Non-singular Terminal Second Order Sliding Mode with Time Delay Estimation for Uncertain Robot Manipulators

Yassine Kali¹, Maarouf Saad² and Khalid Benjelloun¹

¹Department of Electrical Engineering, Ecole Mohammadia d’Ingénieurs, Mohammed V University, Rabat, Morocco
²Department of Electrical Engineering, Ecole de Technologie Supérieure, Montreal, QC H3C 1K3, Canada

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Abstract: In this paper, a second order sliding mode with time delay estimation based on non-singular terminal sliding surface is presented for high-accuracy tracking trajectory of uncertain robot manipulators. The design of the proposed controller is based on a non-singular terminal sliding surface that overcomes the problem of singularity and the restrictions of the exponent in classical terminal sliding surface. Then, a second order sliding mode control scheme with time delay estimation is proposed to eliminate the chattering phenomenon and to estimate the uncertainties and disturbances. Lyapunov theory is used to prove the finite-time convergence of the sliding surface and its derivative. Finally, simulation results are presented to illustrate the effectiveness of the proposed method.

1 INTRODUCTION

Nowadays, robot manipulators become increasingly used in different industrial applications. For this reason, many research has been proposed for control of robot systems. The main control objective of robotic system is to ensure high accuracy tracking trajectory. However, robot manipulators suffer from various model uncertainties (Craig, 1989), caused by friction, unmodeled dynamics, disturbances, and payload parameters. In literature, many control algorithms have been developed for uncertain robotic manipulators, including adaptive control (Seraji, 1987), intelligent controller such as fuzzy control (Yi and Chung, 1997; Guo and Woo, 2003) and neural network (Hsia and Jung, 1991; Khalil, 1992), Sliding Mode Control (SMC) (Utkin, 1992; Utkin et al., 1999).

Sliding mode control, due to its robustness with respect to uncertainties and to the simplicity of control law design, has received a wide attention from the research community. The basic idea of SMC design is to select a user chosen sliding surface and to then design a control law that forces the system’s trajectory to reach and remain on the sliding surface. However, SMC has many drawbacks, the major one is the chattering phenomenon which comes from high frequency switching of the control signal input (Fridman, 2001; Boiko and Fridman, 2005).

To solve this problem, numerous works have been proposed. The first proposition was to replace the signum function by a smooth continuous function (Slotine and Li, 1991). However, this proposition affects robustness and accuracy. Another approach is to use the observer-based sliding mode (Liu and Wang, 2012; Cao and Chen, 2014) where the goal is to provide exact and robust estimation in order to allow chattering reduction by a small choice of the switching gain matrix. However, the control performance can be reduced if the estimation is not accurate.

In (Levant, 1993; Fridman and Levant, 2002; Sh Tesel et al., 2014), a Higher Order Sliding Mode (HOSM) control has been proposed which provides less chattering and better precision compared to classical SMC. HOSM operates on the higher order derivative of the sliding surface unlike the conventional SMC that acts on the first derivative. Here, the switching action appears in the higher derivatives of the control and the control signal becomes continuous. Therefore, the chattering phenomenon is attenuated. However, this technique doesn’t compensate uncertainties growing in time or with the state variables.

Motivated to deal with all these problems, this work will propose a robust controller for uncertain
robotic manipulators. The proposed controller in this paper is a combination of a new non-singular terminal second order sliding mode and Time Delay Estimation (TDE). Although, TDE has a very simple structure, its effectiveness to robotic manipulators has been demonstrated through many applications (Youcef-Toumi and Ito, 1990; Hsia and Gao, 1990; Kali et al., 2015). TDE provides an estimation of uncertainties by observing the inputs and the states of the robot one step into the past while the new non-singular terminal second order sliding mode will be used to ensure fast transient response, finite-time convergence of the sliding surface and its derivative to zero and to reduce chattering.

The rest of the paper is arranged as follows. Section 2 introduces the dynamic equation of n-link robot manipulators and the control objective. In Section 3, the proposed non-singular terminal second order sliding mode and time delay estimation is designed and the stability analysis using Lyapunov theory is established. In Section 4, simulation results for 3-DOF ANAT robot arm are provided to prove the effectiveness of the proposed controller. Finally, the conclusion is drawn in section 5.

2 PRELIMINARY

2.1 Robot Dynamics

Consider the dynamics of n-DOF robot manipulator in the following matrix equation (Craig, 1989):

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = \tau + \tau_d \]  

where \( q, \dot{q}, \ddot{q} \in \mathbb{R}^n \) are the joint position, velocity and acceleration vectors, respectively, \( M(q) \in \mathbb{R}^{n \times n} \) is the inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) is the centrifugal and Coriolis matrix, \( G(q) \in \mathbb{R}^n \) is the gravitational vector, \( \tau_d \in \mathbb{R}^n \) denotes the disturbance and torque input vectors, respectively.

Introducing a constant diagonal matrix \( \bar{M} = \text{diag}(m_1, \ldots, m_n) \), the dynamic equation of the robot manipulator given in partitioned form in equation (1) can be rewritten as follows:

\[ \bar{M} \ddot{q} + H(q, \dot{q}, \ddot{q}) = \tau \]  

where:

\[ H(q, \dot{q}, \ddot{q}) = (M(q) - \bar{M}) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) - \tau_d \]  

For simplicity, let us denote \( H(t) := H(q, \dot{q}, \ddot{q}) \).

2.2 Problem Formulation

The control objective is to design a robust controller to guarantee the finite-time convergence of the track-

3 CONTROLLER DESIGN

In this section, a new non-singular terminal second order sliding mode with time delay estimation will be designed to force the states to move along the sliding manifold. Figure 1 shows the architecture of the closed-loop system.

3.1 Non-singular Terminal Sliding Surface

Let \( q_d \in \mathbb{R}^n \) be the desired position trajectory and \( \epsilon = q - q_d \) be the tracking error. Then, the proposed
NTSM surface in (Tran and Kang, 2015) is given by:

$$
\sigma = \dot{\varepsilon} + \lambda_1 \varepsilon + \lambda_2 e^{-\beta t} (\varepsilon^T \varepsilon)^{-\alpha} \dot{\varepsilon}
$$

(4)

where \( \lambda_1 = \text{diag}(\lambda_{1i}) \) and \( \lambda_2 = \text{diag}(\lambda_{2i}) \) for \( i = 1, \ldots, n \) are diagonal positive matrices, \( 0 < \alpha < 1 \) and \( \beta > 0 \). In (4), the term \( e^{-\beta t} \) will decrease to zero when \( t \to \infty \) and the proposed non-singular terminal sliding surface will become linear. By choosing a suitable \( \beta \), the proposed surface will combine between the classical non-singular terminal sliding surface and the classical linear one.

![Diagram of proposed non-singular terminal sliding surface in phase plane.](image)

Then, using equation (2), the time derivative of \( \sigma \) is as follows:

### 3.2 Second Order Sliding Mode with TDE

The sliding set of order \( r-th \) associated to manifold is defined in (Levant, 1993) by:

$$
\sigma = \sigma = \cdots = \sigma^{(r-1)} = 0
$$

(5)

Notice that the above equation represents an \( r- \)dimensional condition on the system dynamics (\( r \) denotes the relative degree of the system). For second order dynamic system which is the case of our system, \( r = 2 \). Then, the first time derivative of \( \sigma \) is:

$$
\dot{\sigma} = \dot{\varepsilon} + \lambda_1 \dot{\varepsilon} + \lambda_2 E
$$

$$
= \ddot{q} - \dot{q}_d + \lambda_1 \dot{q} + \lambda_2 E
$$

$$
= \bar{M}^{-1}[\tau - H(t)] - \dot{q}_d + \lambda_1 \dot{q} + \lambda_2 E
$$

(6)

where \( E \) is given by:

$$
E = -\beta e^{-\beta t} (\varepsilon^T \varepsilon)^{-\alpha} \dot{\varepsilon} + e^{-\beta t} (\varepsilon^T \varepsilon)^{-\alpha} \varepsilon
$$

$$
-2\alpha e^{-\beta t} (\varepsilon^T \varepsilon)^{-\alpha-1} (\varepsilon^T \varepsilon) \dot{\varepsilon}
$$

(7)

Now, lets define the new system formed by \( \eta_1 = \sigma \) and \( \eta_2 = \ddot{\varepsilon} \), then:

$$
\begin{align*}
\eta_1 &= \sigma \\
\eta_2 &= \bar{M}^{-1}[\tau - H(t)] - \dot{q}_d + \lambda_1 \dot{q} + \lambda_2 E
\end{align*}
$$

(8)

In Eq. (8), the time derivative \( \tau \) would be designed to act on the second order derivative of the sliding surface. Here, the time derivative, \( \tau \) would be designed as a discontinuous signal, but its integral \( \tau \), would be continuous by eliminating the high frequency chatter.

To determine a second order sliding mode, a new surface is defined for the system given in Eq. (8) as:

$$
S = \eta_2 + \lambda_3 \eta_1
$$

(9)

where \( \lambda_3 = \text{diag}(\lambda_{3i}) \) is a diagonal positive matrix and \( S \) satisfies:

$$
\dot{S} = -K \text{sign}(S)
$$

(10)

where \( K = \text{diag}(K_1, K_2, \ldots, K_n) \) denotes the positive diagonal switching gain matrix and \( \text{sign}(S) = \text{sign}(S_1), \text{sign}(S_2), \ldots, \text{sign}(S_n) \)^T \) with:

$$
\text{sign}(S_i) = \begin{cases} 
1, & \text{if } S_i > 0 \\
0, & \text{if } S_i = 0 \\
-1, & \text{if } S_i < 0
\end{cases}
$$

(11)

Resolving Eq. (10) by using Eq. (8), the time derivative \( \tau(t) \) is obtained as:

$$
\tau(t) = H(t) + \bar{M} \int_0^t u(t) dt
$$

(12)

Since \( H(t) \) has uncertain part, the control performance will be affected. Then, based on Assumption 3 given in Section 2, \( H(t) \) can be estimated using a TDE (Yousef-Toumi and Ito, 1990) as:

$$
\dot{H}(t) \approx H(t - L) \equiv \tau(t - L) - \bar{M} \ddot{q}(t - L)
$$

(15)

where \( L \) is the estimation time delay. Clearly the accuracy of \( \dot{H}(t) \) improves as \( L \) decreases. In practice, the smallest possible value of \( L \) is the sampling period. The time delayed \( \ddot{q}(t - L) \) signal can be obtained by one of the following approximation:

$$
\ddot{q}(t - L) \approx \frac{q(t - L) - 2q(t - 2L) + q(t - 3L)}{L^2}
$$

(16)

$$
\ddot{q}(t - L) \approx \frac{q(t - L) - q(t - 2L)}{L}
$$

(17)
Theorem 1. The proposed non-singular terminal second order sliding mode with time delay estimation for the uncertain robot system in Eq. (1) is given by:
\[
\tau(t) = \hat{H}(t) + \overline{M} \int_{0}^{t} u(t) dt
\]
\[
= \tau(t - L) - \overline{M} \left[ \dot{q}(t - L) - \int_{0}^{t} u(t) dt \right]
\]  
(18)
where \( u(t) \) is defined in Eq. (13) and the switching gains \( K_{i} \) of Eq. (10) for \( i = 1, \cdots, n \) satisfy:
\[
K_{i} > \delta_{i}
\]  
(19)
where \( \delta_{i} \) are positive constants that represents the bounds of the derivative of the TDE error. In addition, the proposed controller ensures the convergence of the sliding surfaces \( S_{i} \) to zero in a finite-time:
\[
t_{r}(0) = \frac{\left| S_{i}(0) \right|}{K_{i} - \delta_{i}} \quad \text{for } i = 1, \cdots, n
\]  
(20)
Proof. For the stability analysis of the overall system, we have to ensure that \( S \) converges to zero. To that end, the following Lyapunov function is selected:
\[
V = \frac{1}{2} S^{T} S
\]  
(21)
Then, calculating its time derivative and substituting the derivative of the control law \( \tau \) calculated from Eq. (18) gives:
\[
\dot{V} = S^{T} \frac{d}{dt} S
\]
\[
= S^{T} \left( \overline{M}^{-1} \left[ \dot{H}(t) - \dot{H}(t) - \dot{q}_{d}^{(3)} + \lambda_{1} \dot{E} + \lambda_{2} \dot{E} + \lambda_{3} \dot{E} \right] \right)
\]
\[
= S^{T} \left( \overline{M}^{-1} \left[ \dot{H}(t) - \dot{H}(t) - K_{i} \text{sign}(S_{i}) \right] \right)
\]
\[
= \sum_{i=1}^{n} \left( S_{i} \frac{1}{m_{i}} \dot{H}_{i}(t) - K_{i} |S_{i}| \right)
\]
\[
\leq \sum_{i=1}^{n} |S_{i}| \left( \frac{1}{m_{i}} |\dot{H}_{i}| - K_{i} \right)
\]  
(22)
Where \( \Delta H_{i} = \dot{H}_{i}(t) - \dot{H}(t) \) denotes the derivative of the TDE error. Otherwise, based on Assumption 3 \( \Delta H_{i} \) is bounded as follows:
\[
|\Delta H_{i}| < \dot{\delta}_{i}
\]  
(23)
Then, Eq. (22) becomes:
\[
\dot{V} \leq \sum_{i=1}^{n} |S_{i}| \left( \delta_{i} - K_{i} \right)
\]  
(24)
where \( \delta_{i} = \frac{1}{m_{i}} \dot{\delta}_{i} \). Hence, to ensure \( V \) negative-definite for Lyapunov stability, the following condition must be satisfied:
\[
K_{i} > \delta_{i}
\]  
(25)
In addition, to prove the finite-time convergence of the proposed controller, let us recall Eq. (24). Then, eliminating the sum, dividing by \( |S_{i}| \) and integrating both sides between 0 and \( t \) gives:
\[
\int_{0}^{t} |S_{i}| dt \leq \int_{0}^{t} (\delta_{i} - K_{i}) dt
\]
\[
|S_{i}(t) - |S_{i}(0)| \leq (\delta_{i} - K_{i}) t
\]  
(26)
Assuming that \( t_{r} \) is the time required to reach \( S_{i} \) such as \( |S_{i}(t_{r})| = 0 \), one has:
\[
t_{r} \leq \frac{|S_{i}(0)|}{(K_{i} - \delta_{i})}
\]  
(27)
This completes the proof.
Remark 1. In real-time, the measured signals are contaminated by noise. The noise effect might be amplified when \( \dot{q}_{d} - L \) is obtained using one of the approximations in Eq. (16) and Eq. (17). To solve this problem, a LowPass Filter (LPF) may be used before implementing the controller. However, the attenuation of noise without using a LPF is possible by choosing small values for \( M \) (Jin et al., 2011). If a digital LPF with the cutoff frequency \( \gamma \) is adopted, the control law can be modified as follows:
\[
\tau_{f} = \gamma L (1 + \gamma L)^{-1} \tau_{r} + (1 + \gamma L)^{-1} \tau_{f}(t - L)
\]  
(28)
where \( \tau_{r} \) denotes the calculated input before the filter and \( \tau_{f} \) denotes the filtered control input. Substituting \( \tau_{r} \) by its expression in (18), gives:
\[
\tau_{f} = \tau_{f}(t - L) + \gamma L (1 + \gamma L)^{-1} \overline{M} \int_{0}^{t} u dt - \dot{q}_{d} - L
\]  
(29)
Comparing Eq. (29) with the controller in Eq. (18), then:
\[
\overline{M}' = \gamma L (1 + \gamma L)^{-1} \overline{M}
\]  
(30)
Since \( \gamma L (1 + \gamma L)^{-1} < 1 \), then, for very small value of \( \overline{M} \), the same effect as using a digital LPF will be obtained.

4 SIMULATION

In this section, the proposed controller is used for trajectory tracking of an uncertain rigid manipulator system. The finite-time convergence is illustrated in this example. In addition, a comparison with another robust controller is presented to prove the effectiveness of the proposed non-singular terminal sliding mode control with time delay estimation.
4.1 Robot System

The considered robot here is the 3-DOF ANAT robot arm shown in Fig. 3. ANAT stands for Articulated Nimble Adaptable Trunk. The dynamic model is further specified in (Fallaha et al., 2011) by the well-known equation for rigid manipulators in Eq. (1). The system parameters are specified in (Kali et al., 2015). The initial values of the joint position and velocity are chosen as

\[ q_1(0) = 0.15 \text{rad}, \quad q_2(0) = -0.1 \text{rad}, \quad q_3(0) = 0 \text{rad} \]

and

\[ \dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = 0 \text{rad/s} \]

The robot model used for simulation verifies the properties and assumptions given in Section 2. The disturbances \( \tau_d \) are considered in this paper as:

\[
\tau_d = \begin{bmatrix}
2\sin(t) + 0.5\sin(200\pi t) \\
\cos(2t) + 0.5\sin(200\pi t) \\
\sin(t) + 0.5\sin(200\pi t)
\end{bmatrix}
\]  

(31)

4.2 Controller Setting

The proposed controller has five diagonal matrices with positive constant elements and two positive constant elements that make the controller setting simple. The steps are briefly described as follows:

- Select the matrices \( \lambda_1, \lambda_2 \) and the positive coefficients \( \alpha \) and \( \beta \) of the non-singular terminal sliding surface in Eq. (4).
- Select the matrix \( \lambda_3 \) of the second order sliding surface in Eq. (9).
- Select \( L \) as small as possible (equal to the sampling time interval \( T_s \)).
- Tuning the matrix \( \mathcal{M} \) such as \( \|I - M(q)^{-1}\mathcal{M}\| < 1 \). The elements of \( \mathcal{M} \) are chosen to be small positive values and increased gradually, while checking the control performance by trial and error.
- The switching gain matrix \( K \) in Eq. (10) is chosen such as the condition of stability in Eq. (25) is verified, \( K \) should be further tuned to achieve the optimal performance.

4.3 Results

The controller gains are chosen such the stability condition is met:

\[ \lambda_1 = \text{diag}(5, 5, 5), \quad \lambda_2 = \text{diag}(2, 2, 2), \quad \lambda_3 = \text{diag}(30, 30, 30), \quad \mathcal{M} = \text{diag}(0.15, 0.15, 0.15), \quad K = \text{diag}(2, 2, 2), \quad \beta = 0.4, \quad L = 0.01s \]

The results are shown in Fig. 4 through Fig. 6. The proposed controller ensures the finite-time convergence of the joint position to the desired position with high accuracy, due to a good estimation of uncertainties and disturbances as shown in Fig. 4 and confirmed by the small joint space tracking error in Fig. 5. Furthermore, it can be seen from Fig. 6 that the control torque inputs are chattering free.

To evaluate the performance of the proposed controller, it is compared to the classical sliding mode
The controller gains are chosen as:

\[ \lambda = \text{diag}(5, 5, 5), \]
\[ \bar{M} = \text{diag}(0.15, 0.15, 0.15), \]
\[ K = \text{diag}(2, 2, 2), L = 0.01s \]

The obtain results are given in the figures below.

Comparing the results for both controllers, the tracking performances are similar as depicted in Fig. 4 and Fig. 7 and confirmed in Fig. 5 and Fig. 8. However, it is clear that the chattering is still present in the second method as shown in Fig. 9, while it is completely removed in the proposed method due to the integration of the discontinuous signal of the \( \text{sign} \) function as can be seen in Fig. 6.

5 CONCLUSION

For an n-DOF uncertain robot manipulator, a new non-singular terminal second order sliding mode with time delay estimation is presented in order to achieve the control objective. A sufficient condition of stability is established using Lyapunov theory. The proposed controller allows uncertainties estimation, chattering reduction and finite-time convergence, while the sliding surface ensures faster convergence in comparison with the classical linear sliding surface and solves the problem of singularity of the classical terminal sliding surface. Simulation results and com-
parative study on a 3-DOF ANAT robot manipulator show the effectiveness of the proposed controller.

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