Developing a Mathematical Model of Oil Production in a Well That Uses an Electric Submersible Pumping System

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Keywords: Genetic Programming, Electro Submersible Pump Efficiency, Identification Problem, Well Production.

Abstract: This paper presents an approach to get a mathematical model for describing the production of an Electro Submersible Oil Pumping System (ESP). The determination of this mathematical model considering the variables of the process, is a very hard task. In this case, we propose the determination of this model using a data approach, in order to exploit the large quantity of data about the process obtained from its sensors. Our approach based on data uses the genetic programming techniques for the identification of the mathematical model, which follows an evolutionary process to solve the problem. Our approach has been implemented using an extension of R program. The training data were collected from an oil well. This paper presents the results of the training phase and of the generated models after several iterations. Additionally, the paper analyses the differences between the generated models, according to the number of variables considered, the complexity of the expressions, and the error.

1 INTRODUCTION

Due to the demands of productivity, safety and profitability in the industry, process automation demands sophisticated and complex Information Communication and Automation Technology (ICATs), which must operate on different platforms and exploit diverse data and information sources. Thus, the number, complexity, and variety of ICAT applications for process automation continue to grow.

In previous works, we have proposed a mechanism for integrating the different intelligent techniques, to conform what we have called an Intelligent Autonomous System for Petroleum Processes (SAI²P, for its acronym in Spanish “Sistema Autonómico Inteligente para Procesos Petroleros”), which has the abilities to adapt to new situations, and possesses attributes of reasoning to generalize and discover knowledge from the data, among other things, to make decisions about the processes (Lozada et al., 2017). In this way, SAI²P involves concepts, paradigms and adaptive algorithms, which allow to generate appropriate actions in complex and changing environments.

Specifically, one of the tasks of SAI²P is the optimization of operational performance, and in order to perform this task, it is necessary to determine the mathematical expression that calculates the efficiency of the production process of a well, based on the process variables. This mathematical function allows several things, from predicting the behavior of the well, until detecting possible problems in the production process.

SAI²P has been instance in an oil production process based on the Electric Submersible Pumping system (ESP) (Ronning, 2011), which is an economical and efficient method of artificial crude lifting, capable of handling large volumes of fluids. In this paper, we propose an approach to define the mathematical model of the efficiency of the production of a well, which uses an Electric Submersible Pumping System, based on genetic programming.

The genetic programming has been used to solve problems of system identification, based on the data about the system in study. Genetic programming follows an evolutionary process of the species, and uses a population of solutions (in our case, mathematical models) in order to search the best one, according to the goals of the problem to reach (in our...
We consider a real case of oil production. The application range of this lifting method is between 300 and 80,000 Barrels by day (BPD) for a variety of operating conditions, in oil crudes ranging from 8.5 to 40° API and viscosities of up to 5000 cp (centipoise). Additionally, the depths are between 1000 and 14,000 feet with bottom temperatures up to 350° F.

In general, the cost for installing an ESP system is high, so it requires an adequate operational management if its useful life is to be extended. That is why keeping this system within its operational parameters is the main task of real-time supervising systems. However, getting an optimal set point of operation is usually based on the experience of the field operator. On the other hand, a lot of data from these systems are being stored in the operational databases of real-time monitoring systems. This situation represents a great opportunity for the development of mathematical models, to describe the efficiency of the pump of an ESP system, according to its operational parameters, with the aim of maximizing production. (Bermúdez, 2015), (Osman et al., 2005).

In this paper, we propose an approach to determine the mathematical model that describes the efficiency of the pump of an ESP system, using the data stored in the operational databases. Our approach is based on the genetic programming, and it is able to determine the mathematical model of the efficiency of the pump in function of its operational parameters, with the final objective of optimizing the production of the system.

Also, this paper presents the theoretical framework to develop the model, and the parameters considered as well as the procedure for the creation of the model. Finally, it presents several operational scenarios, as well as the results and their analysis.

2 CONTEXT

2.1 Well Problem

This article focuses on the development of a mathematical model, which describes the efficiency of a ESP, using its operational variables, such as (Pwf) flowing pressure of the well, (Php) Pressure in the Production Head, (Thp) Temperature in the Production Head, and Produced Flow (Qprod), which are captured in the field by temperature and pressure sensors, and are integrated to control devices, and then transmitted wirelessly to the control rooms, where production operators adjust the values. The behaviour of the ESP system can be described by a serie of characteristic curves, such as load vs flow, efficiency vs flow, power vs flow, supplied by the system manufacturer taking into account its model, engine frequency, speed of rotation, and the specific gravity (Ramírez, 2004).

Determining the point of operation of the pump will be a task where factors such as the total system load and the load vs. flow curve will play a fundamental role, and the intersection of each curve marks the point of operation of the system (See Figure 1), (Baiechi et al., 2006). The Figure 1 also shows the curves of the operational parameters of the system, and the green zone determines the most efficient region of operation of a well based on an ESP system.

In general, the flow regulation in this type of system is performed by adjusting the motor rotation speed, this results in a change in the flow rate, and this change causes a new point of operation of the system.

![ESP characteristic curves](image-url)
Likewise, the changes in the physical conditions of the fluid in which the system works bring as a consequence a change in the curves, and therefore, another point of operation of the system.

Keeping the system at an optimum efficiency point is a task that must be constantly carried out whenever there are changes in the operating conditions of the system. In order to achieve this, it is necessary to have a mathematical model that describes the efficiency behaviour, in function of the operational variables of the system.

### 2.2 SAi2P

In previous works, we have proposed an *Intelligent Autonomous System for Petroleum Processes* called SAi2P, which can generate and discover knowledge from the data in the process, to make decisions about it (Lozada et al., 2017). SAi2P uses a set of concepts, paradigms and adaptive algorithms, which allow to generate appropriate actions in complex and changing environments.

SAi2P is defined by a closed loop of data analytical tasks, which allow exploiting the data about the oil process, in order to carry out an intelligent supervision of it (Camargo et al., 2014). In general, an "Autonomic Cycle of Data Analysis" defines a set of tasks of data analysis, whose common goal is to achieve an improvement in the process under study (Aguilar et al., 2016). They interact with each other, and have different roles: observe the process, analyze and interpret what happens in it, or make decisions in order to improve the process. The integration of data analytics tasks allows solving complex problems that have far been impossible to study by the amount of knowledge required for resolution.

Particularly, in this paper, we consider SAi2P, an autonomic cycle of data analytical tasks for oil production. The autonomous cycle defines a closed loop of tasks of analysis of data, which supervises permanently the oil process. It is a supervision cycle of processes based on LA tasks, in order to permanently improve them. Specifically, the data analysis tasks in the autonomous cycle are:

- Monitoring of events affecting the world oil market.
- Identification of production cost.
- Monitoring of the trends of variables.
- Identification of patterns of oil prices.
- Economic profitability assessment.
- Recognition of fault patterns.
- Diagnosis of the operational problems.
- Optimization of the operational performance.
- Maximization of the production at the lowest cost.

Specifically, in this paper, we are interested in the "optimization of the operational performance" task of SAi2. In order to perform this task, it is necessary to determine the mathematical expression that calculates the efficiency of the production process of a well, based on the process variables. This paper proposes an approach for that.

### 2.3 Genetic Programming

Genetic Programming is a technique that automatically solves problems, without requiring the user to know the structure of the solution in advance (Aguilar et al., 2001); (Langdon et al., 2013).

Fig. 2 is a flowchart that shows how the evolutionary process works. The initial step is to create a population of program trees that are randomly assembled from the available inputs. This allows a broad sampling of possible inputs, and begins the search for productive combinations.

![Figure 2: Control Flow for Genetic Programming.](image)

The initial population will probably have a poor performance overall, some individuals will perform better than others because of their features, or the way how they have combined the parameters. It is improved in the next generations, using an evolutionary process, where the best individuals are selected to generate new individuals, which must be better with respect to the individuals of the previous generations.

In the Genetic Programming, the programs are usually expressed as syntax trees. The variables and constants in the program are leaves of the tree. The arithmetic operations (+, *, etc.) are internal nodes called functions.

The steps of the Genetic Programming are:

- Initialization. The individuals in the initial population are typically randomly generated.
- Selection. Genetic operators are applied to individuals that are probabilistically selected
based on their fitness. That is, better individuals are more likely to have more kids than inferior individuals.

- Apply genetic operators. The genetic operations that are used to create new programs from existing ones are:
  - Crossover: The creation of a child by combining randomly chosen parts from two selected parent program.
  - Mutation: The creation of a new child by randomly altering a randomly chosen part of a selected parent program.

![Genetic Programming syntax tree representing a mathematical expression.](image)

Figure 3: Genetic Programming syntax tree representing a mathematical expression.

### 2.4 Genetic Program in System Identification

There are some works about the application of genetic programming in the identification problem. (Iba et al., 1995) presents a genetic programming, which integrates a local parameter tuning mechanism employing statistical search, for the system identification. More precisely, they integrate the structural search of traditional genetic programming with a multiple regression analysis method to establish an adaptive program, called STROGANOFF (STructured Representation On Genetic Algorithms for NON-linear Function Fitting). The fitness evaluation is based on a minimum description length (MDL) criterion, which effectively controls the tree growth in GP.

In (Witczak et al., 2002) it is provided a system identification framework based on genetic programming technique. Moreover, a fault diagnosis scheme for non-linear systems is proposed. In particular, they show how to use the genetic programming technique to increase the convergence rate of the observer. (Patelli, 2011) approaches the nonlinear system identification problem, by suggesting several original genetic programming based algorithms enhanced with various mechanisms designed to increase run time performance.

(Kötzing et al., 2011) analyses the Genetic Programming in the well-known PAC learning framework and point out how it can observe quality changes in the evolution of functions by random sampling. This leads to computational complexity bounds for a linear genetic programming algorithm for perfectly learning any member of a simple class of linear pseudo-Boolean functions. Finally, (Madar, 2005) proposes a method for the structure selection of Linear-in-parameters models, which uses genetic programming to generate nonlinear input–output models of dynamical systems that are represented in a tree structure. The main idea of the paper is to apply the orthogonal least squares (OLS) algorithm to estimate the contribution of the branches of the tree to the accuracy of the model.

Our approach presents an innovative application of genetic programming in the field of continuous processes, in particular, the identification of systems in petroleum processes using data stream. To our knowledge, there are no such applications in this field.

### 3 SYSTEM IDENTIFICATION BASED ON GENETIC PROGRAMMING

In this section, we present our approach for the determination of the mathematical expression that calculates the efficiency of the production process of a well, based on the process variables. The steps of our approach, based on genetic programming are:

1. Definition of the parameters of the problem.
2. Determination of the quality of the data.
4. Test and validation of the results

#### 3.1 Definition of the Parameters of the Problem

The operational parameters used to develop the mathematical model for describing the EPS system used in the oil production are:

- $P_{wf}$ (psi): Background Fluent pressure (referring to the vertical midpoint of the perforations).
- $P_{hp}$ (psi): Pressure in the Production Head.
- $F_{rec}$ (hz): Frequency of pump rotation.
- $T_{hp}$ (psi): Oil temperature on the Production Head.
- $Q_{prod}$ (barrels/day): Oil production.
Data were collected from an oil well. The number of measures were 9107.

3.2 Determination of the Quality of the Data

The quality of the data were determined applying basic statistics to find anomalies.

Figure 4 shows the results of the data analysis with maximum and minimal values for each variable.

3.3 Implementation of the Genetic Programming Model

To apply the Genetic Programming to a problem, we need preparatory steps, in which are defined the set of parameters of the genetic programming:

- The terminal set, composed of variables, functions with no arguments and/or constants (specified or randomly generated).
- The function set (arithmetic, mathematical or Boolean functions)
- The fitness measure (error between the individual output and the desire output)
- The parameters that control the run (probabilities of the genetic operators, etc.)
- The termination criterion (e.g. maximum number of generations).

3.4 Tests and Validation of the Results

The details of this phase are shown in Section 4.

4 EXPERIMENTATION

For the implementation of our approach, we have used the software RGP based on the R environment. This software support Symbolic Regression using Genetic Programming, and provides a basic set of genetic operators: mutation, crossover and selection (Flasch et al., 2010).

In table 1, we show the mean errors of the models generated in a different set of experiments. In the first experiment, we use a source of data that has anomalous data, which represent failures in the system. The second set of experiments uses a source of data where have been filtered the anomalous data. In the third set of experiments we use only correct data, and the concept of individuals’ elite. We find the best models in the third set of experiments, with the lowest mean errors (0.26%), but the models found are complex and have only one variable (see next Tables).

<table>
<thead>
<tr>
<th>Conditions</th>
<th>Mean errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without filter</td>
<td>0.6%</td>
</tr>
<tr>
<td>With filter</td>
<td>0.29%</td>
</tr>
<tr>
<td>With filter and Elite individuals</td>
<td>0.26%</td>
</tr>
</tbody>
</table>

In table 2, we present some of the mathematical models generated by the first experiment, where we have not filtered the data, with the corresponding errors. We obtain very complex individuals, and the error is due to that the mathematical equations generated in this case, consider conditions when the system has failed. That means, the genetic programming tries to generate a model that considers the normal and abnormal behavior.

Table 3 shows the best results of the second experiment. In this case, we improve the quality of the error due to that we only consider data without anomalous data, that means, data that do not represent situations with faults. Again, we obtain very complex individuals, but these mathematical equations can be simplified, because there are constants that can be eliminated.

Table 4 shows the best mathematical equations generated in the third experiment, where we reduce the error in one important level, but the number of variables considered in the equations are very limited.
To determine the best mathematical equation, we need to combine several criteria, such as the error, number of variables and complexity of the expression. We like a mathematical equation with little error because we need to follow the system, with low complexity because the mathematical equation must be easy to interpret, and which considers the largest number of variables in the modeling system.

Table 5 shows these results from the different experiments according to these criteria. The first row is from the first experiment, and we see that this individual is very good, but the expression is very complex and consider only one variable. The second equation is from the second experiment, and this equation is very interesting because the error is not high, it considers three variables of the process, and the final equation is very simple.

One important remark is that the mathematical models generated by the second and third experiments can be used in the context of the detection of failures in our system (Araujo et al., 2003). These mathematical equations follow the normal behaviour of the system, and can predict its future behaviour. However, when the behaviour of the real system is different to the estimated by our models it is because the real system has an abnormal behaviour. This can be due to failure in the system.

<table>
<thead>
<tr>
<th>Mathematical Model</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>sqrt(Frec_Hz * sqrt((7.85157934296876)) + sqrt(Thp_Psi * Thp_Psi * ln(7.6389035070135)) * ln(sqrt(Thp_Psi * Thp_Psi * ln(8.86126786364849.490767270709))))</td>
<td>0.86%</td>
</tr>
<tr>
<td>sin(Thp_Psi) + (8.43441986013204 + exp(6.77450031973422)) * (exp(7.85157934296876))</td>
<td>1.42%</td>
</tr>
<tr>
<td>sqrt(sqrt(sqrt(Thp_Psi + exp(7.85157934296876)) + (Frec_Hz + exp(6.59457715693861) + exp(6.59457715693861))) + Frec_Hz)</td>
<td>0.15%</td>
</tr>
</tbody>
</table>

Table 3: RGP Models with filters and without elite individuals, ranked by error values.

<table>
<thead>
<tr>
<th>RGP Mathematical models</th>
<th>Error &lt;1%</th>
<th>Variables number</th>
<th>Complexity of the structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>sqrt(Pwf_Psi) + Thp_Psi + Pwf_Psi + Frec_Hz</td>
<td>0.36%</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Frec_Hz + (Thp_Psi + Pwf_Psi + sqrt(Pwf_Psi))</td>
<td>0.36%</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>sqrt(Pwf_Psi) + (Frec_Hz + (Pwf_Psi + Thp_Psi))</td>
<td>0.36%</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Pwf_Psi + 4.1718687170998 * Frec_Hz</td>
<td>0.26%</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>sqrt((Thp_Psi + (Thp_Psi + Pwf_Psi)) + (Thp_Psi + Pwf_Psi) + sqrt((Thp_Psi + Pwf_Psi) + sqrt(Pwf_Psi) + Thp_Psi))</td>
<td>0.29%</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4: RGP Models with filter and elite individuals, ranked by error values.

<table>
<thead>
<tr>
<th>RGP Mathematical models</th>
<th>Error &lt;1%</th>
<th>Variables number</th>
<th>Complexity of the structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>exp(sqrt(5.115427799651 * sqrt(5.115427799651 + 6.8138162797129)) + (5.115427799651 * sqrt(5.115427799651 + 6.8138162797129)) + (Frec_Hz + exp(6.59457715693861) + exp(6.59457715693861))) + Frec_Hz</td>
<td>0.15%</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>exp(6.75399861298501) + sqrt(Frec_Hz)</td>
<td>0.22%</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>5.80232170720959 * Thp_Psi</td>
<td>0.46%</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Pwf_Psi + (Frec_Hz + exp(5.90506734503806))</td>
<td>0.46%</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9.91066501475871 + sqrt(sqrt(5.115427799651 * sqrt(5.115427799651 + 6.8138162797129)) + (5.115427799651 * sqrt(5.115427799651 + 6.8138162797129)) + (Frec_Hz + exp(6.59457715693861) + exp(6.59457715693861))) + Frec_Hz</td>
<td>0.63%</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 2: RGP Models without filters, ranked by error values.
Table 5: Best mathematical models, based on the error, number of variables and complexity.

<table>
<thead>
<tr>
<th>RGP Mathematical models</th>
<th>Error</th>
<th>Complexity of the structure</th>
<th>Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>exp(sqrt(sqrt(5.1154277799651 + 6.8138162791729) + (5.1154277799651 + 6.8138162791729) + (Frec_Hz + exp(6.59437715693861) + exp(6.59437715693861)))) + Frec_Hz</td>
<td>0.15%</td>
<td>High</td>
<td>Frec</td>
</tr>
<tr>
<td>Frec_Hz + (Thp_Psi + Pwf_Psi + sqrt(Pwf_Psi))</td>
<td>0.36%</td>
<td>Low</td>
<td>Frec, Thp, Pwf</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

SAi2P is a closed loop of data analytical tasks, which allow integrating different analytical tasks in order to give an intelligent supervision of an oil process. This framework requires of complex data analytical task in order to improve the production process. In this paper, we have studied one of them, to determine the model of production of a well, in order to define the model of optimization of its production.

Genetic Programming has been the intelligent technique used, and it has been an appropriated approach to get the mathematical model from data of the process. We have carried out different tests, with different data, in order to analyze different criteria about the mathematical equations obtained. These criteria were the error, number of variables and complexity of the expression.

The mathematical models obtained are very interesting, because their quality are very different, according to the criteria used. Additionally, some of the models obtained can be used in the context of fault detection, because they can follow the normal behaviour of our system. When the real system change its behaviour due to a failure, a detection system based on our mathematical equations can detect it (Araujo et al. 2003).

With respect to previous works, in the literature (Patelli, 2011), (Cerrada et al., 2001) have been proposed several approaches for the identification problem using genetic programming. The main differences with our approach, it is that our approach can be used in real time, and it forms part of SAi2P, an autonomous loop of data analytical tasks that allows the intelligent supervision of an oil process.

ACKNOWLEDGMENT

Dr. Aguilar has been partially supported by the Prometeo Project of the Ministry of Higher Education, Science, Technology and Innovation (SENESCYT) of the Republic of Ecuador.

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