Optimal Combination Rebate Warranty Policy with Second-hand Products

Sriram Bhakthavachalam[1], Claver Diallo[1], Uday Venkatadri[1] and Abdelhakim Khatab[2]

[1]*Department of Industrial Engineering, Dalhousie University, 5269 Morris Street, Halifax, Canada*

[2]*Laboratory of Industrial Engineering, Production and Maintenance (LGIPM), Lorraine University, National School of Engineering, Metz, France*

Keywords: Second-hand Products, Warranty Policy, Consumer Perspective, Remaining Useful Life.

Abstract: With the increased awareness for sustainability, many engineered products are being recovered and reconditioned for secondary useful lives. These second-hand products can serve as replacement products to honour warranty pledges. This paper presents two mathematical models to determine the optimal combination rebate warranty policy when refurbished products are used for replacements from both the manufacturer and consumer point of views. Several numerical experiments are conducted to derive useful managerial knowledge.

1 INTRODUCTION

A warranty is a contractual agreement offered by the manufacturer at the point of sale of a product (Blischke, 1995), (Blischke, 1993). The use of warranties is universal and serves numerous purposes. It helps the buyers to rectify all the failures occurring within the warranty period at lower or no cost. Whereas, for manufacturers, it acts as a promotional tool to increase sales and revenue (Blischke, 1995). American manufacturers spend over 25 billion dollars to service warranty claims which is about 2% of their annual revenue from sales (Chukova and Shafiee, 2013; Shafiee and Chukova, 2013). In the 2009 General Motors annual report, the company had a total revenue of \$104.2 billion and the future warranty cost on sold cars estimated to be \$2.7 billion, about 2.6% of the revenue (Shafiee and Chukova, 2013). When buying a product, the consumer usually faces the difficult task of deciding between buying the warranty or not. And when the decision is made to get the warranty, choosing between different characteristics and warranty policies is another daunting task. When the warranty period is optional, the consumer has to decide if the warranty is worth the additional cost based a very limited knowledge of the product. This is becoming more and more important, since there is a growing trend among the manufacturers to offer extended term warranties. These involve additional costs, and the terms can vary considerably (Blischke, 1995; Blischke, 1993; Yun et al., 2008). Blischke & Murthy gave the example of a warranty or extended warranty that might cover both labor and parts initially and only cover parts later in the warranty period. The consumer has to decide, often at the time of purchase and based on very limited information, whether to opt for an extended warranty or not and to determine the best extended terms for his situation when there are multiple options (Blischke, 1995), (Blischke, 1993). The everyday consumer is not capable of conducting a mathematical analysis before making a choice because the consumer neither has the expertise for such an analysis nor the bargaining power to obtain relevant data from the manufacturer. However, consumer bureaus and regulatory agencies can carry out such analyses and inform the consuming public. Any model developed from the consumer's point of view in this chapter is then assumed to have been done for a consumer agency on behalf of all consumers and with data obtained by the agency from the manufacturers or from established and recognized independent reviewing bodies such as the Consumer Reports magazine.

There are many different types of warranty policies designed to cover the needs of manufacturers, dealers and consumers. A policy which is based on one factor (usually age) alone is said to be one-dimensional, on the other hand a two dimensional warranty is limited by two factors, usually age and a measure of usage of the product. One-dimensional policies are selected for products which are known to last for a fixed time period. This is common in the

Bhakthavatchalam S., Diallo C., Venkatadri U. and Khatab A.
Optimal Combination RebateWarranty Policy with Second-hand Products.
DOI: 10.5220/0006293504910498
In *Proceedings of the 6th International Conference on Operations Research and Enterprise Systems (ICORES 2017)*, pages 491-498
ISBN: 978-989-758-218-9

marketplace for products such as cell-phones, computers, and projectors. Two-dimensional warranties apply to products that display wear and tear, degradation with usage. Automobiles, aircraft and heavy-duty machinery are examples of products with 2-D warranty policies. It is common to see car advertisements stating coverage of 60 months, 120 000 kilometres which ever occurs first.

Some basic warranty types are the Free replacement (FRW), Pro-rata (PRW), and Rebate warranty.

1. **Free replacement warranty (FRW)**: The manufacturer agrees to repair/replace a failed item during the warranty period at no charge to the customer. Example: small household appliances, electronics.
2. **Pro-rata warranty (PRW)**: The customer covers a proportion of the repair cost prorated to the age of the item at failure. Example: Tires.
3. **Rebate warranty**: The seller agrees to refund some proportion of the sale price to the buyer, if the product fails during the warranty period. The refund amount may be a linear or non-linear function of the failure time. Example: Money back Guarantee for electronic components such as hard drives, computer screens, and storage devices.

A basic taxonomy of warranty policies is presented by (Blischke, 1993; Blischke, 1995). An integrated warranty-maintenance taxonomy based on three categories ,i.e. product type, warranty policy, and maintenance strategy, is proposed in (Shafiee and Chukova, 2013).

Hybrid (combination) warranties are designed to utilize the desirable characteristics of the pure warranties and downplay some of their drawbacks (Blischke, 1993; Blischke, 1995). The combination warranty gives the buyer full protection against full liability for later failures, where the buyer has received nearly the full amount of service that was guaranteed under the warranty. It has a significant promotional value to the seller while at the same time providing adequate control over costs for both buyer and seller. An example for hybrid warranty is seen in the FRW/PRW policy offered on Firestone tires. During the first 2 years of service, the tire is replaced free of charge. Beyond year 2, the replacement price is pro-rated based on years of service from the original purchase date. Some advantages of combination warranties are improved protection towards the product, customer satisfaction, higher ownership lifetime for the buyers and higher sales volume to increase profit to manufacturers.

Combination warranty is a good type of warranty for second-hand products (SHPs) as its offers a good protection to both manufacturers and consumers (Chari, 2015). Two main problems faced by the consumers acquiring SHPs are their uncertainty and durability (Shafiee and Chukova, 2013) due to the lack of past usage and maintenance history. In order to reduce the risk and impact of product malfunctioning, dealers offer generous warranty policies. A review of warranty models currently available in the literature for SHPs show that there are very few of them and all deal with the manufacturers perspective (Shafiee and Chukova, 2013; Chari et al., 2016b; Su and Wang, 2016; Diallo et al., 2016). The goal of this article is to address this shortcoming by proposing a warranty policy and develop mathematical models from both the manufacturer and consumer perspectives.

2 OPTIMAL COMBINATION WARRANTY MODELS USING SHPS

For most warranty policies, failed products are repaired or replaced with new components or products. In the context of remanufacturing, second-hand products may be available and can therefore be re-used as replacements when consumers return failed products (Yeh et al., 2005; Yeh et al., 2011; Chari et al., 2016a). In doing so, the manufacturers can lower their costs and consumers can extend their ownership of the products. However, due to the lower reliability of SHP, it is crucial to determine the optimal parameters of the warranty policy to be offered to avoid higher costs to the manufacturer and less than anticipated performance/ownership time for the consumer. In this article, we will develop two mathematical models for a combination rebate warranty policy using SHPs as replacement products.

2.1 Proposed Warranty Policy

Under the proposed warranty policy, a brand new product is sold with a total warranty coverage period of length w. Under this policy, the seller will replace a defective product with:

- A new product if the failure occurs before w_1 (Phase 0);
- A refurbished product of high quality if the failure occurs between w_1 and w_2 (Phase 1);
- A refurbished product of normal quality if the failure occurs between w_2 and w_3 (Phase 2).

It should be noted that $w = w_1 + w_2 + w_3$. The proposed warranty policy is depicted in Figure 1. New products have age $\tau_0 = 0$. Refurbished or second-hand products of high quality have age τ_1 that is greater than 0. Refurbished or second-hand products of normal quality have age τ_2 that is greater than τ_1. Therefore, we have: $0 < \tau_1 < \tau_2$.

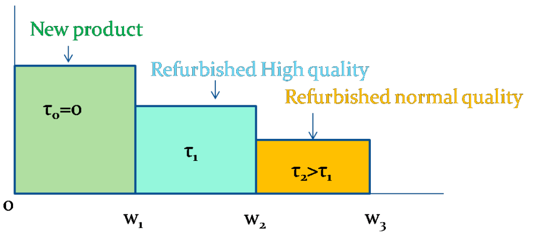

Figure 1: Proposed Warranty Policy.

The policy offered here is Non Renewing Free Replacement Warranty policy (NRFRW). The following notation is used.

2.1.1 Parameters

- C_i: Cost of replacement product in phase i
- C_0: Unit cost for a new product
- C_u: Warranty cost
- a: Price coefficient
- b_i: Warranty coefficient
- d_0: Market demand amplitude factor
- ε, η: Age coefficients for the acquisition cost of reconditioned components
- β: Slope parameter of the Weibull distribution
- θ: Scale parameter of Weibull distribution
- λ: Inverse of the Scale parameter ($\lambda = 1/\theta$)
- m: Number of warranty periods

2.1.2 Functions

- $f(t)$: lifetime prob. density function (pdf)
- $F(t)$: Cumulative distribution (cdf)
- π: Expected unit profit
- $\mathcal{P}(p, w_i, \tau_i)$: Total expected profit for the Seller
- $D(p, w_i)$: Total demand
- EOT: Expected ownership time
- $MTTF_0$: Expected lifetime of the original new product
- $MTTF_1$: Expected lifetime for high quality SHPs
- $MTTF_2$: Expected lifetime for low quality SHPs
- $EOCR_1$: EOT per cost ratio of the product when warranty is purchased
- $EOCR_2$: EOT per cost ratio of the product without warranty

2.1.3 Decision Variables

- p: Unit sale price of the new product
- w_i: Warranty periods
- τ_i: Age of the SHP products offered as replacements in phase i

In the following section, two mathematical models will be developed for the maximization of the manufacturer's expected profit and the maximization of the consumer's ownership time.

2.2 Model 1: Maximization of Manufacturer's Expected Profit

If the product fails within w_1, a full refund of C_0 is given to the customer to buy a new product. When it fails between w_1 and w_2 a refund of C_1 is returned to the customer that is sufficient to buy a high reliability SHP. When the product fails between w_2 and w_3, a lump sum C_2 is given back to the consumer which is sufficient to buy a normal quality SHP. Warranty is not extended when the system fails.

$C(\tau_i)$, the unit cost of a replacement product with age τ_i, is given by Equation (1) where C_0 is the base price and ε, η are positive parameters (Chari, 2015). Parameter ε represents the discount rate offered on used products, and parameter η models the increase in cost due to aging.

$$C(\tau_i) = C_0 \times (1 + \tau_i)^{(-\varepsilon)} + \tau_i^{\eta} \tag{1}$$

A new product will therefore cost

$$C(\tau_0 = 0) = C(0) = C_0 \tag{2}$$

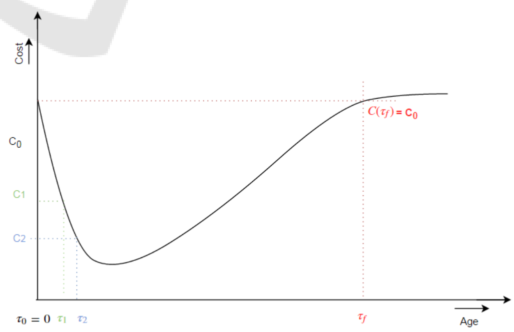

Figure 2: Cost as a function of age.

The profile of $C(\tau_i)$ is depicted in Figure 2. The cost of replacement products initially decrease with age (refurbished cost less) but reaches a minimum then increases with age to account for technical and practical difficulties encountered when trying to disassemble and recondition very old products (availability of parts, obsolescence, corrosion, etc.). Beyond this point $C(\tau_f) = C_0$, customers should buy a new product rather than a second-hand product because the cost of a new product is less than SHP.

$$C(\tau_1) = C_1 = C_0 \times (1+\tau_1)^{(-\varepsilon)} + \tau_1^{\eta} \quad (3)$$

$$C(\tau_2) = C_2 = C_0 \times (1+\tau_2)^{(-\varepsilon)} + \tau_2^{\eta} \quad (4)$$

The probability that a product will fail between w_{i-1} and w_i for $i = 1, \ldots, m$ is given by:

$$[F(w_i) - F(w_{i-1})] \quad (5)$$

where $w_0 = 0$.

The total expected warranty cost (C_u) is given by the weighted average of the replacement costs in each phase i given in Equation (7) shown below.

$$C_u = \sum_{i=1}^{m} C(\tau_i) \times [\text{Prob. failure in phase } i] \quad (6)$$

$$C_u = C_0 \left[\sum_{i=1}^{m} \left[(1+\tau_{i-1})^{-\varepsilon} + \frac{\tau_{i-1}^{\eta}}{C_0} \right] [F(w_i) - F(w_{i-1})] \right] \quad (7)$$

Failure Distribution:

The Weibull distribution is used as the product failure distribution. The lifetime cumulative distribution function of the product is then given by

$$F(x) = 1 - \exp^{-(\frac{x}{\theta})^{\beta}}, \qquad 0 \le x \quad (8)$$

Demand Function:

The market demand function $D(p, w_i)$ for the product is modelled to take into account consumers' preferences for lower prices and longer warranty coverage. $D(p, w_i)$ is modelled as a displaced log-linear function of w_i and p as in (Glickman and Berger, 1976; Chari et al., 2016b).

$$D(p, w_i) = d_0 p^{-a} \Pi_{i-1}^{m} (d_1 + w_i)^{b_i} \quad (9)$$

Parameter a is the rate of decrease of the sales volume with the increasing price of the product. Parameters b_i are the rate of increase of the sales volume with the increasing of the warranty lengths w_i. The factor d_0 is the demand amplitude and d_1 is the warranty displacement constant.

Total Expected Profit:

The total expected profit (TEP) $\mathcal{P}$ is the product of the expected unit profit with the demand as in Equation (10). The expected unit profit is obtained in Equation (11) by subtracting the cost of the original product C_0 and the expected warranty cost C_u from the sale price p of each unit sold.

$$\mathcal{P} = \pi \cdot D(p, w_i) \quad (10)$$

$$\mathcal{P} = (p - C_0 - C_u) \cdot D(p, w_i) \quad (11)$$

2.2.1 Numerical Results

For illustration purposes and without loss of generality, an example with only two decision variables is considered by setting τ_2 as a proportion of τ_1 using: $\tau_2 = k \cdot \tau_1$. For the arbitrarily chosen parameter values given below, we solve for the solution (p, τ_1) which maximizes the manufacturers' total expected profit: $w_1 = 0.5; w_2 = 1; w_3 = 2 : m = 3, \theta = 1.5, \beta = 1.5; C_0 = 15; d_0 = 100,000; a = 2.6; b_1 = 1.9, b_2 = 1.5, b_3 = 1.1; \varepsilon = 3.3; \eta = 0.7; \lambda = 1/\theta; k = 1.5$. Figure 3 shows a 3D-plot of the total expected profit $\mathcal{P}$ as a function of purchase price p and age τ_1. There is a clear optimal solution at $p^* = \$30.68$, $\tau_1^* = 1.43$ and $\mathcal{P}^* = \$3,287$.

Several numerical experiments have been conducted to analyze the behavior of the model when key parameters change. The first experiment consisted in varying the values of k, a parameter that dictates how old the replacement products are in phase 2 in comparison with the replacement products used in phase 1 according to the formula: $\tau_2 = k \cdot \tau_1$. The results obtained are plotted in Figures 4 to 6.

In Figure 4, $\mathcal{P}$ increases until the value of k reaches 1 and after that point, $\mathcal{P}$ decreases. For $k = 1$, the replacement products in phase 1 and 2 are the same. This represents the best case scenario as profit is maximum and price is the lowest. For $k < 1$, phase 2 replacement products are younger than phase 1 products, which is bad because components failing in phase 1 are replaced with older parts and the larger proportion of failures occurring in phase 2 are covered with newer products which are more expensive. This explains why the slope when $k < 1$ is steeper than the slope when $k > 1$. Figure 5 shows that price p behaves in an exact opposition to the behavior of $\mathcal{P}$. For values near and around $k = 1$, it is the cheapest to honour the warranty, so the manufacturer can afford to reduce the price of the product and therefore increase demand, which in return boosts profit.

Figure 6 depicts the relationship between the optimal value τ_1^* and τ_2^*. For smaller values of τ_1^* the model uses larger values of τ_2^* to keep warranty costs under control. When the values of τ_1^* start to increase $(1 < \tau_1^* < 3)$, the model restricts the values of τ_2^* between values of 3 and 1.5 to keep warranty costs low by decreasing the probability of failure in phase 2. For values of $\tau_1^* > 3$ the values of τ_2^* tend to stabilize around 1 and 1.5 for the same reasons as before.

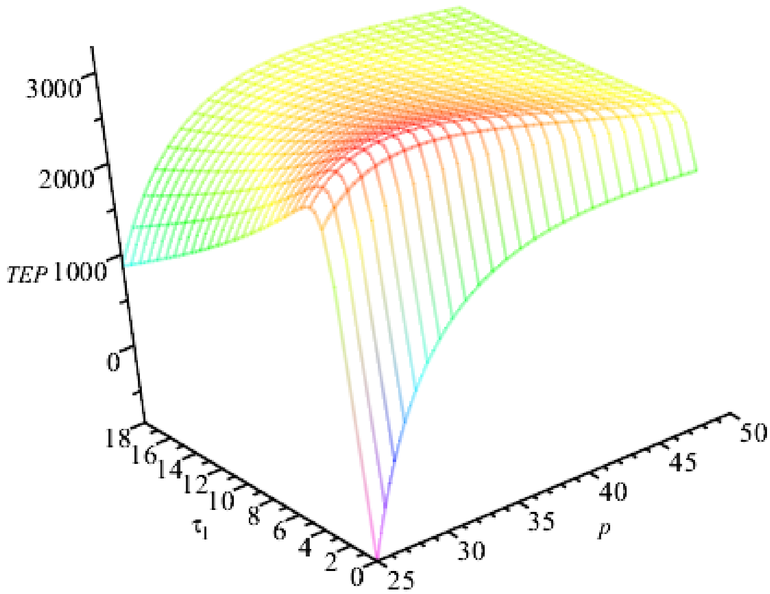

Figure 3: Total expected profit as a function of purchase price p and age τ_1.

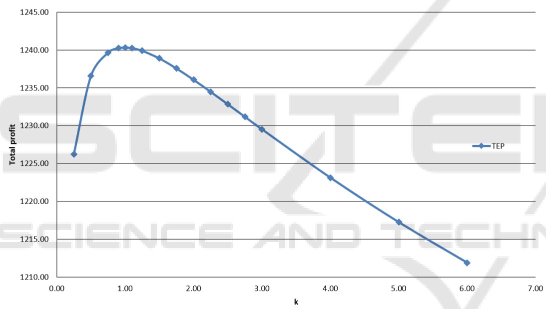

Figure 4: Seller expected profit as a function of k.

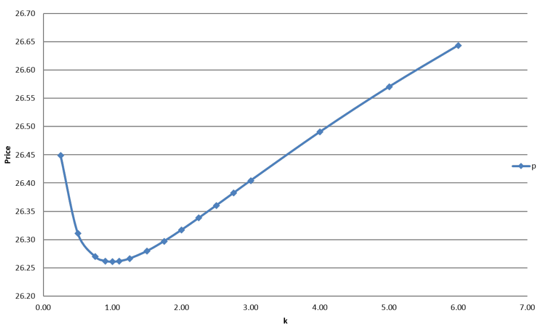

Figure 5: Optimal price as a function of k.

Another set of numerical experiments was conducted by varying β, the shape parameter of the Weibull distribution. The results are plotted on Figure 7. In general, the figure shows an increasing trend and a plateau after $\beta = 4$. Increasing the shape parameter β

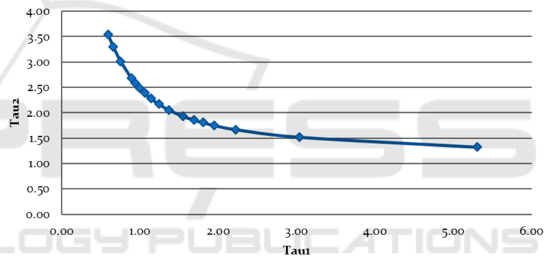

Figure 6: τ_1 vs τ_2 for varying values of k.

newpage
increases reliability of the product so that the warranty cost reduces. There is very little return on investment to improve reliability of the products beyond $\beta = 4$. Figure 8 shows that with improving reliability (increasing β), the warranty costs reduce and therefore the model can afford to reduce the unit price which increases profits. For the same reliability reasons, when β increases, the model can afford to use newer parts which causes τ_1^* to decrease as depicted in Figure 9.

2.3 Model 2: Maximization of Customers' Expected Ownership Time

In the previous model, the focus was on the manufacturers' interests. In this subsection, a model developed from the consumer's perspective will be presented. The warranty policy introduced in sub-section

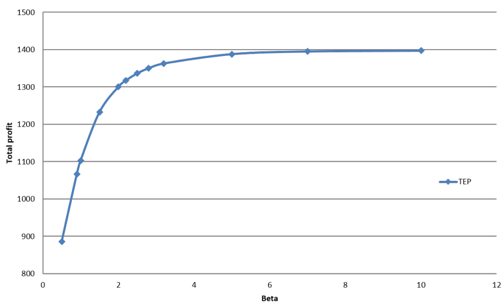

Figure 7: Seller's expected profit as β varies.

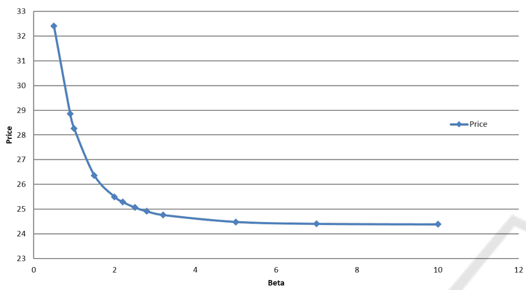

Figure 8: Selling price for varying β.

2.2 is still under consideration here.
At time of purchase, the consumer has two choices:

- Option 1: Buy the original product without warranty at a fraction ρ $(0 \leq \rho \leq 1)$ of the price p set by the manufacturer and determined using model 1 presented above; or
- Option 2: Buy the original product with warranty at price p.

The goal of model 2 is to formulate the Expected Ownership Time per Cost Ratio ($EOCR_i$) for both options ($i = 1,2$) and compare their behaviour through the analysis of their difference Δ:

$$\Delta = EOCR_2 - EOCR_1. \tag{12}$$

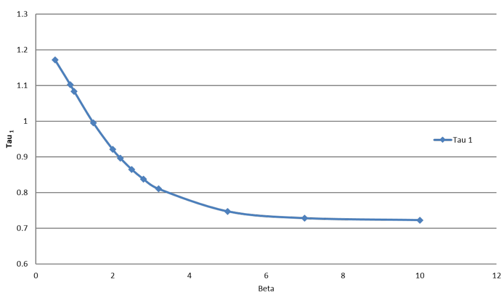

Figure 9: Profile of the optimal τ_1 as β varies.

Option 1: without warranty

$$EOCR_1 = \frac{EOT_1}{\rho \cdot p}$$

$$EOCR_1 = \frac{MTTF_0}{\rho \cdot p}$$

where $MTTF_0$ is the expected lifetime of the original product

$$EOT_1 = MTTF_0 = \theta \cdot \Gamma \left[1 + \left(\frac{1}{\beta}\right)\right].$$

$\Gamma(.)$ is the gamma function. Therefore,

$$EOCR_1 = \frac{\theta \cdot \Gamma \left[1 + \left(\frac{1}{\beta}\right)\right]}{\rho \cdot p}. \tag{13}$$

Option 2: with warranty

$$EOCR_2 = \frac{EOT_2}{p} \tag{14}$$

A consumer enjoys his original new product from purchase time up to the instant of the first failure which has expected duration $MTTF_0$. At failure, the consumer gets a replacement product that will have an expected remaining lifetime of length $RMTTF_i$ if the failure occurred in phase i. The original product fails in phase i with probability $[F(w_i) - F(w_{i-1})]$. Therefore, the expected ownership time for option 2 is given by

$$EOT_2 = MTTF_0 + \sum_{i=1}^{m} RMTTF_{i-1}[F(w_1) - F(w_{i-1})] \tag{15}$$

where

$$RMTTF_i = \frac{1}{1 - F(\tau_i)} \int_{\tau_i}^{\infty} [1 - F(x)].dx \quad \forall i \in 1,2. \tag{16}$$

By definition, $RMTTF_0 = MTTF_0$. Combining Equations (14) and (15), gives the expression for $EOCR_2$:

$$EOCR_2 = \frac{MTTF_0 + \sum_{i=1}^{m} RMTTF_{i-1}[F(w_1) - F(w_{i-1})]}{p} \tag{17}$$

Finally, Equations (12) becomes:

$$\Delta = \frac{1}{p}\left[MTTF_0 + \sum_{i=1}^{m} RMTTF_{i-1}\left[F(w_1) - F(w_{i-1})\right] - \frac{\theta}{\rho} \cdot \Gamma \left[1 + \left(\frac{1}{\beta}\right)\right]\right] \tag{18}$$

The obtained mathematical model is solved for various scenarios in order to derive decision making policies for the consumer organizations.

2.3.1 Numerical Results

Experiment #1: Change in w_1 and varying β

The first set of experiments is designed to analyze the recommendations made by the model for 4 warranty policies when β changes. The 4 warranty policies differ in their value of w_1. The values of w_2 and w_3 are the same for all policies. Table 1 presents the results obtained.

Table 1: Values of Δ for various w_1 and varying β.

β	$w_1 = .5$ $w_2 = 2$ $w_3 = 3$	$w_1 = .75$ $w_2 = 2$ $w_3 = 3$	$w_1 = 1$ $w_2 = 2$ $w_3 = 3$	$w_1 = 2$ $w_2 = 2$ $w_3 = 3$
0.5	0.06	0.05	0.04	0.03
0.9	0.02	0.02	0.02	0.02
1.0	0.02	0.02	0.02	0.02
1.5	0.01	0.01	0.01	0.02
2.0	0.01	0.01	0.01	0.01
3.0	0.00	0.01	0.01	0.02
4.0	0.00	0.00	0.01	0.02
5.0	0.00	0.00	0.01	0.02
6.0	-0.01	0.00	0.01	0.02

The results are also plotted on Figure 10 from where two clear zones can be defined. The zone delimited by the red dotted outline depicts the area where products can be bought without warranty. Products falling in the zone above the red zone need to be purchased along with one of the 4 warranty policies offered. The following other observations can be made:

- The general profile of each plot of Δ as a function of β shows a fast decrease for low values of β and a stabilization for higher values. Δ is higher for $\beta << 1$ because early failures make the purchase of warranty more valuable. Δ stabilizes when increasing β because of the resulting increase in reliability which decreases the likelihood of failure and therefore the purchase of warranty does not add significant value to the consumer.
- Different warranty policies have different slopes of the same profile.
- A clear crossover point can be noticed on Figure 10. Policies with shorter Phase 0 (shorter w_1) that are preferred before the crossover point perform poorly after the crossover point when the products have higher reliability. Conversely, policies with longer Phase 0 (longer w_1) perform better after the crossover point. In other words, a policy that is good for newly designed products do worse with seasoned or proven products with good reliability.
- Warranty policies with longer w_1 coverage are less sensitive to increase in β values. It can be seen on Figure 10 that the 4th policy has the smallest amplitude over the complete range of β.

Experiment #2: Change in ρ and varying β

Here, numerical results are generated for two values of ρ (0.95 and 0.85) to analyze the impact of the selling price over the decision to purchase the warranty or not. The results obtained are in Table 2.

Table 2: Values of Δ for varying β.

$w_1 = .75$	$w_2 = 2$	$w_3 = 3$
β	ρ=0.95	ρ=0.85
0.5	0.06	0.05
0.9	0.03	0.02
1.0	0.02	0.02
1.5	0.02	0.01
3.0	0.01	0.01
4.0	0.01	0.00
6.0	0.00	0.00

As in the previous experiment, Δ shows a decreasing trend with increasing β. The higher the price without warranty (or the lower the warranty cost over premium ratio) the higher the return or incentive to buy the warranty.

3 CONCLUSIONS

This paper presented two mathematical models to determine the optimal combination rebate warranty policy when refurbished products are used for replacements from both the manufacturer and consumer point of views. One model was developed from the manufacturers' point of view to maximize the total expected profit and the second model dealt with the maximization of the consumer's expected ownership time. Numerical experiments showed that appropriate optimal decisions can be reached in the reuse of second-hand products in honouring warranty. Both the manufacturer and consumer groups can use these models to improve profitability levels and increase ownership durations.

The authors are currently investigating a joint analysis that considers both the seller and buyers' perspectives by formulating a multi-objective model to integrate key factors such as brand loyalty and incentives. Case studies from an appliance remanufacturer will be conducted to validate the theoretical results obtained. A

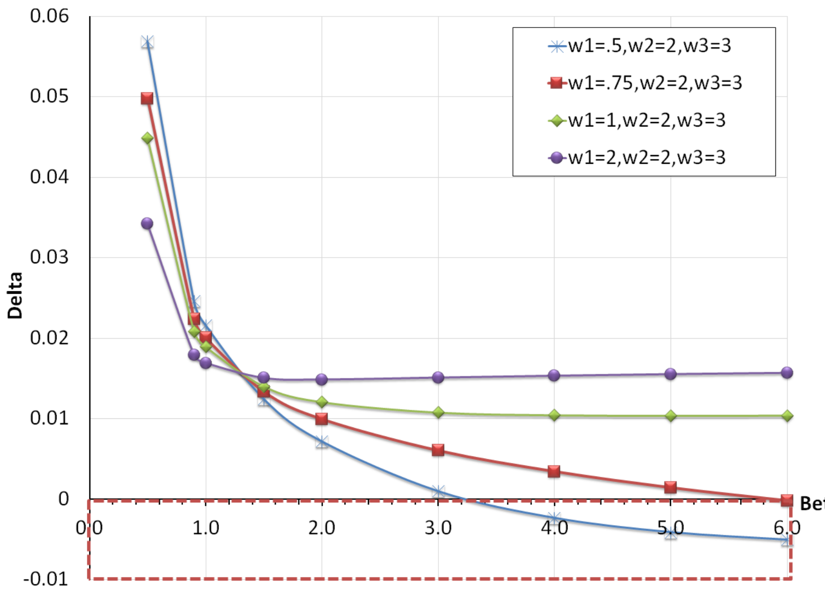

Figure 10: Profile of Δ for varying β.

variability analysis on a reduced set of selected solutions (with high expected values) will also be performed to test the robustness of the solutions. Future extensions of this study can also cover new warranty models suited for remanufactured products such as pro-rata and hybrid pro-rata policies, and integration of reconditioned products of different quality levels.

REFERENCES

Blischke, W. (1993). *Warranty cost analysis*. CRC Press.

Blischke, W. (1995). *Product warranty handbook*. CRC Press.

Chari, N. (2015). *Thematic Development of Recovery, Remanufacturing, and Support Models for Sustainable Supply Chains*. PhD thesis, Dalhousie University, Halifax.

Chari, N., Diallo, C., Venkatadri, U., and Aït-Kadi, D. (2016a). Production planning in the presence of remanufactured spare components: an application in the airline industry. *The International Journal of Advanced Manufacturing Technology*, pages 1–12.

Chari, N., Diallo, C., Venkatadri, U., and Khatab, A. (2016b). Modeling and analysis of a warranty policy using new and reconditioned parts. *Applied Stochastic Models in Business and Industry*, 32:539–553.

Chukova, S. and Shafiee, M. (2013). One-dimensional warranty cost analysis for second-hand items: an overview. *International Journal of Quality & Reliability Management*, 30(3):239–255.

Diallo, C., Venkatadri, U., Khatab, A., and Bhakthavatchalam, S. (2016). State of the art review of quality, reliability and maintenance issues in closed-loop supply chains with remanufacturing. *In Press, International Journal of Production Research*, 0(0):1–20.

Glickman, T. S. and Berger, P. D. (1976). Optimal price and protection period decisions for a product under warranty. *Management Science*, 22(12):1381–1390.

Shafiee, M. and Chukova, S. (2013). Maintenance models in warranty: A literature review. *European Journal of Operations Research*, 229(3):561–572.

Su, C. and Wang, X. (2016). Optimal upgrade policy for used products sold with two-dimensional warranty. *Quality and Reliability Engineering International*.

Yeh, R. H., Chen, G.-C., and Chen, M.-Y. (2005). Optimal age-replacement policy for nonrepairable products under renewing free-replacement warranty. *IEEE Transactions on Reliability*, 54(1):92–97.

Yeh, R. H., Lo, H.-C., and Yu, R.-Y. (2011). A study of maintenance policies for second-hand products. *Computers & Industrial Engineering*, 60(3):438–444.

Yun, W. Y., Murthy, D. N. P., and Jack, N. (2008). Warranty servicing with imperfect repair. *International Journal of Production Economics*, 111(1):159–169.