Optimal 3D Kinematic Analysis for Human Lower Limb Rehabilitation

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Abstract: The majority of the kinematics analysis carried out on the human body are usually available only for use in the sagittal plane. Limited studies were interested in this analysis in all three planes (sagittal, transverse, and frontal) where motions of all joints occur.

The aim of this paper is to develop a new optimal kinematic analysis of human lower limbs in three-dimensional space for a rehabilitation end. The proposed approach is focused on optimizing the manipulability and the human performance of the human leg, as being a physiologically constrained three-link arm. The obtained forward kinematic model leads to define the feasible workspace of the human leg in the considered configuration. Using an effective optimization-based human performance measure that incorporates a new objective function of musculoskeletal discomfort, the optimal inverse kinematic (IK) model is obtained.

1 INTRODUCTION

Nowadays several neurologic injuries such as neuromuscular diseases, spinal cord injury cerebellar disorders, stroke, or impaired functions of the member musculature lead to the joint disorders. Indeed, the lower limb is usually including chronic pain, atypical gait patterns, reduced range of motion (ROM), weak strength, and increased joint stiffness, as well as severe functional limitations, and thereby reducing patient's quality of life.

To remedy these problems, we use the rehabilitation process, based on physical therapy to restore patient's strength, mobility and fitness. Traditionally, limb physical therapy sessions were carried out manually with assistance of therapists. However, the poor performances in terms of duration, strength and task orientation of the training, and the inconsistency in therapy sessions from one session to another have been noted, as principles issues, to encouraged many researchers to require the robotic, where a good repeatability, and a precisely controllable assistance, providing quantitative measures of the subject's performance and reducing the required labor of physical therapists are carried out. Therefore, the use of robotics in this context needs to take into account the different biomedical constraints imposed in the study of this system (H. Faqihi and Kabbaj, 2016).

Generally, robotic is largely used in many applications such as medical, physical and industrial, where high accuracy, repeatability, and stability of the operations are required. For different robotic studies, the developpement of control laws is commonly executed in joint space. However, the motion planning is given in the task space, especially when it comes to real applications as rehabilitation, where the desired input is usually the end effector position in task space. Hence the necessity to resorting to the Inverse Kinematic (IK) task to find a configuration at which the end-effector of the robot reaches a given point in the task space.

Several researches have been provided to derive the IK problem, especially for redundant articulated robotic arm, such as a parts of human body, where the complexity is enhanced with the increased Degrees Of Freedom (DOF). Thereby, solving the IK problem is quite a challenging task, where its complexity lies in the robots geometry and nonlinear relation between cartesian and joint space.

The most popular IK methods developed in the literature are algebraical, geometrical methods (W. M. Spong and Vidyasagar, 2006), and the analyt-

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ical methods (A. Ochsner, 2014), using the pose (position and orientation) as a given goal, (V. Kumar and Shome, 2015), (S. Tejomurtula, 1999), (J.M.Porta, 2005). They are usually designed for a very specific task, and remain very limited for the higher DOF robots. They do not guarantee closed form solutions, and they are entirely sensible to the starting point and singular configuration problem.

Taking into account the dynamic of motion, the Jacobians method can also be used to resolve the IK problem, but it has been pointed out that it does not provide all credible solutions. Additionally, these traditional solution methods may have a prohibitive computational cost because of the high complexity of the geometric structure of the robotic manipulators.

To summarize, the application of the classical IK methods for the human body, besides its complexity, remains just viable mathematically, do not take into account the physiological feasibility and biofidelity of human posture, and suffer from numerical problems (K.Abdel-Malek, 2004) .
Optimization based approaches can be suitable ways to overcome the above mentionned problems. It refers to predict the realistic posture of human limb in its feasible workspace. As any optimization problem, for the posture prediction problem, the joint angles of the human leg are considered as the design variables, the constraints are considered according to physiological feasibility and motion precision, and for the objective function, the human performance measures are used.

There are many forms used in the literature to define the human performance measure, such as physical fatigue defined as reduction of physical capacity. It is mainly the result of three reasons: magnitude of the external load, duration and frequency of the external load, and vibration (Chen, 0004). However, for the movements required low speed such as rehabilitation exercises, the physical fatigue is not so significant. Indeed, the required movements can lead to some human discomfort (K.Abdel-Malek, 2004), where its evaluation may vary from person to person, such as potential energy (Z. Mi, 2009), torque joints, muscle fatigue, or perturbation from a neutral position (W. M. Spong and Vidyasagar, 2006).

This study seeks to introduce a general optimization-based formulation for posture prediction of human lower limb exclusively in all sagittal, transverse, and frontal planes with seven degrees of freedom. Refering to the published studies, the proposed kinematic analysis is the first one developed in 3D plane. A new objective function incorporating three factors that contribute to musculoskeletal discomfort is developed as human performance measure.

To better illustrate these aspects, the remainder of the paper is organized as follows: In section II, the human leg modeling will be presented. The forward kinematics has been developed in the three planes where motions of the human lower limb occur with seven degrees of freedom. According to that, in section III, the feasible workspace have been established. In section IV, the new optimal posture prediction has been described and thereby applied on the human lower limb for the provided motion configuration. To check the effectiveness of the proposed approach, in section V, a simulation model has been developed using Matlab package. To sum up, the results of the study are outlined in section VI.

1.1 Human Lower Limb Description

The human body is a complex system, its biomechanical modeling represents a simplification of its real operating. The introduction of assumptions is necessary in this order, which are selected according to the desired performances.

The model adopted for the lower limb represents a system of articulated links connected by joints, based on three segments to model its anatomical structure: thigh, shank and foot considered as the length between ankle and metatarsal.

The connection of all three segments is ensured naturally by ligaments and muscles, and should be kinematically redundant to ensure biofidelity of the human leg motion (S. Tejomurtula, 2005).

For a static analysis, the human leg is modeled by a kinematic chain of rigid bodies, interconnected by kinematic joints, which can be either simple or complex according to required physiological behavior, and thus the degree of freedom associated with the possible joints. The principal joints are hip, knee, and ankle (H. Faqihi and Kabbaj, 2016).

According to the special rehabilitation use, we are interested in this study, to the human leg motion provided in three planes of the space, where the motion of the human leg is provided for sevens degree-of-freedom, defined as: 3 DOF hip (extension-flexion degree-of-freedom, abduction-adduction degree-of-freedom, and inversion-eversion degree-of-freedom), 1 DOF knee (extension-flexion degree-of-freedom), 3 DOF ankle (extension-flexion degree-of-freedom, abduction-adduction degree-of-freedom, and inversion-eversion degree-of-freedom), as depicted in figure 1.

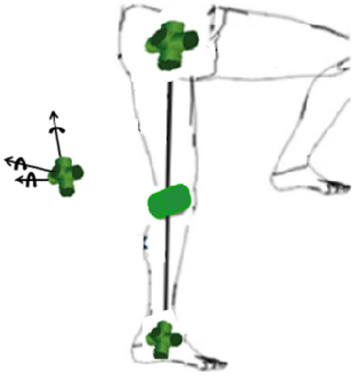

Figure 1: Coordinate systems on 7 DOF of the human leg.

1.2 Anthropometric Parameters

The modeling of body segments must take into account some anthropometric parameters.

In order to customize the model, accurate measurements of the anthropometric parameters are required and can be obtained from statistical tables proposed in (S. Tejomurtula, 2005). It refers to adopt a proportional anthropometric model, based on statistical regression equations to estimate these segmental inertial parameters (PIS).

For this study, physical length segments will be used. They can be computed using total body height (H).

1.3 Range of Motion

Due to the biomechanical constraints of human body motion, the bounds of the joint variables are fixed, which define the Range of Motion (ROM).

Defining ROM of the human lower limb model, is not limited to the designed mechanical structure, but also to the human physiological factors, such as the age, body build, gender, health condition (D.B. Chaffin, 1992). Generally, the ROM of human legs, based on a previous study by (C.S. Hernandez and Rodrguez, 2011) is given in the table I.

1.4 Comfort Zone

To ensure the comfort motion of the human body, each joint variable can be defined by its comfort zone, which must belong to the range of motion (ROM) of the associated joint variable.

Referring to the literature, the comfort zone represents 35% of the range of motion (ROM). The center of the comfort zone, q_{ic}, is calculated by the following expression (S. Glowinski, 2016):

Table 1: DH human leg parameters for 5dof.

Joint(i)	α_i	a_i	d_i	q_i
1	$-\frac{\pi}{2}$	0	0	q_1
2	$\frac{\pi}{2}$	0	0	q_2
3	0	a_1	0	q_3
4	0	a_2	0	q_4
5	0	a_3	0	q_5
6	$-\frac{\pi}{2}$	0	0	q_6
7	$\frac{\pi}{2}$	0	0	q_7

$$q_i^C = 0.5(q_{icz}^u + q_{icz}^l) + q_i^h \tag{1}$$

where q_{icz}^u, and q_{icz}^l are respectively, the upper and lower angles of the comfort zone, for the i^{th} joint variable associated. q_i^h is the home position angle of the i^{th} joint variable. Generally, the home position angle can differ from tested tasks (standing, recumbent, seating,...).

2 FORWARD KINEMATIC ANALYSIS

The forward kinematic (FK) model in n plane, can determine the pose of the end-effector ($x_j, j = 1\ldots n$), from given joint variables ($q_i, i = 1\ldots DOF$). It is a necessary step in the kinematic anlysis process.

$$x_j = f(q_i) \tag{2}$$

For the rigid bodies robotic systems, several methods can be used to resolve this problem.

Despite of the human body complexity, for all practical purposes, it has been shown that approximated modeling of gross human motion, in order to ensure human motion simulation, ergonomic analysis, or rehabilitation process, can be achieved using homogenous transformation matrices method, and the Denavit Hartenberg (DH) representation, based on appropriate kinematic coordinates.

Indeed, the DH method provides an adequate, and systematic method for embedding the local coordinate systems for each link.

The forward kinematic human leg model is developped from the DH parameters (Dombre and Khalil, 2007) where each degree-of-freedom can be modelled as a revolute joints. The DH parameters are depicted in table 1, from the defined kinematic coordinates.

From the provided D-H parameters, the forward kinematic model can be computed using the transfomation matrix, given in relation 3. Generally, the transformation matrix is the relationship expression between two consecutive frames $i-1$ and i, which depends on the described parameters (q_i,α_i,a_i,d_i) given in the Table II.

$$^{i-1}T_i = \begin{pmatrix} cq_i & -sq_i c\alpha_i & sq_i c\alpha_i & \alpha_i cq_i \\ sq_i & cq_i s\alpha_i & -cq_i s\alpha_i & \alpha_i sq_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3}$$

The global transformation matrix for seven DOF can be expressed by:

$$^0T_7 =^0 T_1.^1T_2.^2T_3.^3T_4.^4T_5.^5T_6.^6T_7 \tag{4}$$

From $^0T_7$ the forward kinematic model is given by the following equation:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_2c_{1234} + a_1c_{123} + a_3c_{1234}c_4 - a_3s_{1234}s_5 \\ a_2s_{1234} + a_1s_{123} + a_3s_{1234}c_4 + a_3c_{1234}s_5 \\ -a_1s_{23} - a_2s_{234} - a_3s_{234}c_4 - a_3c_{234}s_5 \end{pmatrix} \tag{5}$$

where: $c_{1234} = cos(q_1 + q_2 + q_3 + q_4)$, $s_{1234} = sin(q_1+q_2+q_3+q_4)$, $c_{123} = cos(q_1+q_2+q_3)$, $s_{123} = sin(q_1+q_2+q_3)$, $c_{12} = cos(q_1+q_2)$, $s_{12} = sin(q_1+q_2)$, $c_4 = cos(q_4)$, $s_4 = sin(q_4)$, and $s_5 = sin(q_5)$.
In terms of velocitiy and acceleration, the forward kinematic is given by:

$$\dot{x}_j = J\dot{q}_i \quad , \quad \ddot{x}_j = J\ddot{q}_i + \dot{J}\dot{q}_i \tag{6}$$

where $\dot{x}_j$, $\ddot{x}_j$, represent respectively the end-effector velocity, the end-effector acceleration in task space, J and $\dot{J}$, represent the jaccobian matrix of the system and its deerivative. Finally, and $\dot{q}_i$ and $\ddot{q}_i$ represent respectively the end-effector velocity and acceleration in joint space.

2.1 Feasible Workspace

In order to analyze the feasible workspace associated to the human lower limb, where we plot the different possible positions of foot which can be achieved, the direct kinematic model $f(q_i)$ is used. Thus, the workspace can be defined as the set of all the possible positions in the task space according to the ROM, as following:

$$E_p = \{q_i \in ROM / \sum_j x_j = f(q_i)\} \tag{7}$$

Using the appropriate forward kinematic model given in equation 5, and the ROM described in the table I, the feasible workspace can be plotted for the motion provided according to the used configuration.

3 PROPOSED OPTIMIZATION APPROACH

3.1 Problem Formulation

The optimal posture prediction is considered to be a constrained optimization problem (CO), using a constraint to find a realistic configuration.
Generally, the CO problem can have equality and/or inequality constraints according to the described problem, and the objective function, which requires some assumptions according to the continuity and differentiability. In that fact, in the following the optimization problem model are described:

3.1.1 Design Variables

The design variables represent in this case, the joint variables q_i, $i = 1 \ldots DOF$, following the used configuration.

3.1.2 Constraints

The first constraints consider the difference between the current end-effector position, velocity, and acceleration, and the given target position, velocity and acceleration respectively in cartesian space, as following:

$$||x_j^{computed}(q_i) - x_j^{desired}(q_i)|| \leq \varepsilon_1$$

$$||\dot{x}_j^{computed}(q_i) - \dot{x}_j^{desired}(q_i)|| \leq \varepsilon_2 \tag{8}$$

$$||\ddot{x}_j^{computed}(q_i) - \ddot{x}_j^{desired}(q_i)|| \leq \varepsilon_3 \tag{9}$$

where $||.||$ define the euclidean norm. The end-effector position $x_j^{computed}$ hits a predetermined target point $x_j^{desired}$ in cartesian space, within a specified tolerance ε_1 a small positive number that approximates zero, similarly to end-effector velocity and acceleration.

It should be noted that, determining the end-effector position, velocity and acceleration are ensured using the forward kinematic model (equations 5, 6 and 7).

On the other hand, each joint variable is constrained to lower and upper limits, represented by q_i^l and q_i^u, respectively. These limits ensure that the human posture does not assume an unrealistic position to achieve the target point.

To more rigorous biofidelity end, we can choose that each joint variable is constrained to lie between upper and lower angles of the comfort zone, designed by q_{icz}^u and q_{icz}^l respectively.

$$q_{icz}^l \leq q_i \leq q_{icz}^u \tag{10}$$

Finaly, the constraints in term of velocity and acceleration limits are used, as following.

$$\dot{q}_i^l \leq \dot{q}_i \leq \dot{q}_i^u \tag{11}$$

$$\ddot{q}_i^l \leq \ddot{q}_i \leq \ddot{q}_i^u \tag{12}$$

where $\dot{q}_i^l$ and $\ddot{q}_i^l$ are the lower limit of velocity and acceleration, $\dot{q}_i^u$, and $\ddot{q}_i^u$ are the upper limits of velocity and acceleration respectively. These limits are fixed according to biomedical studies, where the physiological and health state of patient are taken into account.

3.1.3 Cost Function

As described previously, the posture prediction requires a human performance measure, where its rigorous choice ensures the optimal realistic posture.
To this end, the modeling musculoskeletal discomfort is used as human performance measure, which can be somewhat ambiguous, as it is a subjective quantity, thus its evaluation may vary from one person to another (K.Abdel-Malek, 2004).

According to the last described researches (K.Abdel-Malek, 2004; Z. Mi, 2009), in this order, different forms of human performance measures have been adopted, but it often results in postures with joints extended to their limits, and thus to some uncomfortable positions.

As remedy, we can add factors associated with moving while joint variables are near their respective limits in terms of position, velocity and acceleration. In this respect, it is possible to incorporate different factors that contribute to discomfort. The first factor, is referred to the tendency to move different segments of the body sequentially. The second factor, is referred to the tendency to gravitate to a reasonably comfortable position. Finally, the discomfort associated to the motion while joints are near their ROM in term of position, velocity and acceleration, expresses the third factor.

According to the previous studies, the proposed objective function is similar to that adopted by (J.Yang, 2004) applied for the upper limb. However, there is currently no research focused on prediction of human leg posture, by applying this form of discomfort, with the restriction particularity of 15%, taking into account the end-effector position, velocity and acceleration.

In order to incorporate the first factor, we can find several strategies which induce motion in a certain order, or with higher weighted joints than others. Consider q_{ic} the comfortable position of i^{th} joint variable, measured from the home configuration defined by $q_i^h = 0$. Then, conceptually, the displacement from the comfortable position for a particular joint position is given by: $|q_i - q_{ic}|$.

However, to avoid numerical difficulties and non-differentiability, we can use: $(q_i - q_{ic})^2$.

Generally, terms should be combined using a weighted sum w_i, to emphasize the importance of particular joints depending on the characteristics of each patient. Thereby, the joint displacement function is given as follows:

$$f_{displacment} = \sum_i w_i (q_i - q_{ic})^2 \tag{13}$$

The weights are used to approximate the lexicographic approach (R.T. Marler, 2009).

In order to incorporate the tendency to gravitate to a reasonably comfortable neutral position, each term in equation (13) is normalized, as described in the following:

$$\Delta q_i = \frac{q_i - q_{ic}}{q_i^u - q_i^l} \tag{14}$$

Each term of $(\Delta q_i)^2$ of this normalization scheme is considered as a fitness function with each individual joint and has normalized values, which lie between zero and one.

The principal limitation of this approach often results in postures with joints limits extended, thereby an uncomfortable joint. In this order, the third factor is introduced, which defines the discomfort of moving while joints are near their respective limits. This factor requires to add some designed penalty terms to increase significantly the discomfort where joint values are close to their limits.

Generally, the new designed penalty term $P(d)$ is a barrier penalty function (H. A. Eschenauer, 1989), of d argument, expressed by:

$$P(d) = (sin(a.d + b))^p \tag{15}$$

The $P(d)$ function is adapted to penalize any number d, considered as normalized parameters, which is approaching zero at some number value.

The proposed idea is that the penalty term remains zero until the d value reaches $d \leq 0.15$, which defines the desired curve data.

Thereby, the parameters a, b, and p of the basic structure of barrier penalty function are fitted to reach the desired curve data.

According to the three described factors, the consequent discomfort function is obtained as follows:

$$f_{dicomfort} = \sum_{i=1}^{DOF} [w_i(\Delta q_i)^2 + P(R_{pui}) + P(R_{pli}) \tag{16}$$
$$+ P(R_{vui}) + P(R_{cli}) + P(R_{aui}) + P(R_{ali})]$$

where $P(R_{pli})$ and $P(R_{pui})$, $P(R_{vli})$ and $P(R_{vui})$, $P(R_{ali})$ and $P(R_{aui})$ are the penality terms with joint values that approach their lower limits, and their upper limits, respectively for end-effector position, velocity and acceleration.

$$
\begin{aligned}
R_{pui} &= \frac{q_i^u - q_i}{q_i^u - q_i^l}; \quad R_{pli} = \frac{q_i - q_i^l}{q_i^u - q_i^l} \qquad (17)\\
R_{vui} &= \frac{\dot{q}_i^u - \dot{q}_i}{\dot{q}_i^u - \dot{q}_i^l}; \quad R_{vli} = \frac{\dot{q}_i - \dot{q}_i^l}{\dot{q}_i^u - \dot{q}_i^l}\\
R_{aui} &= \frac{\ddot{q}_i^u - \ddot{q}_i}{\ddot{q}_i^u - \ddot{q}_i^l}; \quad R_{ali} = \frac{\ddot{q}_i - \ddot{q}_i^l}{\ddot{q}_i^u - \ddot{q}_i^l}
\end{aligned}
$$

The penalty term that depends on parameter d, remains zero as long as the upper or lower joint value does not reach 15% of its range, as depicted in the Figure 3.

3.1.4 Constrained Optimization Model

From the described design variables, constraints and cost function, the final optimization problem can be formulated, as the following:

$$
\begin{cases}
min \ : f_{Discomfort}(q_i) \\
subject \ \ to: \ ||x_j^{computed}(q_i) - x_j^{desired}(q_i)|| < \varepsilon_1 \\
\qquad ||\dot{x}_j^{computed}(q_i) - \dot{x}_j^{desired}(q_i)|| < \varepsilon_2 \\
\qquad ||\ddot{x}_j^{computed}(q_i) - \ddot{x}_j^{desired}(q_i)|| < \varepsilon_3 \\
\qquad q_{icz}^l \le q_i \le q_{icz}^u \\
\qquad \dot{q}_i^l \ \le \dot{q}_i \le \dot{q}_i^u \\
\qquad \ddot{q}_i^l \ \le \ddot{q}_i \le \ddot{q}_i^u
\end{cases}
$$

3.2 Propose Optimal Solution

The inverse kinematic optimization problem formulated previously is a Nonlinear Optimization Problem (NLP). Several numerical solutions of constrained nonlinear optimization problems have been presented in the literature. For resolution feasibility Sequential Quadratic Programming (SQP) is considered to be suitable method (P. Gill and Saunders, 2002) to resolve the proposed optimization problem. The algorithm resolution is divided into three main steps, as following:

– Step1: The first step begins by the initialization, where it is necessary to determine the total body height of a subject and to calculate the length of thigh, shank, and foot segments. Then, we fix the initial joint position, velocity and acceleration variables, and calculate the initial guess as being initial effector position, velocity and acceleration by using forward kinematics, according to the used configuration. Next, taking into account the joints constraints, the workspace and the comfort zone of each joint are computed.

– Step2: Giving the end position, velocity and acceleration coordinates, in the second step, we check if the position is in the workspace, else it is necessary to find new coordinates.

– Step3: SQP optimization technique is applied in this step to find the optimal postures. This is performed using Matlab fmincon constrained function. The obtained solution is a matrix with position, velocity and acceleration angles. Taking into account the comfort zone of each joint, the obtained angles are the most comfortable.

4 TRAJECTORY GENERATION

In rehabilitation robots, the reference trajectory must be predefined as a human limb motion practiced during activities of daily live. Indeed, motion therapy can be carried out in different modes including passive, active, active-resistive, active-assistive, and bilateral exercises, which differ depending on the degree of patient involvements. Selecting the proper mode strategy requires an appropriate rehabilitation robot choice, with concerned patients.

To determine the appropriate trajectory for the movement of the rehabilitation robot, there are several methods such as a prerecorded trajectory obtained by gait analysis, and a prerecorded trajectory during therapist assistance, which require data use, and modelling the trajectory based on normative movements which can be based on kinematics and/or dynamics constraints during the path motion in terms of fitting more realistic motion (Rastegarpanah and Mozafar, 2016).

Generally, it is desirable to use reference trajectories ensuring the feasibility and biofidelity of rehabilitation session. Researchers have found, from observations of healthy voluntary limb movement in joint space, that normal human movements follow a smooth trajectory that minimizes the jerk (Hogan, 1984), defined as the time derivative of acceleration and therefore the third-time derivative of position.

$$jerk(x) = \frac{d^3x}{dt} \qquad (18)$$

The minimum jerk criterion is therefore very suitable for the reference trajectories formulation of a rehabilitation robot. Firstly introduced using a point-to-point trajectory for the lower human limb.

The minimum jerk trajectory of an end-effector is obtained by the minimization of the integral of the squared jerk over time. This corresponds to minimize the function I, where T is the terminal time at which the target position x_T , velocity $\dot{x}_T$ and acceleration

$\ddot{x}_T$ are to be achieved when starting with the initial position x_0, velocity $\ddot{x}_0$ and acceleration $\ddot{x}_0$:

$$I = \frac{1}{2}\int_0^T (\ddot{x}(t))^2\,\mathrm{dt} \tag{19}$$

The condition for the trajectory x to minimise I is given as (Xie, 2016):

$$x_t^{(6)} = 0 \tag{20}$$

Therefore, the minimum jerk trajectory occurs when the sixth-time derivative of the trajectory function x is equal to zero. The possible solution of this equation can be taken as a fifth-order polynomial in time (equation 21), where a_0; a_1;a_2; a_3; a_4 and a_5 are constants to be determined from the initial and terminal conditions.

$$x(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \tag{21}$$

$$\dot{x}(t) = a_1 + 2a_2 t + 3a_3 t^2 + 4a_4 t^3 + 5a_5 t^4 \tag{22}$$

$$\ddot{x}(t) = 2a_2 + 6a_3 t + 12a_4 t^2 + 20a_5 t^3 \tag{23}$$

Using the initial conditions $t = 0$, the first three constants are obtained:

$$\begin{cases} a_0 = x_0 \\ a_1 = \dot{x}_0 \\ a_2 = \ddot{x}_0 \end{cases} \tag{24}$$

Using the terminal conditions $t = T$, the last three constants are obtained. Therefore, using the trajectory generation in joint space and forward kinematic, the trajectory in task space is obtained.

5 SIMULATION RESULTS

In order to show the effectiveness of the proposed approach, this section presents a simulation example of the developed optimal posture prediction algorithm.

For a person of 1.80m height, using the anthropometric associated data, the defined forward kinematic model and the range of motion, the workspace is plotted for the three-dimensional space as shown in Figure 2.

First the penality function P(d) is computed using equation (26) according to the desired curve data as explained previously, by using Curve Fitting Package in Matlab, as depicted in Figure 3. The obtained parameters of the penalty function are given by : $a = 2.5$, $b = 7.855$, $p = 100$.

Thereby the final expression of the penality term is given by:

$$P(d) = (sin(2.5d + 7.855))^{100} \tag{25}$$

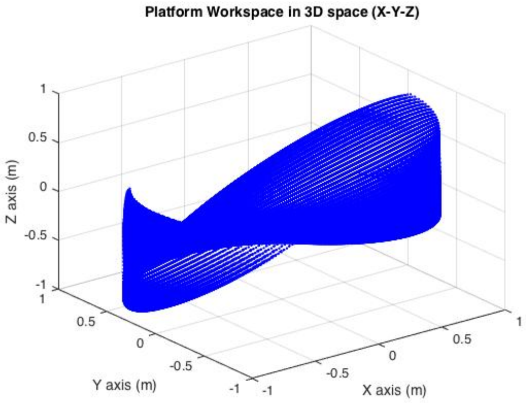

Figure 2: Workspace for 3D space.

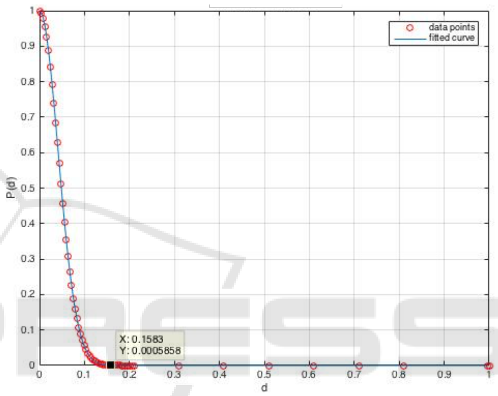

Figure 3: Penality term P(d).

To show the effectiveness of the optimal posture algorithm, the initial value is fixed in joint space according to the feasible biomechanical posture for the used configuration as $q_{initguess} = [0,0,0,0,10,0,0]$ (in degree). $\dot{q}_{initguess} = [0,0,0,0,0,0,0]$ and $\ddot{q}_{initguess} = [0,0,0,0,0,0,0]$, where the result of the optimization is usually sensitive to the initial guess.

The remain parameters are given by $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.0001$, the weight w_i for the joints variables of the lower limb, are defined in (R.T. Marler, 2009).

The optimum kinematic a parapmeters is obtained using the desired motion in terms of end effector position, velocity as depicted in Figure 4, which belongs to the defined workspace, and optimization designed routine, where the objective function, and constraints are defined, as described previously.

From the provided checking function, the end-effector coordinates of desired motion are validated, and then the optimal position, velocity and acceleration joints are predicted as shown in figure 5, and 6.

One of the most important factors in the development of any optimization problem is the selection of the suitable fitness function and well defined con-

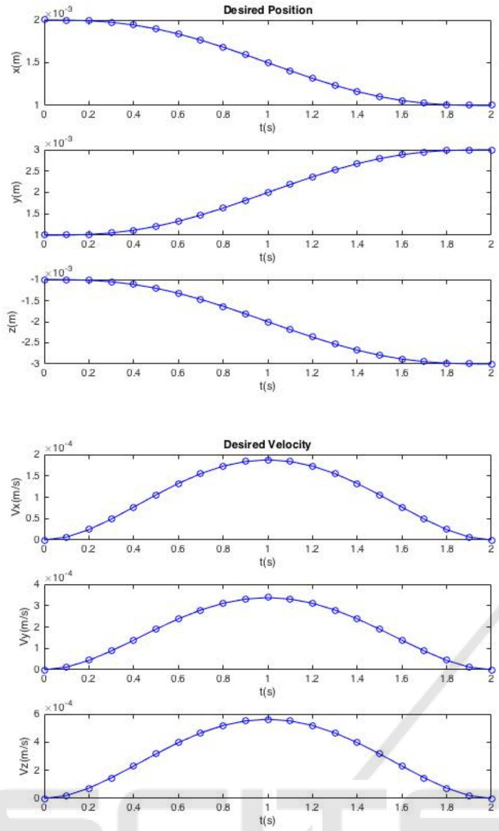

Figure 4: Desired Cartesian-space Trajectory.

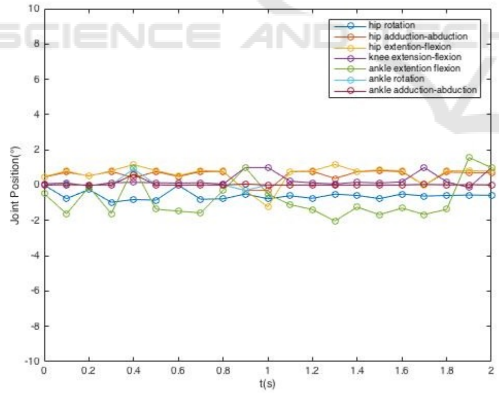

Figure 5: Obtained Joint-space Position.

straints according to the problem complexity as described in this paper. Using SQP algorithm resolution for the proposed optimization problem formulation the inverse kinematic model of 3D space human leg configuration have been predicted, and represent to our knowledge the first study carried out in the all three plane of human leg configuration.

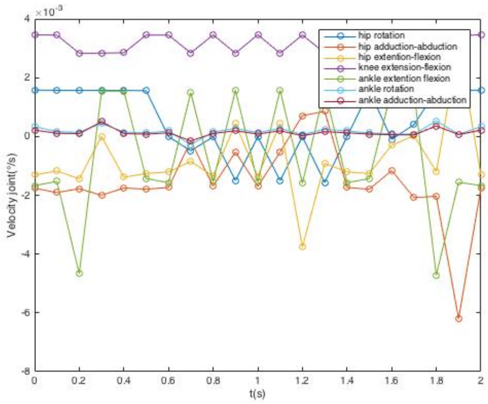

Figure 6: Obtained Joint-space Velocity.

6 CONCLUSION

A new general optimization-based formulation for optimal kinematic analysis for the human lower limb has been develope, in this study. The proposed method is developed specialy in three dimensional space, in terms of the kinematic parameters, using an objective function incorporating three factors that contribute to musculoskeletal discomfort as human performance measure.

The performance measure is referred to the tendency to move sequentially all segments of the human leg, the tendency to move leg while joints are near their ROM, and the discomfort associated with gravitating around a reasonably comfortable position.

This new form of objective function is developed principally for rehabilitation use, where physiological patient constraints need to be taken into consideration.

In this order, according to the physiological constraints and the developed forward kinematic model, the feasible workspace is presented. To validate the feasability and the effectiveness of the proposed kinematic method to predict the inverse kinematic model of 3D space human leg configuration, a reference trajectory is generated to be suitable in rehabilitation case, and thereby, applied in the proposed algorithm solution.

The simulation results, were present an optimal joint space parameters defining the inverse kinematics.

However, this study still valid for the static purpose, and can be improved according to a large definition of discomfort according to dynamical parameters in term of muscle, fatigue... which can be developed in the future research.

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