Some Vector Fk Sequence Spaces Generated by Modulus Function

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Abstract: In this paper, some vector value sequence spaces G_f (X) and L_f (X) using modulus function are presented. Furthermore, we examined some topological properties of these sequence spaces equipped with a paranorm.

1 INTRODUCTION

Let X be a vector space and R be the set of real numbers. A function f : R^+ [f0g ! R^+ [f0g is called modulus function if following condition of f satisfying:

1. f is vanishing at zero
2. f satisfies triangle inequality
3. f is an increasing function i.e. f ( ) "

The function f must be continuous for every element x in (0;¥). The space of all real number sequences (x_n) such that the infinite series of absolute modulus function is finited denoted by `(f) (Ruckle, 1973)

$$\overset{¥}{\underset{n=1}{1.}}\ f\,(jx_nj) < ¥$$

The space `(f) becomes a FK-space under the F-norm.

$$p(x) = \overset{¥}{\underset{n=1}{å}} f\,(jx_nj) < ¥$$

(Adnan, 2006) examined the FK-space properties of an analytic and entire real sequence space using modulus function. (Adnan, 2006) showed the characterization to matrix transformation of Ruckle's space `(f) into analytic FK-space. For the theory of FK-space we refer to Banas and Mursaleen[2].

Through the article W(X), G_f (X), L_f (X) denoted by the space of vector value sequences, entire vector value sequence space and analytic vector value sequence space. The vector value sequence space studied by some authors (Herawati et al., 2016; Gultom et al., 2018; Kolk., 2011; Leonard., 1976; Das and Choudhury., 1992; Et., 2006; Et et al., 2006; Tripathy et al., 2004; Tripathy et al., 2003), and many others. Further, the concept of sequence space using modulus function was investigated by (Bilgin., 1994; Pehlivan and Fisher., 1994; Waszak., 2002; Bhardwaj., 2003, Altin., 2009), and many others.

Recently, (Herawati et al., 2016) studied the geometric of the vector value sequence spaces defined by order-j function under Lattice norm. Further, (Gultom et al 2018) studied some topologies properties of a finite arithmatic mean vector value sequence space denoted by W_f (X) for X is a linear space and f is a j-function.

A functional is called paranormed if satisfies the properties p : X ! R that satisfies the properties p(q) = 0, with q is the zero vector in X, non-negative, p satisfies triangle inequalities, even and every real sequence (l_n) with jl_n lj ! 0. The space X with paranorm p is called paranormed space, written as

$$X = (X; p). \text{ (Nakano, 1951; Simons, 1965)}$$

In this work, we define the space of vector value sequences G_f (X) and L_f (X) called entire and analytic vector valued sequence spaces generated by modulus function and study the topological properties of the sets equipped with paranorm.

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2 MAIN RESULTS

In this main result section, firstly, we introduce para-norm on this space and examine some topological properties such as complete properties. Let X be a Banach space and f be a modulus function. Let y(n) = f (kx(n)kX) 2 R for all natural numbers n, then we get a sequence y = (y(n)). We define the sets

Gf (X) = x = (x(n))n2N : x(n) 2 X and (y(n)) n ! 0;

n ! ¥

L f (X) = x = (x(n))n 2 N : x(n) 2 X and

sup f(y(n)) 1 n g < ¥
n2N

Theorem 2.1. The sets Gf (X) and Lf (X) are vector spaces.

Proof. Let x;z be any elements in G_f (X), then

lim (y(n)) 1 n = 0 and lim (w(n)) 1 n = 0
n!¥ n!¥

for n ! ¥, with y(n) = f (x(n) and w(n) = f (z(n)) for each natural number n. We will apply the following inequality : if a_n;b_n 2 R and 0 q_n sup q_n = H for each natural number n, then

$ja_n + b_nj^{qn}$ $M(ja_nj^{qn}) + jb_nj^{qn}$

where M = maxf1;2^H 1g. Therefore,

(y(n) + w(n)) 1 n = ((y(n)) 1 n + (w(n)) 1 n

Since (q_n) sup), then H = = 1. Thus

(y(n) + w(n)) 1 n (y(n)) 1 n + (w(n)) 1 n

Since (y(n)) 1 n ! 0 and (w(n)) n for n ! ¥, then

(y(n) + w(n)) 1 n ! 0 for n ! ¥. Therefore, we obtain x + y 2 G_f (X). Further, for element x 2 G_f (X) and a 2 R, then

(y(n)) 1 n ! 0;n ! ¥

Because of an increasing function f and the positivity of jaj, then from the Archimedean properties, there exists natural number n_0 with

f (jajkx(n)k) f (2^{n0} kx(n)k)

Since f satisfies 4_2-condition, we get

(f (2^{n0} kx(n))) 1 n K 0 = (f (kx(n)k)) 1 n ! 0

for each natural number n. It shows that ax 2 G_f (X). Because x + z 2 G_f (X) and ax 2 G_f (X) for each x;y 2G_f (X) and each a 2 R, we get G_f (X) is a vector or lin-ear space and the proof of the theorem is finished. In the same way, it can be shown that L_f (X) is a vector space.

Theorem 2.2. A functional p : Gf (X) ! R defined by

p(x) = sup n 1(y(n)) 1 n

is a paranorm.

Proof. Let x be an element in Gf (X). It is clear that the functional p is non-negative, p(q) = 0, with q is the zero vector in X and even, for each x 2 Gf (X). Now, we will show that p satisfies the triangle in-equality. To do that, take any x;z 2 Gf (X), then

lim (y(n)) n = 0 and lim (w(n)) n = 0
n!¥ n!¥

for n ! ¥, with y(n) = f (x(n)) and w(n) = f (z(n)) for each n 2 N. we obtain

up (y(n) + w(n)) 1 sup (y(n)) 1 + (w(n)) 1

Therefore, there's vector sequences of x;y 2 Gf (X), we get p satisfies the triangle inequality. Next, we will show that p satisfies the continuity of scalar mul-tiplication. To do that, take any real sequence (ln) and (x(n)) 2 Gf (X) with jln lj ! 0 for n 2 ¥. We have

(f (kx(n))kX)) 1 n =(f (klkx(n) lx(n)k)) 1 n
=(f (k(ln l)x(n)k) + kl(x(n) x)k)) 1 n
((f jln ljkx(n)k + jljk(x(n) x)k) 1 n

1

p(lnx(n) lx(n)) = sup f(f (klnx(n) lx(n)k)) n g

jln ljp(x(n) + jljp(x(n) x) ! 0

Hence, p(lnx(n) lx(n)) ! 0. The proof of the theorem is finished.

Theorem 2.3. The vector spaces of Gf (X) and Lf (X) are complete paranormed sequence space under the paranorm defined in Theorem 2.2.

Proof. Take any Cauchy sequence (x^i) in Gf (X) with $x^i = (x^i(n)) = (x^i(1);x^i(2);:::;)$. Therefore, for any positive real number e, there exists i0 2 N, for all j i i0, we get

p(x j x i) = sup (f (kx j(n) x i(n)k)) n < e

1 < e, we have

Since sup (f kx J(n) x I(n)k)) n (f (kx J(n) xI(n)k)) 1 < e for e > 0. Since f is a modulus function, then kx j(n) x i(n)k = 0 for each natural number n. In other words, kx j(n) x i(n)k < e.

It shows that for each natural number n of the sequence (x J(n)) is a Cauchy. Since X is a complete normed space, then (x J(n)) converges to x(n) 2 X. Hence, lim x J(n) = x(n) for all n. Therefore, there's j!¥ sequence x = (x(n)) = (x(1);x(2);:::;) such that

su k k p i!¥ k 1

sup (f (x i)) n f(lim x i k k) n sup i!¥

lim(f (x i k k x) n i!¥

= lim sup (f (x i x) n

for every i i0. By using the definition of paranorm, we get

p(x x i) = sup (f(kxx i k)) n e 1

It shows that x^i ! x for i ! ¥. Then it will be shown that x 2 Gf (X). Using the continuous property of f , we get

(f (x k) n = (f (lim x i k ¥) n

= lim(f (x i k) n

for i ! ¥. Hence, x 2 Gf (X). The proof of this theorem is finished. □

3 CONCLUSIONS

According to the main results, it can be concluded Gf (X) and Lf (X) are complete paranormed sequence space under the paranorm.

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