008; Fan and Liu, 2016), multi-tank systems (Precup et al., 2013), evolving systems (Blažič et al., 2014), node localization (Derr and Manic, 2015; Wang et al., 2017), turbojet engines (Fozo et al., 2017), routing problems (Osaba et al., 2017) and neural networks (Dumitrache et al., 1999; Alique et al., 2000; Fioriti and Chinnici, 2017; Saadat et al., 2017;
5 CONCLUSIONS

This paper has presented the design of gain-scheduling control approaches viewed as adaptive control approaches developed to deal with the nonlinearities of the electromagnetic actuator and to ensure the switching between PI controllers. The simulation results prove that the GS-based control systems guarantee the performance improvement (zero steady-state control error, small settling times and small overshoots) with respect to staircase changes of the reference input.

Future research will be focused on the improvement of the performance indices by designing of CSs with PI(D) fuzzy gain-scheduling controllers, with model predictive controllers and hybrid structures applied to mechatronics systems.

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