Substantial Differences between Fuzzy Set and Soft Set Theories

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Abstract: Set theories like fuzzy set, rough set, and soft set for dealing with vague or uncertain data have their own scopes. Fuzzy set theory was considered as most suitable tool for vague data before the concept of soft set theory, but after its presentation, soft set is considered as most appropriate among all. Beside hundreds of applications, the main superiority and reason of most appropriateness of soft set theory over fuzzy set theory is still lying as hidden and unclear as the vagueness of uncertain data itself. In this paper, we reveal the main differences between soft set and fuzzy set theory which discloses the appropriateness of soft set theory.

1 INTRODUCTION

In today's scientific era, data is one of the most crucial-key-instruments being dealt in more than tetra bytes every hour. There exists both crisp as well as vague data for everyday data dealing in computer science, engineering, medical, social science and every field of life. Crisp or clear data is processed using traditional simple or complicated mathematical tools and techniques but vague data can't be processed for obtaining meaningful knowledge, information and decisions using those common tools and techniques of crisp data. Special tools and hundreds of applications are used for vague data based on elementary theories like the theory of probability, interval mathematics, fuzzy set, rough set and soft set [1-8]. Most recent and appositely considered theory of soft set claims that fuzzy set theory is most appropriate between all previous theories. But fuzzy set theory has its own limitations which are covered by soft set theory. But no one has shown the clear difference between fuzzy set and soft set theories and it is becoming difficult for every new researcher to conclude it from thousands of articles e.g. [9-24] in the field of vague data and soft computing. Every scholar including Molodtsov in his pioneer work of soft set has used the word "possibly" for adequate parametrization being used by soft set and lacked by previous theories, especially fuzzy set theory.

In this paper, we present the main difference and similarity of fuzzy set and soft set theories. We use a common vague data example for describing both theories in a short systematic manner. We also introduce a method for deriving fuzzy set membership functions from soft set.

Remaining of our work is organized as follow. Section 2 contains the basic definitions and examples of vague data, basic review of fuzzy and soft set theories, furthermore an initial application of soft set. In section 3, we present main difference and similarity of fuzzy set and soft set theories. Finally, we conclude our work in section 4.

2 RUDIMENTARY

Data having no ambiguity or uncertainty is called crisp or clear data. For example, a university data base containing student's record is crisp data and after certain processing through specific tools and techniques it also yields the output in crisp form as student GPA, requirements fulfilled and due fees. In contrast to crisp data, the fuzzy, uncertain, vague and unclear data contains uncertainty and ambiguity. We can't process such data with ordinary tools of crisp data and if still processed will result in unexpected, very small, too big or misleading result. For example the word birds (penguin, bat?) tall man, beautiful women, creditworthy customer, responsible person and trusty friend. We need special tools for processing of such datum or its combination.

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2.1. Fuzzy Set Theory

Let X be a Universal set (objects/space of points) with its members x, i.e. $X = \{x\}$. A fuzzy set A in X is represented by characteristic function $f(x)$. Such that $f(x)$ associates with each point of X through interval [0,1], X takes a real value in this interval for each of its membership association level e.g. $f(x)=1$ if $x \in A$ and $f(x)=0$ if $x \notin A$, closer the value of x to 1 means higher grade of membership and closer the values of x to 0 means lower grade of membership, e.g. we can have membership functions $f(x)$ of A like $f(1)=0.03$, $f(2)=0.21$, $f(3)=0.17$, $f(101)=0.73$, $f(996)=0.84$ and $f(1000)=1$. In contrast to fuzzy set, the ordinary set (simply set), crisp set or "set" takes only two values i.e. either 1 or 0 for completely belonging or completely not-belonging to X.

2.2. Definition of Soft Set Theory

Let U be a Universal set and let E be a set of parameters then a fair (F, E) is called to be soft set over U if and only if F is a mapping of E into the set of all subsets of U i.e. the soft set is a parameterized family of the subsets of the set U. Every fuzzy set can be considered a special case of soft set.

2.3. Initial Application of Soft Set Theory in Fuzzy Data

[25] implemented soft set in tabular form and indicated that how it can be used in decision making. Let $U=\{h_1,h_2,h_3,h_4,h_5,h_6\}$ be a set of houses and E = {expensive, beautiful, wooden, cheap, in the green surroundings, modern, in good repair, in bad repair} be a set of parameters. Consider the soft set (F,E) which describes the attractiveness of the houses, given by

(F,E) = {Expensive houses $(e_0)=\varphi$, beautiful houses $(e_1)=\{h_1,h_2,h_3,h_4,h_5,h_6\}$, wooden houses $(e_2)=\{h_1,h_2,h_6\}$, cheap houses $(e_3)=\{h_1,h_2,h_3,h_4,h_5,h_6\}$, in the green surroundings houses $(e_4)=\{h_1,h_2,h_3,h_4,h_6\}$, in good repair houses $(e_5)=\{h_1,h_3,h_6\}$, modern houses $(e_6)=\{h_1,h_2,h_6\}$ in bad repair houses $(e_7)=\{h_2,h_4,h_5\}$ }.

The (F,E) is represented in tabular form as shown in Table 1. All objects are shown by rows and parameters by columns, for an object having certain parameter present is shown by putting its value equal to 1, otherwise zero.

Table 1: Representation of soft set (F,E) in Tabular form

U	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7
h_1	0	1	1	1	1	1	1	0
h_2	0	1	1	1	1	0	1	1
h_3	0	1	0	1	1	1	0	0
h_4	0	1	0	1	1	0	0	1
h_5	0	1	0	1	0	0	0	1
h_6	0	1	1	1	1	1	1	0

Suppose Mr. X is interested in buying house on the bases of parameter having subset P = {beautiful, wooden, cheap, in green surrounding, in good repair} $=\{e_1,e_2,e_3,e_4,e_5\}$. Then the tabular representation for this choice should be as given in Table 2. Choice of Mr. X is calculated by simply adding all parameters value in last column as shown by d_i for each object (house).

Table 2: Decision value or choice value calculation from soft set

U	e_1	e_2	e_3	e_4	e_5	d_i
h_1	1	1	1	1	1	5
h_2	1	1	1	1	0	4
h_3	1	0	1	1	1	4
h_4	1	0	1	1	0	3
h_5	1	0	1	0	0	2
h_6	1	1	1	1	1	5

It can be observed from Table 2 that h_1 and h_6 have highest d_i value, therefore either of them is best choice or optimal choice for Mr. X, while h_2 and h_3 having second highest value are the sub-optimal choices and h_5 is the worst choice having lowest value among all.

3 COMPARISON OF FUZZY SET AND SOFT SET THEORIES

Consider above example of Mr. X house choice. Choice values of all houses for soft set are re-entered from Table 2 into new Table 3, plus fuzzy set membership function values are calculated by dividing the sum of belonging parameters by number of total parameters. Total parameters are 5 in this example, therefore every object's belonging parameters are divided by 5 in this case. For example h_5 has only two parameters belonging among total five, therefore $f(h_5) = 2/5 = 0.4$.

Table 3: Fuzzy set and soft set values description for same vague data

U	Fuzzy set	Soft Set				
		e_1	e_2	e_3	e_4	e_5
h_1	1	1	1	1	1	1
h_2	0.8	1	1	1	1	0
h_3	0.8	1	0	1	1	1
h_4	0.6	1	0	1	1	0
h_5	0.4	1	0	1	0	0
h_6	1	1	1	1	1	1

It is obvious from Table 3 that description of fuzziness for all houses in fuzzy set is narrow and restricted to one value only, while same is described comprehensively in soft set. For further illustration, h_2 and h_3 are treated same having same accumulated value equal to 0.8 in fuzzy set, but in soft set it is different and described with detail of parameters i.e. h_3 is not wooden but in good repair condition, while h_2 is wooden but in bad repair status. This difference of fuzzy set and soft set is further revealed if a parameter is deleted from or inserted into this comparison Table 3, [26-28]. Based on this example, we state below differences between fuzzy set and soft set theory.

a. Soft set describes fuzzy data in term of each parameter presence or absence while fuzzy set describe it in term of all parameter's accumulative weight only.
b. Each parameter is described in crisp from i.e. one or zero in soft set while fuzzy set has no description for parameter's description at all.
c. Fuzzy set membership function values are in $[0, 1]$ range while in soft set, membership is calculated for each object by adding parameters weights.

In addition to above differences between soft set and fuzzy set, there is a big similarity between both theories.

> "*Decision making or calculating maximal choice has same results for both soft set and fuzzy set theories*"

Like soft set maximal choice in Table 2, h_1 and h_6 are maximal choices of Mr. X in fuzzy set as well in Table 3. It is because both have maximum values equal to 1 among all other houses. Similarly, h_2 and h_3 are sub optimal choices having value equal to 0.8 and h_5 is the worst choice.

4 CONCLUSION

In this paper, we revealed the main difference between two famous theories of uncertain data named fuzzy set and soft set. We used existing application for representation of soft set and derived its equivalent membership function for fuzzy set. In comparison of both theories through same example, their significant differences and similarity are exposed with a reasonable brief clarification.

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