Transposition Based Blendshape Direct Manipulation

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Abstract: The blendshape approach is a predominant technique for creating high quality facial animation. Facial poses are generated by altering the corresponding weight parameters manually for each key-frame by using traditional slider interfaces. However, authoring a production quality facial animation with this process requires time-consuming, labor intensive and iterative work the artists. Direct manipulation interfaces address this problem with a “pin-and-drag” operation inspired by the inverse kinematics approaches. The mathematical frameworks of the direct manipulation techniques are mostly based on pseudo-inverse of the blendshape matrices which include all target shape’s vertex positions. However, the pseudo-inverse approaches often give unexpected results during the facial pose editing process because of its unstable behavior. To this end, we propose the transposition approach to enhance the direct manipulation by reducing unexpected movements during weight editing. Our approach extracts the deformation directions from the blendshape matrix, and directly maps the sparse constrained point movements to the extracted directions. Our experiments show that, instead of pseudo-inverse based formulations, transposition based framework gives more smooth and reliable facial poses during the weight editing process. The proposed approach improves the fidelity of the generated facial expressions by keeping the hazardous movements in a minimum level. It is robust, efficient, easy to implement and operate on any blendshape model.

1 INTRODUCTION

Computer generated characters have recently become ubiquitous in movies, cartoons, advertisements and computer games. With the advances of technology and computer graphics techniques, it is currently possible to create lifelike characters. During the process of creating character animation, facial animation plays a key role to carry emotions and personality to the spectators. Especially, the increasing demand of the entertainment industry for high quality results drives the researchers to investigate new techniques to automate the animation workflow for realistic 3D characters. Despite the extensive use of 3D facial animation, the evolution in animation packages and workflows have been rather slow, because the adaptation of advanced methods to the current animation pipelines requires a long time period. As a result, creating high-quality facial animation is still a labor intensive and time-consuming process.

Blendshape is a widely adopted technique for high-quality facial rigging (Lewis et al., 2014). The blendshape model forms a specific facial expression as a sum of linear combination of target faces. These target faces are obtained either by a skilled artist manually sculpts from a neutral facial mesh, or by scanning a human face with various expressions. Most commercial animation and modeling software packages support blendshape models with their weight editing interfaces. These interfaces, nevertheless, allow the animators to perform the pose editing process only on external modules, where each pose is represented by a slider. However, creating facial animation using blendshapes requires a significant effort in production environment for realistic models that may consist of 100 or more target shapes.

Direct manipulation methods address the difficulty of the pose editing process by offering a mathematical framework with a practical pin-and-drag operation (Lewis and Anjyo, 2010). The user simply selects arbitrary points on the surface of the 3D facial model, and freely drags them until the desired facial pose is obtained for each animation frame. Beneath the direct manipulation interface, the mathematical framework solves an inverse problem of weight determination for the arbitrary selected point movements. However, solving the inverse problem is generally a challenging task, because of the problem’s
excessive under-constrained behavior. Besides, inverse kinematics algorithms perform similar to direct manipulation, nevertheless constraints and number of unknowns are notably less than the direct manipulation approaches. For example, human arm movement can be expressed roughly 7 degrees of freedom for the joint angles (Tolani and Badler, 1996), but in a production quality blendshape model, the number of unknown for weights can exceed 100.

A discrete function has to be defined to satisfy all constraints to solve the inverse problem for direct manipulation. According to blendshape direct manipulation (Lewis and Anjyo, 2010), moved points determine the resultant face as a simple linear combination of predefined target shapes,

\[ m \approx Bw \quad (1) \]

where \( m \) is the vector of resulting vertex positions that are already moved, \( w \) is the corresponding weight vector and \( B \) is the blendshape matrix which includes vertex coordinates of all target shapes at its columns. The solution of Equation (1) shows that there is an obvious pseudo-inverse relationship between the blendshape weights and manipulated points. The general pseudo-inverse approach gives the best possible approximated solution to \( w \) by minimizing the moved point to its correspondent target shapes (\( \min_w \|Bw - m\|^2 \)). However, pseudo-inverse based mathematical frameworks are known with their unstable and unintuitive results. Altering an arbitrary side of the face model causes unexpected little changes in the remaining parts of the model. The reason for this instability is the fact that pseudo-inverse tends to assign many non-zero values to its columns which cause unintended moved point projections for the irrelevant weights.

To prevent these unexpected movements during weight editing, we introduce a new direct manipulation algorithm. Our approach is inspired from the Jacobian Transpose method (Welman, 1993) and provides the direct projection of the moved points onto \( B \) which avoids the instability of the pseudo-inverse approach. During the drag operation, employing the transpose of \( B \) instead of its pseudo-inverse allows the user to manipulate desired part of the face without any little alterations in the remaining parts of the 3D model (see Figure 1). In terms of computing the weight updates directly using the transpose of \( B \), we analyzed the deformation (vertex displacements) directions to prove that our transposition based direct manipulation is a modified version of the classic pseudo-inverse approach. By considering each column of \( B \) is a deformation vector for the base neutral pose, we extract the deformation directions from the columns of \( B \) by using Gram-Schmidt process and store these columns as a new orthonormal matrix, which shares the same deformation directions as \( B \). After applying the pseudo-inverse algorithm to the new matrix, the resultant mathematical framework has appeared similar to the proposed transposition based framework with an additional step-size matrix.

In our experiments, we compare the proposed approach with two different pseudo-inverse based algorithms for blendshape weight updates. The first version is the classic pseudo-inverse method, and the other is the commonly used pseudo-inverse with Tikhonov regularization (Lewis and Anjyo, 2010). Against both methods, our approach produces more intuitive and reliable posing results. Further, we discuss the advantages and disadvantages of both approaches and present a hybrid approximation by combining only the powerful sides of pseudo-inverse based approach and our transposition based approach. Finally, the resulting approach is simple to implement, efficient (based on solving only linear systems), and can be adapted easily to the existing blendshape deformation frameworks.
2 RELATED WORK

Blendshape is a commonly used rigging technique to create realistic facial animation for over many years, (Orvalho et al., 2012). Performance capture or keyframe animation techniques have been employed for obtaining blendshape animation. After its practical usage in industry, (Pighin et al., 1998) explored blendshapes as a research topic. They offered a set of techniques which allows to create the textured target shapes rapidly from pictures of a human face with a painterly interface to blend the parts of the morph targets. After this prior research, blendshape was integrated nicely to other fields of facial animation such as modeling (Pighin et al., 1998), facial performance capture (Joshi et al., 2003), facial animation retargeting (Deng et al., 2006), and facial rigging (Li et al., 2010).

Several techniques exist for character animation and facial deformation based on Inverse Kinematics approach. Some significant techniques include pose-space deformation (Lewis et al., 2000), shape by example (Sloan et al., 2001), style-based inverse kinematics (Grochow et al., 2004), mesh-based inverse kinematics (Sumner et al., 2005), differential inverse kinematics (Lewis and Dragošavac, 2010), and partial differential equations based animation (González Castro et al., 2010). Besides, robotics research often employs inverse kinematics approaches such as (Nakamura and Hanafusa, 1986), (Nakamura et al., 1987), (Chan and Lawrence, 1988).

Direct manipulation methods allow to create deformations by directly selecting and moving points on the surface of the 3D models. This type of approach has been common in inverse kinematics methods for character animation, and recently got popular for facial animation. Blendshape direct manipulation methods solve an inverse problem to find the weight values that are most appropriate for the desired movement. (Joshi et al., 2003) proposed an automatic segmentation for localized blendshapes with a direct manipulation technique. (Zhang et al., 2004) offered a method that obtains the target shapes from a high quality stereoscopic data on a template mesh, and provides direct editing by using adaptive local radial basis blends.

For the first time, (Lewis and Anjyo, 2010) solved the problem of direct manipulation for blendshape models. They developed a novel solution by formulating direct manipulation as a soft constrained inverse problem of finding the underlying weights for the selected point movements. The solution includes Tikhonov regularization parameter (Tikhonov and Arsenin, 1977) for the optimum results during interactive editing. (Seo et al., 2011) improved the regularization term of the inverse problem, and offered a new matrix compression scheme for complex models based on hierarchically semi-separable representations with an interactive GPU-based animation. (Tena et al., 2011) proposed to control more local modification by dividing the PCA based facial models into independently trained regions which share the boundaries with each other. This method allows the user to directly interact with the model using the pin-and-drag operation. (Anjyo et al., 2012) presented an algorithm, which provides rapid and better edits that learned from the previous animation.

Recently, (Neumann et al., 2013) introduced sparse localized deformation method, which decomposes the global deformations to localized components from the performance capture data using a variant of sparse PCA. (Holden et al., 2015) developed a real-time solution to the inverse problem of rig function, which enables mapping of the animation data from character rig to the skeleton of the character. That method can be applicable to whole character animation as well as bone based facial animation. (Cetinaslan et al., 2015) proposed an approach to locate the manipulators onto the facial model intuitively by using a sketch-based interface, and apply the drag operation after the sketching procedure. (Yu and Liu, 2014) proposed a method to reproduce the facial poses based on blendshape regression, which uses an optimization procedure to improve the quality of the facial expressions. We refer to (Lewis et al., 2014) as a comprehensive report about practical and theoretical aspects of blendshape facial models.

In the proposed approach, we have taken advantage of the listed prior work. However, pseudo-inverse solutions dominated the final mathematical frameworks of the prior works. In contrast to the previous research, we provide a transposition based solution to direct manipulation for more robust and intuitive results.

3 METHOD

In this section, we first explain the algebraic perspective of blendshape models and their pseudo-inverse relation. After, we describe the details of the proposed transposition based direct manipulation.

3.1 Blendshape Direct Manipulation and Pseudo-inverses

Blendshape model can be defined as a linear vector sum of predefined target shapes as illustrated in Equa-
\[ f = \sum_k b_k w_k \]  
(2)

where \( f \) is the produced final face in a vector form which includes all vector positions of the mesh in the order of \( xyzxyz... \), \( b_k \) are the blendshape targets in a vector form, and \( w_k \) are the corresponding weights. Equation 2 can also be denoted as \( f = Bw \). However, one particular target shape, which is the neutral face, can be considered different than the other shapes. Because, all other targets are the offsets from the neutral target shape \((f_0)\). Therefore, it is added to Equation 2, \((f = Bw + f_0)\). This new formulation is called delta form of blendshape model, and provides many zero or almost zero entries to each target vector. Current commercial modeling and animation software packages most often use the delta form for blendshape applications.

According to direct manipulation scheme, the manipulators (or pins) are located on the surface of the model by selecting some arbitrary vertices. After moving these pins on the surface of the face, mathematically the resultant face can be explained as \( \bar{f} = m + f_0 \), where \( m \) denotes for the moved pins. The delta form and the new face form with moved pins direct us to Equation 1.

By using the aforementioned mathematical model, the weight updates can be simply approximated as in Equation 3:

\[ w = B^+ m \]  
(3)

where \( B^+ \) denotes for the pseudo inverse of \( B \), \((B^+ \) is also defined as \((B^T B)^{-1}B^T \). However, in some cases the inverse operation in Equation 3 may fail because of the singularity of \( B^T B \). In these cases, blendshape matrix may not provide a full rank.

(Lewis and Anjyo, 2010) addressed this problem with the Tikhonov regularization term to update the weights with the most optimal fit to the point movements. Another advantage of applying regularization term is keeping the equilibrium of the system by fitting the new weight values according to the desired pose during point movements. Equation 4 shows the weight update from (Lewis and Anjyo, 2010) after applying regularization to the system defined in Equation 3

\[ w = (B^T B + \alpha I)^{-1} (B^T m + \alpha w_0) \]  
(4)

where \( \alpha \) is the regularization term which is defined by user, and \( w_0 \) are the previous weights. \( \alpha \) term is added to the diagonals of \( B^T B \) matrix, and forces the system to obtain the most close weight values with the corresponding pin drags. In general, \( \alpha \) is chosen as a small value, such as 0.001 or 0.0001, therefore it can not dominate \( B^T B \) matrix with a significant change. After setting the \( \alpha \) term to such a small number, the term \( \alpha w_0 \) of Equation 4 has an almost zero effect to determine the final weights. If \( \alpha \) is selected as zero, Equation 4 transforms to the classic pseudo-inverse form, which is shown in Equation 3. While dragging the constraint points on the surface of the model, Equation 3 or 4 constantly updates the weight values for each coordinate unit.

### 3.2 Transposition Approach

Direct manipulation formulation is based on pseudo-inverse of \( B \) with or without regulation term. However, pseudo-inverse approaches have a well-known instability problem, by which unexpected and unintuitive movements can be observed during the pin-and-drag operation. For example, while the user drags a pin in the mouth region of the face, eye brows may move slightly. The problem can be understood intuitively as follows: Denote the rows of Equation 2 corresponding to the moved pin as \( m = B \vec{w} \). Expanding \( B \) with an SVD, we have \( m = USV^T \vec{w} \). We see that the largest entries of \( S \) are those that (when rotated by \( U \)) are most effective for reaching the moved pin \( m \). But in forming the corresponding pseudo-inverse (Equation 3), \( w = VS^+ U^T \vec{m} \), the singular values are inverted, so the directions that are least effective in reaching \( m \) are scaled by the largest values. In practice this means that distant blendshape targets that have little effect on a particular pinned point will tend to move more than is desirable.

Inspired from the Jacobian transpose approach of differential inverse kinematics (Lewis and Dragosavac, 2010), we reformulate the blendshape direct manipulation in a more stable and efficient perspective. The inverse kinematics problem is explained as \( \min_q \| p - f(q) \|^2 \) which was solved by (Zhao and Badler, 1994) with a general nonlinear programming. According to their approach, nonlinear \( f() \) turns into a linear function about the current position with an update equation:

\[ \dot{p} = Jq \]  
(5)

where \( \dot{p} \) is the vector from the point on the inverse kinematics handle to the target point, \( J \) is the Jacobian of \( f() \). Equation 5 has to be solved for the parameter update \( \dot{q} \) with the most optimal approximation. The resultant system is nothing but \( \dot{q} = J^+ \dot{p} \). The problem in Equations 1 and 5 nicely fits to each other. However, Welman (Welman, 1993) offers the Jacobian Transpose algorithm for differential inverse kinematics to avoid the unstable behavior of pseudo-inverse approach. According to this algorithm, the parameter...
update is defined as \( \dot{q} = J^T \dot{p} \) instead of \( \dot{q} = J^+ \dot{p} \). In our transposition approach, we adapt Jacobian Transpose algorithm to the blendshape direct manipulation in an elegant way to reduce the global deformation impact to a minimal level. Therefore, the weight updates are defined as:

\[
w = B^T m
\]  

(6)

The transition from pseudo-inverse based weight updates to our transposition based approach is given in Appendix section. From an intuitive perspective, Equation 6 provides a direct projection of the point movements onto the \( B \). Therefore, point displacements directly determine the exact deformation with the correcting rows of \( B^T \). This type of approach prevents the unexpected and unintuitive deformation effects on the facial model during the artistic pose editing. We accomplish this by appending the constrained points to \( m \), and corresponding rows to \( B \). Thereby, each pinned points adds three columns to \( B^T \) and rows to \( m \). For example, while \( n \) arbitrary number of pins are located on the 3D facial model with \( k \) blendshape targets, the system described in Equation 6 can be represented in the matrix form in Equation 7:

\[
\begin{pmatrix}
w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_k
\end{pmatrix} = \begin{pmatrix}
x & y & z & \cdots & z \\ x & y & z & \cdots & z \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b_1^T & b_2^T & b_3^T & \cdots & b_{3n}^T \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{1k} & m_{1y} & m_{1z} & \cdots & m_{1z} \\ m_{2k} & m_{2y} & m_{2z} & \cdots & m_{2z} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ m_{3k} & m_{3y} & m_{3z} & \cdots & m_{3z}
\end{pmatrix}
\]  

(7)

It should be noted that in Equation 7, \( B^T \) is \( k \times 3n \) matrix that consists of the coordinates of blendshape targets over the selected vertex, and \( w \) vector includes all weight values to form the resultant face. It is clear that each weight value is computed as the inner product of the relevant \( B \) vector and the constraint points vector \((w_i = b_i \cdot m)\). Therefore, this statement reveals that each pin displacement is only projected on its corresponding blendshape targets, and this behavior keeps the system stable, intuitive and prevents the unexpected movements.

Strictly speaking Equation 6 gives the same answer as Equation 3 in the case where the problem is linear and Equation 6 is iterated to convergence. However, in interactive use, the pin \( m \) is constantly moving, and Equation 6 is not iterated to convergence. In this case the Jacobian transpose approach naturally moves each weight in proportion to the amount that the corresponding blendshape target is useful in reaching the goal, and there is no need to rigidly threshold the singular values to separate what directions are part of the nullspace.

### 3.3 Pseudo-inverse with Transposition Approach

The pseudo-inverse and the proposed transpose approaches have their own complementary advantages and disadvantages. Therefore, it would be desirable to combine them for obtaining a smooth and promising hybrid approach. The instability problem of pseudo-inverse approaches is already mentioned in the previous section. However, this negativity can be used as an advantage during the pose editing. Because in some particular cases, the movement of target shapes naturally requires the movement of other shapes. For example, the specific movement of inner eyebrow for the “anger expression” is only possible with the slight movement of the mouth. During pose editing, this type of coupling may seem unexpected but not necessarily unrealistic. Besides, in rare cases, the gross pin movements may overshoot the pose with the transpose approach. By taking these possibilities into account, we define a simple parametric representation for the weight updates that is shown in Equation 8:

\[
w = (\gamma B^T + (1 - \gamma) B^T) m
\]  

(8)

where \( \gamma \) is an arbitrary scalar which keeps the balance of the system. The value of \( \gamma \) would vary between \([0,1]\) where error stays in a minimal state. Unfortunately, in this context there is no exact definition of
error, since pseudo-inverse part of Equation 8 already produces a slight error. However, this error can be considered as the unexpected movements of the face that are within the tolerable limits. The further details of Equation 8, and the $\gamma$ value will be discussed in the next section.

4 EXPERIMENTAL RESULTS AND ANALYSIS

Our approach and its interface have been implemented as a plugin for Maya 2016 using Python. Numpy package has been used for matrix calculations. We demonstrate the comparison of our transposition approach with the other existing techniques on the various blendshape facial models. All direct manipulation techniques including our transposition approach respond in real time, and rapidly produce the interactive visual feedback to the user. Our main motivation has been keeping the user away from the technical aspects of the method during the implementation. For that reason except the mouse, the interface does not include any control components. All the tests were performed on 4-core Intel Core i7-2600 3.4 GHz machine with 8 GB of RAM and an nVidia GTX 570 GPU.

Comparison to Pseudo-inverse based Approaches

Despite the instability problem of pseudo-inverse based approaches, over the years they have dominated the research on direct manipulation. However, our experiments show that our transposition approach deforms the desired area of the face with a more smooth blending. Besides, the unexpected movements are almost completely eliminated. Figure 2 visually compares our approach with the pseudo-inverse technique under the single pin editing scenario. The model consists of 18 blendshape targets, and the same pin movements produce thoroughly different facial poses. The slider interface of Figure 2 is shown in Figure 3. The pseudo-inverse approach clearly updates the least corresponding weights more than it should be, and the resultant pose stays far away from the expected pose. On the other hand, our transposition approach directly maps the displacement of the pin movement onto the corresponding blendshape targets. The resultant pose is in the direction of the expectations. Besides, it should be noted that both approaches operate in the deformation space defined by the preconfigured blendshape targets. Therefore, it is not possible to express poses beyond the limits of what blendshapes allow to the user.

Further, we created an experimental pose (Figure
Figure 5: Comparison of our proposed approach with other pseudo-inverse based techniques on a production quality blendshape model with more than 100 target shapes. Our transposition based approach in (a) generates visually plausible and desired results with same pin drag direction. The other methods (in b and c) produce unintuitive results with many unexpected global deformations.

4a) for an experienced user, and asked briefly produce the same pose using our transposition based approach, pseudo-inverse based approach and the method of (Lewis and Anjyo, 2010). The test model consists of 40 blendshape targets and we measured each target shape value as shown in Figure 4 below. The most correct pose was intuitively obtained by our transposition approach. However, all three approaches produce errors, which are reported using the root mean square deviation per weight:

\[ E_{\text{RMS}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} |w_i - \bar{w}_i|^2} \]  

where \( N \) is the number of target shapes, \( w_i \) is the weight value of the \( i^{th} \) target shape, and \( \bar{w}_i \) is the corresponding weight value of the \( i^{th} \) target shape of the experimental pose (Figure 4a). The error values for each target shape is compared in Figure 6. According to the error analysis, the pseudo-inverse approach and the method of (Lewis and Anjyo, 2010) produce several irrelevant weight values during the pose generation. Besides, both approaches generate error values on the relevant weights which cause unintuitive end results. On the other hand, our transposition based approach produce very few error values, which mostly occur on the relevant weight values. This difference is due to the precise mouse movements, and is almost not observable on the model. Figure 6 demonstrates these error differences in scatter and line graph styles.

We compare all three approaches for posing with a production quality model (number of blendshapes > 100) in Figure 5, which is a more realistic scenario for the artists to create facial expressions during the animation process. We observe that method of (Lewis and Anjyo, 2010) (Figure 5c) produces visually more plausible poses than the classic pseudo-inverse approach (Figure 5b), nevertheless they both produce unexpected movements on the regions that are other
Table 1: Summary of the processed datasets. The face model order follows the order in Figure 1 (from left to right). For example, Face 1 is the left most model, Face 3 is the cartoon model in the middle, Face 5 is the right most model. From left to right, columns show in order, the model used, number of vertices, number of edges, number of faces, number of blendshape targets, \(T_{\text{pin}}\) is average timing for pin generation, \(T_{\text{edit}}\) is exact timing for pose editing. All computation times are in seconds.

<table>
<thead>
<tr>
<th>Model</th>
<th>#Vertex</th>
<th>#Edge</th>
<th>#Face</th>
<th>#Targets</th>
<th>(T_{\text{pin}})</th>
<th>(T_{\text{edit}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Face 1</td>
<td>6720</td>
<td>19984</td>
<td>13264</td>
<td>40</td>
<td>0.15</td>
<td>0.0099</td>
</tr>
<tr>
<td>Face 2</td>
<td>9562</td>
<td>28416</td>
<td>18848</td>
<td>18</td>
<td>0.03</td>
<td>0.0099</td>
</tr>
<tr>
<td>Face 3</td>
<td>11926</td>
<td>35576</td>
<td>23648</td>
<td>36</td>
<td>0.13</td>
<td>0.0099</td>
</tr>
<tr>
<td>Face 4</td>
<td>5692</td>
<td>11282</td>
<td>5584</td>
<td>127</td>
<td>1.75</td>
<td>0.015</td>
</tr>
<tr>
<td>Face 5</td>
<td>5792</td>
<td>11625</td>
<td>5834</td>
<td>136</td>
<td>1.46</td>
<td>0.019</td>
</tr>
</tbody>
</table>

Figure 6: The error analysis of the weight deviation from Figure 4. The error values are calculated by using Equation 9. The scatter plot of error values are presented in the top graph, and the comparison of error values are shown in the bottom graph. Our transposition approach produces the minimum error during the pose generation.

than the desired deformation region of the face. Our transposition based approach (Figure 5a) provides almost fully local deformation which allows the artists to edit only the desired manipulation area without any visible global deformation impact. It should be noted that in Figure 5, all sub-figures are intended to produce the same pose with the same number of pins.

Figure 8 illustrates the application of our transposition based approach on different facial models. Table 1 summarizes the processed data, model details, and computation timings for the proposed approach.

Parametrization of the Hybrid Approach

Assigning a fixed parameter value to \(\gamma\) value in Equation 8 can be considered as the most straightforward approach. In that case, it would be most likely to choose a small heuristic value (such as 0.1) and allow the transposition approach to dominate Equation 8. Instead, we apply the singular value decomposition to \(B^T\) where \(B^T = VSU^T\) and choose the smallest singular value \(\sigma_1\) for \(\gamma\). The singular values smoothly approach to zero when the number of pins increase. On the other hand, since \(B^+ = VS^+U^T\), the singular values of pseudo-inverses grow toward infinity.

As the user applies few number of pins to the model (such as 1 or 2), the pseudo-inverse side of Equation 8 will be intrinsically effective. Besides, by intuition few number of pins denotes that the user does not fully constrain the model, and execute only a simple weight update. Another possibility can be an unstable pose needed for a particular case. Nevertheless, the general practice is based on applying the high number of pins to the model (such as 5 to 15), and controlling all of them without any artifact. In these cases, our transposition approach dominates Equation 8, and provides smooth results. Figure 7 demonstrates an example case to the usage of Equation 8. Some poses are necessary for irregular expressions, and it is possible to create with 1 or 2 pins that can be seen in the first and second columns of Figure 7. In these cases, pseudo-inverse approach can be considered to generate the desired irregular pose. However, when the number of pins increase on the model, transposition based approach dominates Equation 8 (last column of Figure 7).

Limitations. Our transposition approach works without any limitation. However, there exists one limitation in our hybrid approach which comes from the fact that the \(\gamma\) value in Equation 8 is computed according to the located number of pins on the model. Therefore, it is ideal to locate the desired number of pins on the model before applying the posing operation.
Figure 7: While the transposition approach generally produces good results, in certain cases the hybrid approach from Equation 8 is superior. The parametrization in Equation 8 is calculated according to singular values of SVD. When the user locates few number of pins on the model, SVD produces a high value for the $\gamma$ parameter and pseudo-inverse side of Equation 8 dominates. On the other hand, with high number of pins on the model, SVD produces a very small number and transposition part of Equation 8 dominates.

5 CONCLUSIONS

In this paper, we develop a practical transposition based blendshape direct manipulation for 3D facial animation. Our approach has focused on the precise projection of the pin displacements onto the corresponding deformation vectors. Unlike the previous pseudo-inverse based methods, our approach provides a direct control of the deformation space by purely sticking its dimensions. This behavior reduces the unexpected movements during the artistic editing, and almost completely reduces the deformation influence area to the local geometry where the pins are located. While the artifacts of the pseudo-inverse approach can be reduced with regularizers, our new approach is more stable intrinsically. The difference between the two approaches is particularly noticeable while dragging the mouse, and when the matrix $B^+$ in (3) has a large nullspace. In addition, we offer a hybrid approach, which incorporates our proposed transposition approach with the pseudo-inverse solution. In the results, we demonstrate how our approach provides intuitive and stable artistic edits on blendshape models almost without any artifacts. Our approach is an original, simple, and powerful direction that can be easily adapted to the existing direct manipulation frameworks.

Hybrid combination of transposition and pseudo-inverse approaches, as offered by our method, is a promising direction that we would like to pursue in the future. Applying an arbitrary number of pins within the blendshape deformation space is parametrized by employing the smallest singular value in our current form. We plan to extend this approach to formulate a balanced form that reduces the differences between pseudo-inverse and transposition in the final result. Another promising research direction is to explore a step-size formulation for the proposed transposition approach to prevent possible overshooting during the gross pin movements.

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The transposition algorithm can be motivated by expressing the pseudoinverse in terms of an orthonormalized basis. We orthonormalize the $B^T$ matrix, and extract the deformation directions from the blendshape matrix, because both share the same vector spans. Therefore, we apply the Gram-Schmidt process (Cheney and Kincaid, 2009), and store the extracted orthonormal matrix in $Q$. After, we substitute the new $Q$ matrix instead of $B^T$ in Equation 3 with a scaling step-size matrix $R$ for the point movements.

The new pseudoinverse equation becomes:

$$w = (QQ^T)^{-1}Q(Rm)$$  \hspace{1cm} (10)

In Equation 10, $Q$ is an orthonormal matrix. Thereby, $QQ^T$ becomes an identity matrix. The new update function turns into $w = Q(Rm)$. According to Jacobian Transpose suggestion, weight update can be represented as $w = B^T m$. To minimize the scaling step-size of the point movements we apply the following minimization:

$$\min_R |B^T - QR|^2$$  \hspace{1cm} (11)

The solution of Equation 11 shows us $R = Q^T B^T$. Alternatively, least square process can be replaced with a simple qr factorization. Then, $R$ is substituted to Equation 10 and the final weight update function becomes:

$$w = B^T m$$  \hspace{1cm} (12)

Reviewing these steps, we see that when the basis is orthogonal, the pseudoinverse can be reduced to the transposition approach. In turn, the bad behavior of the pseudoinverse can be understood in terms of the lack of orthogonality of the basis.

**APPENDIX: Transition from Pseudo-inverse to Transposition**
Figure 8: Sequential posing using our transposition based blendshape direct manipulation approach. Our approach can be applicable to all blendshape models without any pre-configuration, and it takes less than 3 minutes to create the final expression from the neutral pose. The details of the models are reported in Table 1.