A Framework for Automatic Generation of Fuzzy Evaluation Systems for Embedded Applications

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Abstract: Fuzzy logic is a powerful modelling approach to build control applications and to generate knowledge-based evaluation indices. In both cases, however, the applicability to complex systems is limited by the effort required to formulate the rules, whose number grows rapidly with the number of input variables and membership functions. This work presents a framework that implements the F-IND fuzzy model to simplify the formulation of fuzzy indices, where the rules are automatically generated on the basis of the specification of best and worst cases on the membership functions of each input variable. The paper discusses the method and presents the organization of the framework that allows automatic code generation, targeting the efficient execution of the calculations on an embedded system. The framework has been tested and validated on real hardware.

1 INTRODUCTION

Fuzzy logic is widely used to represent systems characterized by high complexity and uncertainty such as those encountered in biology, sociology or economy. Applications of fuzzy logic have been proposed in several scientific domains, from ecology (Zhu et al., 1996; Pouv and Kwiatkowska, 2013) to engineering (Feng, 2006), medicine (Mahlouf et al., 2001) and economics and management (Wong and Lai, 2011). The advantage of using fuzzy logic over other techniques stands in its capability to apply meaning to imprecise concepts and using uncertainty as additional source of information. In particular, fuzzy logic is suitable for developing knowledge-based indices, as it allows for incorporating expert judgement in the index design process, while providing the tools to handle subjectivity with mathematical rigour.

It must be pointed out that a “general” fuzzy model does not exist because the three stages of fuzzy modelling (fuzzification, inference, defuzzification) include a series of decisional steps: selection of relevant variables and their properties (number and shape of membership functions), structure of rule-base, mathematical interpretation of the logical connectives IF...THEN, AND, OR, defuzzification strategy, which can be implemented in several different ways. For example, the published fuzzy models of environmental conditions offer a wide variety of solutions (see (Marchini, 2009) for a review). While guaranteeing high modelling flexibility, the variety of choices that exist at each stage may be seen as an impediment due to the high degree of subjectivity (Silvert, 1997). Determining or tuning good membership functions and fuzzy rules is not always an easy task (Adriaenssens et al., 2004), since optimal parametrization of membership functions, weights, rules, etc., is only possible when a large amount of experimental data is available, which in some disciplines is difficult to achieve. This is especially problematic when many variables and many membership functions are involved in a model: because of the exponentially increasing number of inference rules, the fuzzy rule base has been considered both non-transparent and challenging to apply (Cornelissen et al., 2001).

An approach to overcome these difficulties was proposed by (Marchini et al., 2008), who developed F-IND, a framework to develop fuzzy indices of ecological conditions (quality, vulnerability, etc.) that uses an algorithm for automatic assessment and weighting of inference rules. The requirements to develop a fuzzy expert system with F-IND are: (i) the identification of relevant variables and the cate-
categories that express attributes of the variables (e.g., low, medium, high), designed in the form of fuzzy membership functions; (ii) the identification of the ‘best-case’ and ‘worst-case’ categories, (iii) the assessment of variable weights (optional) and (iv) the definition of output categories. F-IND generates a graphical user interface for a rapid and easy processing of input data, to produce a risk value (model output) between 0 and 100. The main advantage of F-IND is that it does not require the manual definition of fuzzy rules, since such rules are automatically derived from the distances calculated between membership functions. The automatic calculation requires the association of a ‘worst’ and a ‘best’ membership function to each variable. Best and worst functions represent the references to calculate the weights that are used for the automatic derivation of the rules.

This work describes a framework for the efficient implementation of the F-IND method. The paper firstly recalls the mathematical details of the method introduced in (Marchini et al., 2008). The main contribution consists in the description of the software framework that implements the method. A dedicated configuration format based on the TOML language is used to define the model, i.e., the parameters of variables and membership functions. The description of the model feeds a program that generates the source code implementing the model. The generator program is written in Python language, which allows a straightforward and flexible implementation, while the generated code is standard C language, which favors the portability on any computing platform—with specific attention to resource constrained embedded systems—and efficiency of the execution. The performance of the generated code is evaluated on real hardware (Arduino embedded board).

The automatic generation of rules in fuzzy inference systems is a topic that was tackled from different perspectives. (Angelov and Buswell, 2003) adopts genetic algorithms to derive fuzzy rules from data, providing the benefit of neural networks with the expressiveness of fuzzy systems. Template-based code generation methods are already available (Hasan, 1994). However, on one hand, the existing frameworks do not address the automatic generation of rules provided by F-IND. In (McCarty et al., 2013), a framework for automatic generation of a fuzzy controller is proposed. The focus is on visual tools to configure, model and run the controller. Indications on automatic rule generation are given also in (Byte Craft, 2008) for traditional models.

2 SYSTEM MODEL

The proposed framework implements the F-IND method, i.e., an approach that simplifies the definition of the rules in a fuzzy inference system. The F-IND method is based on the Mamdani model, since it puts into relationship input and output fuzzy sets; the output sets can be defuzzified to determine the crisp value of the index.

The method is designed to simplify the definition of fuzzy indices. The main advantage of F-IND is the possibility to express the inference rules between inputs and outputs by defining a reduced number of parameters. Indeed, the method uses such parameters associated to input variables to calculate a set of weights, thus allowing for an automatic definition of the rules; the designer is thus exempted by the manual definition of the rules.

We consider a fuzzy model with $N_i$ input variables $V_i$, $i \in [1,N_i]$, valid in the interval $[\text{q}_i^{\text{min}}, \text{q}_i^{\text{max}}]$. Each variable $V_i$ is defined as a set of $M_i$ membership functions, denoted as $f_{ji}(\cdot)$, $j \in [1,M_i]$ (e.g., variable $V_1$ has $M_1 = 3$, and $f_1^1 \equiv \text{"low"}$, $f_2^2 \equiv \text{"medium"}$, $f_3^3 \equiv \text{"high"}$).

With this notation, in standard fuzzy models the number of rules to define is:

$$N = \prod_{i=1}^{N_i} M_i$$

The F-IND method reduces the number of parameters to tune to

$$N = 2 \cdot N_i$$

This is an important feature, since rule definition can be both a time consuming and challenging process.

Notice that these parameters only refer to those necessary to define the fuzzy rules. In particular, the parameters that define the membership functions are not accounted, since we assume that the same effort is required in the traditional model as in F-IND to define the membership functions.

There are some requirements to be satisfied to guarantee the correctness of the calculations:

- Membership grades are in the $[0, 1]$ interval;
- For every input variable $V_i$ it must hold:

$$\forall x \in [\text{q}_i^{\text{min}}, \text{q}_i^{\text{max}}], \sum_{j=1}^{M_i} f_{ji}(x) = 1$$

(1)

- For every variable $V_i$, there are at most two non null membership grades for any given point $x$, i.e.:

$$\forall x \exists j : \begin{cases} f_{ji}(x) \neq 0, & j \leq M_i \\ f_{j+1i}(x) = 1 - f_{ji}(x), & j < M_i \\ f_{ki}(x) = 0, & k \neq j, j + 1 \end{cases}$$

(2)
When a variable $V_i$ is evaluated for a given input value $x$, we say that such variable is full degree if $\exists f_i^j : f_i^j(x) = 1$.

This section will describe the methodology behind the F-IND framework, from the definition of inputs to the process of weights assignment and the actual calculation of the index.

### 2.1 Inputs and Outputs

The definition of a fuzzy variable under the F-IND method requires the association of best and worst cases to each variable. The best and worst membership functions define, respectively, the most and the least favourable cases w.r.t. the corresponding variable. The calculation of the weights that define the fuzzy rules is based on the distances between membership functions of a variable. Figure 1 shows the best and worst values of a variable in the range $[V_{min}^i, V_{max}^i]$.

As mentioned before, the number of model parameters required by this method is reduced to $N = 2N_a$, i.e. the definition of best and worst cases for each input variable.

### 2.2 Weights

The F-IND method overcomes the problem of manual rule definition by means of an automatic rules-assessment procedure. This is based on the qualitative concept of best and worst membership functions described in section 2.1. Indeed, the meaning of these concepts is that the maximum value of the fuzzy index is obtained when the membership grade of the best membership functions is 1 for all the variables. Similarly, the minimum value of the index (i.e., equal to zero) is obtained when the membership grade of the worst membership functions is 1 for all the variables.

The weights used in the F-IND model are essentially the distances of the membership functions representing the properties of a variable from the worst function of the same variable. Such a distance depends on the shape of the membership function and their “proximity” along the horizontal axis.

For each variable, the best and worst membership functions must be distinct. Moreover, any of the available functions can be selected as best and worst cases. The procedure to calculate the distances between the membership functions ensures that such distance has always the highest possible value between best and worst functions. This is achieved using the concepts of internal distance and external distance. The actual distance of the considered membership function is either equal to the internal or the external distance depending on the position of the function w.r.t. the worst and best functions.

Since most of the systems are affected by best and worst cases placed on the boundaries of the input interval, only the concept of internal distance is relevant and will be discussed in this section, while both are implemented in the code.

The internal distance is defined as the sum of two values: the internal difference and the internal gap. The former value is defined as the integral of the absolute value of the difference between the two adjacent membership functions. This value is calculated by eq. (3).

$$id_{j,j+1}^i = \int_{q_{min}^i}^{q_{max}^i} \|f_j^i(x) - f_{j+1}^i(x)\| \, dx \tag{3}$$

Figure 2 shows the calculation of $id$ for $f_j^i = A$ and $f_{j+1}^i = B$.

The internal gap $ih_{j,j+1}^i$ is instead defined as the integral of the “hole” between the two adjacent membership functions, as shown in fig. 3, as calculated in eq. (4).

$$ih_{j,j+1}^i = \int_{q_{min}^i}^{q_{max}^i} h_{j,j+1}^i(x) \, dx \tag{4}$$

The term $h_{j,j+1}^i$ in eq. (4) is defined in eq. (5), where $\bar{x} \in [\bar{q}_{min}^i, \bar{q}_{max}^i]$ is the value such as $f_{j}^i(\bar{x}) = f_{j+1}^i(\bar{x})$. 

Figure 1: Examples of desired behaviours of the index for a particular input variable $V_i$: on the left the case of best and worst cases on the boundaries and on the right the case of best case within the valid range of the variable $V_i$.  

Figure 2: Internal difference between A and B.
The internal distance between two adjacent membership functions belonging to the $i$-th variable is derived as in eq. (6).

$$ID_{j,j+1}^{i} = id_{j,j+1}^{i} + ih_{j,j+1}^{i}$$

This way, all the distances between consecutive membership functions can be calculated.

Once the distance between adjacent functions is known, it is possible to calculate the distance of any function from the worst function. The distance between the $j$-th and the $(j+n)$-th functions is the sum of the distances between consecutive functions $f_{k}^{i}$ and $f_{k+1}^{i}$ for all $k \in [i,j+n-1]$ (see eq. (7)).

$$ID_{j,j+n}^{i} = \sum_{k=j}^{j+n-1} ID_{k,k+1}^{i}$$

The calculation of the weight associated to the $j$-th membership function $f_{j}^{i}$ assumes that $ID_{j,w}^{i}$ is the distance between $f_{j}^{i}$ and the worst function, while $ID_{b,w}^{i}$ is the distance between the best and worst functions.

The weight $W_{j}^{i}$ associated to $f_{j}^{i}$ is calculated as in eq. (8).

$$W_{j}^{i} = \frac{ID_{j,w}^{i}}{q_{j}^{\text{max}} - q_{j}^{\text{min}}} \sum_{i=1}^{N_{m}} ID_{b,w}^{i}$$

This normalization process is performed in order to have weights independent from the variable range.

### 2.3 Index Calculation

Once the weights have been calculated, the F-IND approach can be applied to the input values. Given an input, only the membership functions that obtain a non-zero membership grade are considered to define the active rules. For evaluating each rule the and operator is used, implemented as the multiplication operator.

Since for each input the maximum number of membership functions with non null degree is two, as in eq. (2), the maximum number of active rules is $N = 2^{N_{m}}$, with $N_{m}$ being the number of input variables.

Denoting with $V_{k}$ the product of all the membership functions in the active rule $R_{k}$, as:

$$V_{k} = \prod_{i \in \{1,N_{m} \mid f_{j}^{i}(x) \}} f_{j}^{i}(x)$$

and $W_{k}$ the sum of the weights of the same membership functions, as:

$$W_{k} = \sum_{i \in \{1,N_{m} \mid f_{j}^{i}(x) \}} W_{j}^{i}$$

where $f_{j}^{i}$ is the $i$-th membership function, present in the $k$-th active rule.

We define the weight of the $l$-th membership function of the output as:

$$W_{l} = \sum_{k} V_{k} f_{l}(W_{k})$$

Finally, the output index is calculated using eq. (12).

$$I = 100 \cdot \text{div} \cdot \sum_{l} l \cdot W_{l}$$

where $l$ is the index of the membership function and $\text{div}$ is defined as: $\text{div} = (N_{\text{out}} - 1)^{-1}$, with $N_{\text{out}}$ being the number of membership functions in the output variable.

### 3 SOFTWARE IMPLEMENTATION

This section describes the implementation of the F-IND model and the organization of the implemented modules. The software framework is made by three main parts: two libraries of procedures that implement fuzzy systems, written in C and Python languages, and a template engine that automatically generates the source code from configuration and template files.

#### 3.1 Operational Workflow

The software implementation of the F-IND model is based on a configuration file that allows to define the components of the model (input and output variables, membership functions, etc.). Afterwards, the C library is created from the template engine and can be built into an executable program. Finally, the user can

1The project is available for download at https://github.com/lab-robotics-unipv/pyFUZZYgenerator
run it either as a standalone C program or in a Python environment, both on a computer or on top of an embedded platform. The whole process can be used to create and run either a standard Mamdani fuzzy system or a F-IND model. The latter is presented in details in this paper.

3.2 The Python Library

Since the use of the C language to define the model from scratch could be a relatively time consuming operation, a dedicated library has been implemented using the Python language. Due to its simple syntax, powerful and modern features, and the availability of a huge set of libraries, the Python language represents an ideal solution for software prototyping, and to act as abstraction layer on top of more efficient (but less flexible) languages. Since the F-IND Python library generates pure C code, no native Python code is required to be executed for the calculation of index, which eliminates the well known overhead due Python code execution. This is obtained by means of a configuration process and a template engine that will be discussed below.

3.2.1 Configuration of a Fuzzy Model

To simplify the design and the implementation of a fuzzy model, the model can be configured by editing a dedicated configuration text file. In particular, the configuration file uses the TOML

2

syntax. TOML is a minimal text file format that is clear and readable, but still implements flexible data structures such as arrays and dictionaries.

The configuration facility can define either standard or F-IND fuzzy models, depending on the type field in the file. The two cases differ in terms of the information required to define a model: while standard fuzzy systems need the explicit definition of rules beside input and output details, F-IND models require the specification of best and worst functions for each input variable, while the rules are automatically derived as described in section 2.

The syntax used in the configuration file has been inspired by the Fuzzy Control Language (FCL) (IEC 61131-7, 2000). In the FCL the various components, such as variables and rules, are encoded in blocks. A custom configuration file format based on the TOML language was used instead of the standard FCL format for one main reason: the FCL format, despite being standardized, is not really widespread in computer science applications dealing with fuzzy systems.

This is due to its complicate format, which is difficult to parse. For this reason, only few libraries are available to parse FCL files. On the other hand, the proper encoding of fuzzy concepts using the TOML format allows a trivial parsing of the configuration file (this is done with one single instruction in Python), while libraries for parsing TOML files are available for dozens of programming languages, making this file format easily adoptable in other software projects that require the implementation of fuzzy models.

An excerpt of the configuration file format, containing the configuration of a single input F-IND system, is reported in fig. 4. The human readability and the straightforward syntax are evident in the example. The framework allows the use of commonly shaped membership functions, including trapezoids, triangles, crisps, sigmoids and gaussians. It is noticeable that the [[model]] syntax defines an array.

```
[[model]]
name = "Example model"
description = "Example model"
version = "1.0alpha"
type = "f-ind"

[[model.input_var]]
name = "I"
description = "An input variable"
best = "A"
worst = "B"

[[model.input_var.mf]]
name = "A"
type = "trapezoid"
parameters = [0.0, 0.0, 0.7, 0.9]

[[model.input_var.mf]]
name = "B"
type = "triangle"
parameters = [0.7, 0.9, 1.05]

[[model.output_var]]
name = "O"
description = "The index value"
best = "A"
worst = "B"

[[model.output_var.mf]]
name = "A"
type = "triangle"
parameters = [0.0, 0.0, 1.0]

[[model.output_var.mf]]
name = "B"
type = "triangle"
parameters = [0.0, 1.0, 1.0]
```

Figure 4: An excerpt of the configuration format proposed in the framework.

2Tom’s Own Minimal Language: https://github.com/toml-lang/toml.
means that in a single configuration file the user can define more than a single fuzzy model, allowing for the definition and the simultaneous generation of multiple, complex models.

### 3.2.2 Automatic Code Generation

The generation of the C code is done by means of templates that are filled with the proper parameter values to obtain a program that can be successfully built using standard ANSI C compilers.

In this project, the Jinja\(^3\) text-based template engine was used. Thanks to its syntax, it is highly integrated with the Python programming language. Differently from basic template engines, where specific parts of the templates are filled with the right text, Jinja offers more flexible and powerful options including the embedding of pure Python objects and programming statements directly in the template itself, and template inheritance. These features allow a modular implementation of the framework, a clear and concise definition of the templates, which translate to a clear and readable C library.

The code generation follows a hierarchy of three different levels: the models are stored in separate files, one for each model, while the included logic to process each type of fuzzy model (e.g. F-IND or standard model) is included as a library only when there is at least one model requiring it. Finally, common files shared among all models are created and included, such as a `main.c` file containing the `main` function. This latter provides to the user an example on how each defined index can be initialized and executed. Descriptions for each model are generated from the TOML and provided as comments both in each model file and in the sample `main.c`.

The automatic code generation handles all but index calculation, which depends on the input and therefore must be calculated online. In particular, the automatic code generation uses eqs. (3) and (4) to calculate the distance between membership functions as in eq. (7) and evaluates the relative weights (eq. (8)).

Model and variable names defined in the configuration file are used as identifiers in the code, to achieve the desired level of readability of the generated code. Some variable types are also automatically inferred by the model properties.

### 3.3 The C Library

The goal of the proposed framework is to generate an executable program that suitably runs a fuzzy inference system on resource constrained embedded hardware. Therefore, C programming language has been chosen for its resource efficiency. The generated C code is structured in several libraries containing the procedures to compute the supported fuzzy indexes. The source code of each index is written within a dedicated `.c`, along with the necessary parameter values to implement the index. These values include the references to input and output membership functions and the normalized weights. All these values are statically allocated in memory to optimize the memory usage in the constrained environment that characterizes typical embedded applications. Dynamic allocation (the family of `malloc` functions) is not used in the library since several platforms do not support it. Most of the required memory is used to store models parameters; the index calculation only makes use of a few local variables. As a side benefit, this approach allows to estimate the required memory beforehand. Moreover, thanks to eq. (2), we know that only up to two adjacent membership functions have non-null value, and their values are related, since they sum up to 1. Therefore, once the first “active” membership is identified, there is no need to calculate nor to store the next membership values. As a result, exactly one value per input variable is stored, and possible time consuming calculations are eliminated; this is especially relevant when complex functions need to be evaluated.

Valgrind analysis shown that around 90% of the computation time is spent in detecting and evaluating the membership functions having non-null value, and to calculate the automatically generated rules. Therefore, the optimization of the code focused on the evaluation of the rules (see eqs. (9) and (10)). An efficient method to evaluate such equations is to store the indexes of the first active membership function for each variable – and their values – into one-dimensional arrays, which are indexed using a properly encoded bit-mask. In particular, the indexes of the first active MF and their values for each variable are stored and indexed using a properly encoded bit-mask. The advantage is that the update of the bit-mask is done efficiently.

Furthermore, when a variable is detected to be full degree, half of the remaining rules, i.e., those including the adjacent membership function (which has degree equal to 0) can be skipped. An empirical evaluation of this behaviour is reported in section 4.2.

As consequence, the worst case computation time (WCET) corresponds to the evaluation of all the rules.

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\(^3\)Jinja template engine: http://jinja.pocoo.org/.
4 EXPERIMENTAL RESULTS

This section evaluates the performance of the F-IND library on the real hardware of an embedded platform. The goal is to assess the computational performance of the generated C code running on top of an embedded micro-controller. For this reason, the experiments define a simple index based on the measurements from an accelerometer interfaced to the micro-controller. The F-IND index runs on the embedded board and is required to produce an indication of the planarity of the board in the 2D space. The index shall be equal to 100% when the inclination of the board is null; the index value decreases when the two angles increase. The angles along the two orthogonal axes are denoted with $\alpha$ and $\beta$. They can assume values in the interval $[0^\circ, 60^\circ]$. The index is designed such that it is less sensitive to changes in the low range of the $\beta$ angle, while being equally sensible to both angles for high values of the angles. These specifications translate to the membership function depicted in Figs. 5(a) and 5(b), respectively for $\alpha$ and $\beta$.

Figure 5(c), instead, shows the same relationships in a surface plot.

4.1 Hardware Setup

The assessment of the computational performance of the F-IND index was carried out on the Arduino Pro Mini 3.3 V board, which is a low-end development board with rather constrained memory and processing power. It features an ATmega micro-controller without floating point operations unit (FPU)$^4$. The board is connected to the computer by an FTDI to USB connector that is used both for programming and for serial communication between the two devices.

The microcontroller samples acceleration measurements from a ADXL335B accelerometer$^5$. An analogue signal is sampled for each of the three axes. These values are fed into the C function that calculates the index, and both the index and timing information are sent to the computer for logging.

4.2 Results

The full program occupies 555 B of memory, around 27% of the total memory available to the Arduino Pro Mini. Since the serial library alone takes 198 B of memory, it turns out that the fuzzy library alone requires about 357 B of memory using the defined model. The time required for the index calculation ranges from 500 $\mu$s to 2500 $\mu$s, depending on input values (as explained in Section 3.3).

The performance of the generated C code was tested on more complex models to evaluate the scalability of the solution as a function of the number of model parameters (i.e., the number of variables). The test was performed on the same Arduino Pro Mini platform, where increasingly larger indexes were run and fed with random input values. Figure 6 shows the memory usage as a function of the number of input variables, either in case of 3 and 5 membership functions per variable. The memory usage changes almost linearly with the number of input variables, while it increases with the number of membership functions. The memory occupation $\text{mem}$ can be approximated using the following linear regression: $\text{mem} = 29.5 \cdot M + 98.98$ B. The $R^2$ value is 0.99.

As mentioned in section 3.3, the time required to compute the index greatly depends on the number of inputs, while it does not significantly change with the number of membership functions used to define the input variables. Figure 7 shows the time required to calculate the index using two different models, having 4 and 5 input variables respectively; each variable has 5 membership functions in both cases. The models are fed with random input values. As can be seen in the figures, the computation time required by the index calculation is distributed in a number of discrete clusters, depending on the number of full degree variables, which in turn depends on the input values. Defining $N_{\text{full}}$ as the number of full degree variables, and $C$ the time required to evaluate one single rule, it holds that the time required to evaluate the index is around $\text{Time} = 2^{N_{\text{full}}} \cdot N_{\text{full}} \cdot C$, with small approximations due to execution overheads, which lead to small variability within each cluster. As a consequence, the WCET is obtained when $N_{\text{full}} = 0$, i.e. when the number of active rules is maximum ($2^{N_m}$).

5 CONCLUSIONS

This paper presented a framework for the generation of software programs that implement fuzzy indices with automatic generation of inference rules. The latter feature leverages the F-IND model, which was proposed with the aim of simplifying the specification of models to define fuzzy indices.

One relevant contribution was the introduction of a file format to describe fuzzy models that is based on a standard text format, making it straightforward to parse the new format for both read and write opera-
Figure 5: Membership functions for the $\alpha$ and $\beta$ input variable, and resulting 3D surface.

Figure 6: Memory usage on the Arduino Pro Mini versus the number of input variables.

Figure 7: Distribution of execution times with random-sampled inputs for different sized models.

The proposed framework was demonstrated on real hardware.

REFERENCES


