Behaviour of a Hybrid ILS Heuristic on the Capacitated Profitable Tour Problem

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Keywords: Heuristics, Iterative Local Search, Vehicle Routing Problem.

Abstract: In the present paper, we study the behaviour of a hybrid Iterative Local Search heuristic (ILS). A Large Neighborhood Search (LNS) and a Variable Neighborhood Descent with Random neighborhood ordering (RVND) are used in the local search phase of the proposed ILS. The approach is evaluated on a well-known variant of the Vehicle Routing Problem (VRP) called Capacitated Profitable Tour Problem (CPTP). In this variant, the vehicles are no longer required to visit all the customers. However, a specific profit is obtained each time a customer is visited. The goal of the CPTP is to design routes with maximum difference between collected profits and routing costs, which satisfy the capacity constraint of the vehicles. The experimental study consists in comparing different combinations of ILS, LNS and RVND. The computational results show that the hybridization of the three heuristics leads to better solutions. Furthermore, comparisons with a Variable Neighborhood Search and two Tabu Searches from the literature indicates that our hybrid approach is competitive.

1 INTRODUCTION

The Capacitated Profitable Tour Problem (CPTP) can be defined as a variant of the Vehicle Routing Problem (VRP) in which the visit of all customers is no longer mandatory. In particular, a specific profit is collected each time a customer is visited. In addition, each customer is visited at most once by one of the available capacitated and identical vehicles. The goal is to design feasible vehicle routes that maximize the difference between collected profits and routing costs.

Archetti et al. (Archetti et al., 2009) have implemented three methodologies to solve the CPTP namely Variable Neighborhood Search (VNS), Tabu Feasible (TF) and Tabu Admissible (TA). The TF algorithm accepts only feasible solutions, while TA allows the visit of infeasible solutions by favouring those with small infeasibilities. In both tabu searches, the employed movements are inter-routes movements. The first movement is the 1-move. It consists in the relocation of a given customer in another route (a deletion of the customer is also considered). The second movement is the swap-move. Swap-move aims at exchanging the positions of two given customers. For deleting or at least decreasing a solution’s infeasibility, Archetti et al. have proposed a repair heuristic based on series of 1-move. To evaluate the solutions, several functions are used. Those functions deal with difference between total profit and total distance, route duration, number of routes and maximum constraint violation. In the diversification phase, series of 1-move are executed between “good” and “bad” routes. On the other hand, the VNS algorithm uses the Tabu Feasible method with a small number of iterations. This allows the VNS to visit a larger number of regions within the search space.

Some researchers attempted the resolution of the CPTP using exact methods. Among these works one can find the branch and price algorithm of Archetti et al. (Archetti et al., 2009). More recently, Archetti et al. (Archetti et al., 2013) proposed a branch and price algorithm for both the CPTP and another variant of the Vehicle Routing Problem with Profits called the Capacitated Team Orienteering Problem. Finally, Jepsen et al. (Jepsen et al., 2014) presented a branch and cut algorithm to solve the CPTP. For more details on Vehicle Routing Problems with Profits, we refer the reader to (Archetti et al., 2014).

In the present paper, we propose a hybrid Iterative Local Search heuristic (ILS) for the CPTP. The pro-
posed approach makes use of both a Large Neighborhood Search heuristic (LNS) and a Variable Neighborhood Descent with Random neighborhood ordering (RVND) in the intensification phase. The diversification is ensured by the perturbation mechanism of the ILS heuristic. To the best of our knowledge, it is the first time that ILS, RVND and LNS are combined to solve a VRP variant. The approach is evaluated on CPTP benchmark instances from the literature.

In the following, we describe the proposed approach (Section 2). After that, we evaluate the performance of the hybrid ILS (Section 3): First we provide a comparison between different combinations of the components. Then, we contrast our results with those presented in the literature of the CPTP. Finally, we give some concluding remarks (Section 4).

2 THE PROPOSED METHODOLOGY

In the present section, we briefly describe the Iterative Local Search heuristic (ILS), the Variable Neighborhood Descent with Random neighborhood ordering (RVND) and the Large Neighborhood Search (LNS), which are the main components of our approach.

Combinations of ILS and RVND were successfully applied to a variant of the VRP called Vehicle Routing Problem with Simultaneous Pick-up and Delivery services (see (Subramanian et al., 2010), (Subramanian et al., 2013)) and are still competitive with other new approaches proposed for the same problem. The ILS and the Variable Neighborhood Descent (VND) heuristics also provide good quality solutions for other variants of VRPs see (Subramanian and Battarra, 2013), (Erdoğan et al., 2009), (Hernández-Pérez et al., 2009), (Rodríguez-Martín and Salazar-Gonzále, 2012) and (Todosijević et al., 2017). In addition, the two heuristics perform well on some Vehicle Routing Problems with Profits (see (Assis et al., 2013) and (Gansterer et al., 2017)). Furthermore, several versions of VND are used to solve different variants of transportation problems (see (Sifaleras and Konstantaras, 2017), (Samá et al., 2017) and (Hassannayebi and Zegordi, 2017)).

To the best of our knowledge, no paper from the literature proposes a combination of ILS with LNS to solve a VRP variant. However, the two heuristics have been successfully applied independently to several variants of VRPs as well as to some transportation problems (see for instance (Cuervo et al., 2014), (Silva et al., 2015), (Morais et al., 2014) and (Li et al., 2015) for ILS, and (François et al., 2016), (Grangier et al., 2017), (Akpinar, 2016), (Domínguez et al., 2016) and (Canca et al., 2017) for LNS).

The ILS heuristic aims at improving solutions of basic local searches. Indeed, after a local search is performed, a local optimum is found, then ILS perturbs the so obtained local optimum and recall the local search to improve it. This process iterates until stopping criteria are met.

In a RVND heuristic, a list of neighbourhood structures is used in such a way that, at each iteration, a neighbourhood structure is chosen at random and is applied to the current solution. The new solution is accepted or not according to given criteria. After that, the chosen neighbourhood structure is removed from the list and the process continues with the remaining structures. RVND stops when the list of neighbourhood structures is empty.

In comparison to other local search heuristics, LNS allows the visit of larger areas in the search space by changing the structure of the studied solution. Indeed, LNS removes a relatively large number of customers from a current solution, and re-inserts these deleted customers in different positions. This leads to a completely different solution, that helps the heuristic to escape local optima. If the number of deleted customers is set to a small value, LNS can be considered as a simple local search heuristic.

Our approach combines ILS, LNS and RVND in a multi-start heuristic denoted ILS_LNS_RVND. First, a sequential insertion heuristic is used to generate a different initial solution at each iteration. Then, for each initial solution, an ILS heuristic is executed until reaching a given number of iterations without improvement. In ILS, a hybrid LNS_RVND heuristic plays the role of the local search procedure. The perturbation mechanism of ILS is ensured by a deletion/reinsertion procedure that randomly deletes customers and reinserts other ones. In the local search phase, ILS_LNS_RVND accepts only improved solutions (better than the current solution). However, in the perturbation phase, all solutions are accepted. The pseudo-code of ILS_LNS_RVND is given below.

Algorithm ILS_LNS_RVND.
Inputs: A CPTP instance.
Outputs: The best solution found.
Begin
While (stopping criteria are not met)
  do
  Generate an initial solution;
  While (ILS stopping criteria are not met)
    do
      LNS_RVND();
      Update the best solution;
      Perturb();
  End While
End While
End.
In the following subsections, we give more details about ILS, LNS, RVND. We start by describing the sequential insertion heuristic. Then, we present the LNS, the RVND and the LNS, RVND heuristics. Finally, we describe the perturbation mechanism.

2.1 Sequential Insertion Heuristic

To generate an initial solution for the CPTP, we implement a multi-start sequential insertion heuristic based on the I1 heuristic (see (Solomon, 1987)). Initially the I1 heuristic was implemented for the Vehicle Routing Problem with Time Window constraints (VRPTW). First, I1 fills an empty route with a so-called seed customer. Second, I1 evaluates all the possible insertions of the remaining customers into the route. Third, I1 selects the best position for each customer under some criteria. Finally, I1 retains the insertion that optimizes the studied criteria. To adapt I1 to the CPTP, we slightly change its selection criteria. The new criteria deal with profits and distances and are described using Expressions (1)–(4)). These criteria aim at evaluating the insertion of a customer \( r \) between customers \( i \) and \( j \). In Expressions (1)–(4)), \( c_{xy} \) stands for the distance between customers \( x \) and \( y \), \( pr_u \) is the profit of customer \( u \), \( l_0 \) and \( l_i \) refer to the depot (the first and the last points of a studied route), \( \alpha_1 + \alpha_2 = 1 \) with \( \alpha_1 \geq 0 \) and \( \alpha_2 \geq 0 \).

\[
\begin{align*}
    cr_1(i(u),u,j(u)) &= \min_{p=1,...,cr_1(i-1,u,i_p)} cr_1(i-1,u,i_p); \\
    cr_1(i(u),j(u)) &= \alpha_1 cr_11(i,u,j) - \alpha_2 cr_12(i,u,j); \\
    cr_11(i,u,j) &= pr_u; \\
    cr_12(i,u,j) &= c_{iu} + c_{ij} - c_{ij};
\end{align*}
\]

The second criterion of I1, that sets the importance of the distance between customer and depot, is not used in our heuristic. In addition, the seed customer is randomly selected in our heuristic.

2.2 Local Search Phase

2.2.1 Large Neighborhood Search

The LNS heuristic proposed in the present paper uses one removal and one insertion operator. To select the best couple of operators for the CPTP, we implement all those presented by (Pisinger and Ropke, 2007) using the objective function and the constraints of the CPTP. After that, we test all the possible combinations of removal/insertion couples and retain the related removal and the regret-4 heuristic. We recall that the related removal aims at deleting customers that are somehow similar in order to interchange them easily. The similarity \( s_{ij} \) between customers \( i \) and \( j \) is defined by Formula (5), where \( c_{ij} \) is the distance between \( i \) and \( j \), \( pr_i \) and \( pr_j \) are the profits of \( i \) and \( j \) respectively. On the other hand, the regret-4 heuristic aims at inserting a given customer in its 4th best position. The goal is to avoid postponing the placement of difficult customers to the last iterations which may produce local optima. LNS stops when a given number of iterations without improvement is reached.

\[
s_{ij} = |pr_i - pr_j| + c_{ij}
\]

2.2.2 Variable Neighborhood Descent with Random Neighborhood Ordering

The RVND heuristic uses four intra- and three inter-route(s) operators. The latter are randomly chosen at each iteration in such a way that, each operator is executed only once.

The neighbourhood operators used in our RVND heuristic are described below. Examples of neighbourhood movements are given in Figure 1.

2-Opt. This operator connects two customers \( k \) and \( l \) within a same route by reversing the path between \( k \) and \( l \). In Figure 1, customer 1 stands for \( k \), while customer 2 stands for \( l \). Customers 1 and 2 are connected and the path between them is reversed. To maintain the connectivity of the route, customer 4 is connected to customer 5.

2-Opt*. This operator first divides two routes into four sections. Then, the first section of the first route is connected with the second section of the second route and the first section of the second route is connected with the second section of the first route. In Figure 1, the first route is divided by disconnecting customers 1 and 2, and the second route is divided by disconnecting customers 5 and 6. After that, customers 1 and 6 and customers 5 and 2 are connected to create the new routes.

Intra-route 1-0 Exchange. This operator moves customer \( l \) to position \( k+1 \) within a same route. In Figure 1, customer 2 stands for \( l \) and position \( k \) is the position of customer 1, which is position 1. Hence, customer 2 is moved to position \( k+1 \), i.e. position 2.

Inter-routes 1-0 Exchange. This operator moves customer \( l \) to position \( k+1 \) in a different route. In Figure 1, \( l \) is customer 3 and position \( k \) is the position of customer 6, which is position 3. Hence, customer 3 is moved to position \( k+1 = 4 \) within the second route.
Intra-route 1-1 Exchange. This operator swaps the positions of two customers \( k \) and \( l \) within a same route. In Figure 1, customer 2 stands for \( k \) and customer 5 stands for \( l \).

Inter-routes 1-1 Exchange. This operator swaps the positions of two customers \( k \) and \( l \) in two different routes. In Figure 1, customer 3 stands for \( k \) and customer 6 stands for \( l \).

Or-Opt. This operator moves two consecutive customers \( k \) and \( k+1 \) between two other consecutive customers (or a customer and the depot) \( l \) and \( l+1 \). This results in a sequence \((l, k, k+1, l+1)\). In Figure 1, customer 1, customer 2, the depot and customer 3 stand for \( k, k+1, l \) and \( l+1 \) respectively.

2.2.3 Hybrid LNS_RVND

LNS_RVND first executes the LNS heuristic until a given number of iterations without improvement are reached. The best solution found so far is then improved using RVND. This process is repeated for a given number of iterations.

2.3 Perturbation Mechanism

The perturbation procedure is also based on the LNS principle. However, to maintain the diversification
aspect of the perturbation, the random removal described by (Pisinger and Ropke, 2007) is used. Customer insertions are done using the basic greedy heuristic (see (Pisinger and Ropke, 2007)).

3 COMPUTATIONAL RESULTS

In the present section, we assess the performance of ILS\_LNS\_RVND: we run different versions of the heuristic to evaluate the impact of each component (ILS, LNS and RVND) and to study different combinations of these components. Then, we compare our approach with some heuristics from the literature. A brief description of CPTP instances is given in the first subsection.

All the heuristics are implemented in C and run on a laptop with an Intel(R) Core (TM) i7-4600U CPU @ 2.10GHz 2.70GHz with 8.00 Gb RAM and 64-bit operating system. Due to the random aspect, the approaches are run 3 times for each instance. The best encountered solutions are reported.

3.1 CPTP Instances

The studied CPTP instances are proposed by (Archetti et al., 2009). They are derived from the Capacitated Vehicle Routing Problem instances proposed by (Christofides et al., 1979). Archetti et al. generate these instances by modifying the capacity bounds and the number of vehicles of the original instances. The number of customers varies between 50 and 199.

3.2 Study of the LNS Heuristic

We implement all the removal/insertion operators presented by (Pisinger and Ropke, 2007) using CPTP objective function. In order to select the best couple of removal/insertion operators to use in the local search phase of ILS\_LNS\_RVND, we run 30 LNS basic versions \{V1,...,V30\} that use different pairs of removal/insertion operators. Each version starts from an initial solution generated by our construction heuristic using random values of parameters \( \alpha_1 \) and \( \alpha_2 \). The studied LNS versions delete 1 to 3 customers at each iteration. The insertion heuristics try to insert profitable customers. The best LNS version so obtained is used in the ILS\_LNS\_RVND heuristic. LNS stops when 50000 iterations without improvement are reached.

Table 1 gives the pair of removal/insertion operators used in each version. In this Table, \( \text{ind} \) refers to the indices of removal/insertion couples. The symbol “X” indicates whether an operator is used or not.

Figure 2 compares the average deviation (gap) from the solutions of the literature and the average computing time (in seconds) of the 30 versions. All versions are tested on all CPTP instances and they are run until 50000 iterations without improvement are reached. From Figure 2, we remark that the version using the related removal combined to the regret heuristic with a regret number of 4 is the best version in terms of solution quality. It also provides reasonable computing time. Hence, this version is retained for the local search phase of the ILS\_LNS\_RVND heuristic.

3.3 Hybrid ILS\_LNS Heuristic

The hybrid ILS\_LNS heuristic is a multi-start heuristic that executes, at each iteration, an ILS heuristic with LNS in the local search phase.

The LNS heuristic implemented here uses the best couple of removal/insertion operators found in Subsection 3.2. We test four configurations of the number of customers to delete at each iteration. We retain the following configuration: the number \( r \) of customers to delete is randomly chosen from the interval \([1,0.4 \cdot n]\), where \( n \) is the number of customers in the current solution. LNS stops when 50 iterations without improvement are reached.

The perturbation procedure of ILS\_LNS is described in Subsection 2.3. Each time, the perturbation procedure is executed, \( r' \) customers are deleted, where \( r' \in [0,1,0.4] \cdot n \) and \( n \) is the number of customers that belong to the current solution. After that, the basic greedy heuristic tries to insert profitable customers. Note that, a random insertion of customers was also developed but the latter leads to low quality solutions.

ILS\_LNS stops when 50 iterations without improvement are reached.

Table 2 compares the results of LNS using the best couple of removal/insertion and two versions of ILS\_LNS. In the first version, the number \( r \) of customers to delete is randomly chosen from \([1,3]\). While, in the second version, \( r \) is randomly chosen from \([1,0.4 \cdot n]\). We remark that the first version of ILS\_LNS provides better results and needs less computing time than LNS. When \( r \) is chosen from \([1,0.4 \cdot n]\), the heuristic provides better solutions. However, it is more time consuming. As the computing time of all these heuristics is reasonable, we chose to continue the study using the ILS\_LNS heuristic with \( r \in [1,0.4 \cdot n] \).
Table 1: Removal/insertion couples.

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<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
</tbody>
</table>

Table 2: Comparison between LNS with the best couple of removal/insertion and two versions of ILS_LNS.

<table>
<thead>
<tr>
<th></th>
<th>LNS</th>
<th>ILS_LNS</th>
<th>ILS_LNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r \in [1, 3]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>gap</td>
<td>12.16</td>
<td>5.61</td>
<td>5.07</td>
</tr>
<tr>
<td>CPU</td>
<td>23.78</td>
<td>14.75</td>
<td>36.35</td>
</tr>
</tbody>
</table>

3.4 Hybrid ILS RVND Heuristic

Instead of using LNS in the local search of ILS, ILS RVND uses RVND with the same perturbation procedure presented in Subsection 3.3.

As the heuristic is very fast, we increase the number of iterations without improvement of ILS. We remark that ILS RVND with 50 iterations without improvement converges in only 1.26 seconds. However, the gap of this heuristic with respect to the benchmark solutions equals 7.59%, which is larger than the gap of ILS_LNS. When we fix the number of iterations without improvement of ILS to 500, the gap of ILS RVND reaches 4.95% in only 10.57 seconds. This shows that ILS RVND outperforms ILS_LNS in both solution quality and computing time. These results are summarized in Table 3.

Table 3: Comparison between ILS_LNS and ILS RVND.

<table>
<thead>
<tr>
<th></th>
<th>ILS_LNS</th>
<th>ILS RVND</th>
<th>ILS RVND</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r \in [1, 0.4 + n]$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>gap</td>
<td>5.07</td>
<td>7.59</td>
<td>4.95</td>
</tr>
<tr>
<td>CPU</td>
<td>36.35</td>
<td>1.26</td>
<td>10.57</td>
</tr>
</tbody>
</table>

3.5 Hybrid LNS RVND Heuristic

LNS RVND is a heuristic that combines the best version of LNS found in Subsection 3.2 with the RVND heuristic. LNS RVND is not a multi-start heuristic. Indeed, it starts from an initial solution generated by our construction heuristic using random values of parameters $\alpha_1$ and $\alpha_2$. LNS is then executed, and each time $i$ iterations are reached, the current solution is improved by RVND using a given probability $P$. We have studied the configurations $(i = 100, P = 1/10)$, $(i = 1000, P = 1/10)$ and $(i = 1000, P = 1/100)$. We remark that the second configuration provides the best
gap and computing time. However, these results are not promising in comparison with those of ILS LNS RVND as shown in Table 4. We conclude that the multi-start ILS heuristic plays a key role in the achievement of good quality solutions in reasonable computing time.

Table 4: Comparison between LNS RVND, ILS LNS, and ILS RVND.

<table>
<thead>
<tr>
<th></th>
<th>LNS RVND</th>
<th>ILS LNS</th>
<th>ILS RVND</th>
</tr>
</thead>
<tbody>
<tr>
<td>r ∈ [1, 0.4 + n]</td>
<td>500 iterations</td>
<td>gap</td>
<td>CPU(min)</td>
</tr>
<tr>
<td>LNS RVND</td>
<td></td>
<td>11.94</td>
<td>67.10</td>
</tr>
<tr>
<td>ILS LNS</td>
<td></td>
<td>5.07</td>
<td>36.35</td>
</tr>
<tr>
<td>ILS RVND</td>
<td></td>
<td>4.95</td>
<td>10.57</td>
</tr>
</tbody>
</table>

3.6 Hybrid ILS LNS RVND Heuristic

ILS LNS RVND is a multi-start ILS that uses the hybrid LNS RVND heuristic described in 2.2.3. Actually, in ILS LNS RVND, LNS RVND is repeated until reaching a given number of iterations $i_{LNS RVND}$. Then, the perturbation mechanism described in Subsection 2.3 is executed. This process is repeated until reaching a fixed number of iterations without improvement denoted $maxOcc$. We choose to stop LNS RVND when a given number of iterations is reached instead of a given number of iterations without improvement because this led to better results.

The parameters of the approach are set as follow: $i_{LNS RVND}$ equals 7, $maxOcc$ is set to 200, the maximum number of iterations without improvement in LNS $maxOcc_{LNS}$ is set to 20.

Table 5 compares ILS LNS RVND with the previously studied heuristics. From this table, we remark that ILS LNS RVND obtains the best results with reasonable computing time. We also remark that the more a heuristic is hybridized, the better are the results. However, this may cause additional computing time. This is because, if we do not hybridize a heuristic, it will be trapped in a local optimum very quickly.

3.7 Comparison of ILS LNS RVND with Heuristics from the Literature

We compare our approach with other approaches from the literature. Table 6 compares the results of ILS LNS RVND with the Variable Neighborhood Search (VNS), the Tabu Feasible (TF) and the Tabu Admissible (TA) heuristics of (Archetti et al., 2009). In this table, gap refers to the average deviation of each heuristic with respect to the best solutions (solutions of ILS LNS RVND included). CPU(min) is the computing time of each heuristic in minutes. We remark that ILS LNS RVND provides competitive results in reasonable computing time. Note that ILS LNS RVND provides better solutions in comparison to the other heuristics in 6 cases.
Table 5: Comparison between ILS_LNS_RVND and the other heuristics.

<table>
<thead>
<tr>
<th></th>
<th>ILS_LNS</th>
<th>LNS_RVND</th>
<th>ILS_LNS</th>
<th>ILS_RVND</th>
<th>ILS_RVND_LNS</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>12.16</td>
<td>11.94</td>
<td>5.07</td>
<td>4.95</td>
<td>1.57</td>
</tr>
<tr>
<td>CPU (min)</td>
<td>23.78</td>
<td>67.1</td>
<td>36.55</td>
<td>10.57</td>
<td>28.39</td>
</tr>
</tbody>
</table>

Table 6: Comparison between ILS_LNS_RVND and the other heuristics from the literature.

<table>
<thead>
<tr>
<th>ILS_LNS_RVND</th>
<th>VNS</th>
<th>TF</th>
<th>TA</th>
</tr>
</thead>
<tbody>
<tr>
<td>gap</td>
<td>0.66</td>
<td>0.18</td>
<td>0.78</td>
</tr>
<tr>
<td>CPU (min)</td>
<td>9.94</td>
<td>10.3</td>
<td>2.83</td>
</tr>
</tbody>
</table>

4 CONCLUSION

In this paper, we present a hybrid ILS heuristic that makes use of LNS and RVND in the local search phase. We compare several versions of ILS using different levels of hybridization of the components. The proposed heuristic is evaluated on a well known variant of the Vehicle Routing Problem called Capacitated Profitable Tour Problem. The results show that the more we hybridize ILS, the better are the results. Finally, we contrast our results with those obtained in the literature. This shows that our hybrid ILS is competitive in terms of solution quality and computing time. A future work may consist in applying the hybrid ILS heuristic to other variants of Vehicle Routing Problems with Profit.

REFERENCES


