

# A Backpressure Framework Applied to Road Traffic Routing for Electric Vehicles

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**Abstract:** Electric vehicles (EVs) emerged in the transport domain, due to their energy efficiency and clean energy that they utilise. The electric vehicle routing problem is essentially a problem of selecting a set of minimum cost routes, while the demand of the customers is achieved. Route cost metrics include energy consumption and driving time. In this work, we model the electric vehicle routing problem using a wireless network methodology, namely the backpressure framework. The penalty imposed to every route includes the driving time of each road. We derive a weight as a function of the road queue backpressure and the driving time of a car. The next route for our EV is the one that has the highest weight. It turns out that this methodology leads to faster routes in that there are often roads with accidents or traffic jams, even though they are in the shortest path of the route to the destination. We present results via simulations, which verify the fact that backpressure is an efficient algorithm to be applied to electric vehicle routing.

## 1 INTRODUCTION

The use of electric vehicles (EVs) into the transport sector has been introduced (Emadi, 2011) for two reasons. Firstly, there is the need for energy efficiency in transport, since the charging of the EV is not expensive as opposed to the internal combustion engine vehicles. Secondly, there is the reduction of the  $CO_2$  emissions due to the clean sources (battery) employed for the production of electricity. On the other hand, traditional vehicles are associated to the combustion of fossil fuels. Thus, EVs provide a great alternative as a next generation of the transport means in a city.

The main characteristics of EVs play a key role on the design of algorithms utilised by route planners. EV route planning exhibits certain differences from conventional route planning. Initially, the limited capacity of the EV's battery introduces the constraints of driving ranges of approximately 100 km on average. Moreover, charging of EVs take place in stations that do not exist in as many places as gas stations. EV charging is a process that may take hours to complete. Thus, routes may be determined in an economic manner rather than just the fastest or shortest. On the other hand, there are cases where the state of the battery does not necessarily decrease when driving. All of the above sum up to the fact that companies with

EV fleets, require their goods to be delivered on time in often jammed road networks with potential uncertainties, such as road accidents or road works.

The Vehicle Routing Problem (VRP) was proposed in (Dantzig and Ramser, 1959). Thereafter, a plethora of extensions of the problem have been suggested, which included real world constraints. Two works that constitute the most widely investigated extensions are the Capacitated VRP (CVRP), where vehicles have a limited freight capacity and the VRP with Time Windows (VRPTW), where customers have to be reached within a specified time interval (Laporte, 2009) (Nagata et al., 2010). The Electric Vehicle Routing Problem (EVRP) (Touati-Moungla and Jost, 2012; Artmeier et al., 2010) has been formulated as a problem of locating a set of minimum cost routes, in order for the demand of the customer to be accomplished. Often, constraints related to the capacity of the battery of the EV have been investigated.

In this paper we deal with dynamic traffic routing for electric vehicles. For our approach, we employ a methodology utilised in wireless networks, called the Backpressure routing, which essentially builds the routing without routes (Moeller et al., 2010). The aim is to employ infrastructure, such as traffic cameras to estimate the queues created at each traffic light road.

This will provide the driving time of each street and we provide a penalty minimisation used in a weight to be calculated for each road. We provide results based on simulations to show the efficiency of our approach, which results in the successful delivery of companies' goods on a predefined schedule in a single depot. We show the following:

- EV routing may be modelled with the backpressure framework, which exhibits stable queues and they are used to construct a weight to select the next best route
- Road driving time is a metric that may act as a penalty that is minimised.
- Backpressure application results in fastest routes, avoiding any sudden road condition changes in a real-time fashion.
- There are cases that backpressure outperforms Dijkstra's shortest path algorithm in reaching a destination via EV routing.

The paper is structured as follows: Section 2 presents the related work in electric vehicle routing, section 3 provides a description of the backpressure framework utilised, section 4 explains the penalty to be minimised, section 5 gives the simulations of our proposed scheme and section 6 gives the conclusions of our approach.

## 2 RELATED WORK

In (Afroditi et al., 2014), the authors investigate the one-to-many vehicle routing and scheduling problem in EVs. More specifically, problem formulation and constraints in practical scenarios are examined. They highlight that the EVRP is an NP-hard problem and requires significant computational power for the location of near optimal solutions in medium to large scale scenarios. The authors provide a mathematical formulation to model the EVRP due to its capacity, time window and predefined charging level constraints. Furthermore, EVRP trends are examined providing significant information regarding future research on real world scenarios and approximation algorithms.

In (Bruglieri et al., 2015), the authors aim to find the optimal route for EVs in a multi-customer scenario considering recharging requirements during the routes. They formulate routing as a Mixed Integer Linear Programming problem. The battery recharging at every station is a variable, since flexible routes are to be guaranteed. The proposed scheme optimises total travel, waiting and recharging time as well

as the number of the EVs utilised. The problem is solved using Variable Neighbourhood Search Branching (VNSB) in reasonable computational times.

In (Schneider et al., 2014), the authors introduce the electric vehicle-routing problem with time windows and recharging stations (E-VRPTW). The model employs recharging at any of the predetermined stations using a recharging scheme. Furthermore, they consider limited vehicle freight capacities in conjunction with customer time windows, which constitute constraints in real-world transport applications. To solve the aforementioned problem, a hybrid heuristic is presented, which combines a variable neighbourhood search algorithm with a tabu search heuristic.

In (de Weerd et al., 2015) the authors propose an intention-aware routing system (IARS) for electric vehicles. The system provides the ability to EVs to estimate a routing policy, which minimises journey time, while keeping track of other vehicles intentions. Considering other vehicles' intentions is significant since the driver may have to charge the vehicle in the journey and queueing time may be large, in case other vehicles select the same stations. Thus, queueing times are predicted based on the intentions of the other EVs.

In (Abousleiman and Rawashdeh, 2014), the authors attempt to tackle the problem of energy efficient routing for EVs using Particle Swarm Optimisation. They also show that EVs route optimization techniques, such as negative edge costs, battery power and capacity limits, as well as vehicle parameters that are only available at query time, make the task of electric vehicle routing a challenging problem.

In (Baum et al., 2014) the authors investigate route planning applications for electric vehicles. They show that such problems have to consider constraints such as energy consumption. They indicate that recent approaches for EV routing focus on optimizing energy consumption as a single variable. They provide preliminary work towards a holistic framework for computing shortest paths for electric vehicles with limited range. Their scheme comprises driving energy-efficient speed adjustments, realistic modeling of battery charging and the integration of turn costs.

## 3 BACKPRESSURE FRAMEWORK

The dynamic nature of the road network combined with the unwanted but sometimes occurring bottlenecks, provide the necessary means for the emergence of a dynamic routing algorithm, where routing deci-

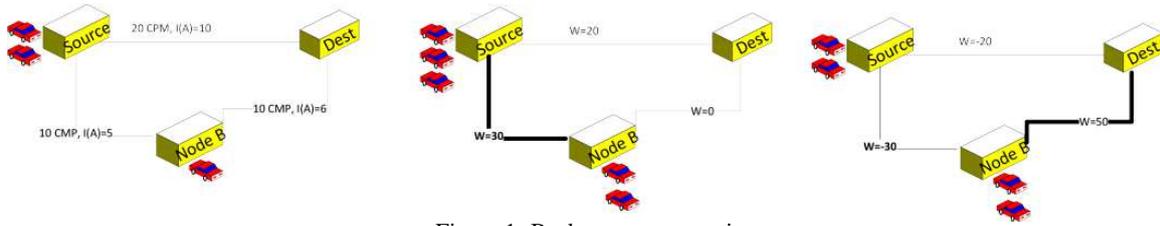


Figure 1: Backpressure operation.

sions will be made dynamically each time a car enters a road leading to a junction. Hence, it is quite useful to employ a wireless network routing algorithm that performs this task, with respect to the road network conditions. Modern road networks utilise traffic cameras to monitor traffic and to prevent speeding above the limit. Hence, it is quite reasonable to assume that they may be used to calculate driving time and number of cars within a road. Furthermore, with the emergence of the Internet of Things (Kopetz, 2011) and Cyber-Physical Systems (Baheti and Gill, 2011), it is clear that infotainment systems in cars maybe connected to devices on junction traffic lights that may exchange traffic information, resulting thus, to a wireless road network. Hence it may be promising to employ a dynamic routing algorithm in a road network, if we consider it as a graph-theoretic entity (Bondy and Murty, 1976) where vertices are considered as junctions and edges as roads.

Backpressure routing does not operate like traditional routing mechanisms, meaning that it does not locate an explicit path estimation from any source to a destination. It performs routing decisions for each car by calculating for each outgoing road a backpressure weight. This weight is a function of localised queue and link state information. In figure 1 we observe the backpressure functionality in a simple road network. Note that *Node B* represents an intermediate routing junction.

We will next provide a definition of a stable network.

We denote the queue at junction  $i$  during time slot  $t$  as  $Q_i(t)$ . A network of queue backlogs is defined as strongly stable if:

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} \mathbb{E}[Q_i(\tau)] < \infty, \forall i \quad (1)$$

Furthermore, we denote as  $f(\vec{x}(t))$  the penalty, which comes as an outcome of routing decisions between queues in time slot  $t$ . We assume that  $f$  is non-negative, continuous, convex and entry-wise non-decreasing, meaning,  $f(\vec{x}) \leq f(\vec{y})$  when  $\vec{x} \leq \vec{y}$  entry-wise. Let  $f(\vec{x})$  be the penalty value of the function that works on the average value of the  $\vec{x}$  vector. We formulate the road network as a stochastic optimi-

sation problem in which routing decisions minimise function  $f$  and keeping strongly stable queues simultaneously. That is:

$$\begin{aligned} \text{minimise: } & f(\vec{x}) \\ \text{subject to: } & \text{Strongly Stable} \end{aligned} \quad (2)$$

Assuming that  $f(\vec{x})$  is a cost metric of the routing process, we can derive the solution of equation (2) using the Utility Optimal Lyapunov Networking framework (Neely et al., 2008; Neely and Ugaonkar, 2008). This can show that we may have the result of routing decisions resulting in a backpressure routing policy. More specifically, each junction computes the weight per outgoing road in every time-slot. The weight is given below:

$$w_{i,j} = (\Delta Q_{i,j} - \tau \theta_{i,j}) \bar{R}_{i,j} \quad (3)$$

where  $\Delta Q_{i,j} = Q_i - Q_j$  is the queue backpressure and  $Q_i, Q_j$  are the backlogs of junctions  $i, j$  respectively,  $\bar{R}$  is the road car driving rate and  $\theta_{i,j}$  a road usage penalty that depends upon the particulars of the utility and penalty functions of (2). The parameter  $\tau$  is a constant trades system queue occupancy for minimising the penalty. We propose a decentralised approach where junction  $i$  calculates the backpressure weight of all its neighbouring junctions. Thereafter, it is used to determine independent routing. For example, junction  $i$  locates the road  $(i, j^*)$ , which has the highest value of the backpressure weight as the next route of the car. We assume, at this point, that the weight is always larger or equal to 0. In the case that the weights are equal, we adjusted the algorithm to select the route with the shortest distance.

## 4 PENALTY MINIMISATION

The main challenge we are facing is to identify the most suitable penalty function  $f(\vec{x})$  in equation (2), in order to provide efficient performance when we use it in a real work road traffic network.

Initially, we give some preliminaries regarding the road traffic network problem. We model the road network using a directed graph  $G(V, A)$ , where  $V$  represents the junctions and  $A$  represents the roads. As

we mentioned at a previous section, we assume that each junction has traffic control cameras, or other devices that may monitor and calculate the rate and driving time of each vehicle. The graph includes two attributes on each road  $a \in A$ . The first is an estimate before hand for its driving time for the solution we request, and the second is the road performance function. We use the road performance function that has been utilised in (Jahn et al., 2005). In particular, the road performance function,  $I_\alpha$ , maps the traffic rate  $x_\alpha$ , to its driving time  $I_\alpha(x_\alpha)$ .

The road performance functions, which will serve as the penalty functions of (2) determine the impedance of roads for a variety of congestion levels. We wish our functions to be nondecreasing and differentiable, as well as  $I_\alpha(x_\alpha)$  to be convex. In our experiments we employ the function derived by the U.S Bureau of Public roads (BUREAU, 1964)

$$I_\alpha(x_\alpha) := I_\alpha^0 \left( 1 + \alpha \left( \frac{x_\alpha}{c_\alpha} \right)^\beta \right) \quad (4)$$

where  $I_\alpha^0$  represents the travel time under no congestion,  $\alpha \geq 0$  and  $\beta \geq 0$  are tuning parameters, and finally,  $c_\alpha$  is the practical capacity (Patriksson, 1994).

We employ the metric (4) to be minimised in the backpressure framework and we have the following penalty function in (2):

$$f(\vec{z}) = \sum_i \sum_{j \in N_i} z_{i,j}(t) \bar{I}_\alpha \quad (5)$$

where  $N_i$  is the set of neighbouring junctions of junction  $i$ ,  $I_\alpha$  is the road performance and  $z_{i,j}(t)$  is the number of routed cars over the road  $i \rightarrow j$ . Notably,  $f(\vec{z})$  satisfies the properties of problem (2) and provides a backpressure weight, calculated by a junction  $i$  to a neighbour  $j$  as follows:

$$w_{i,j} = (\Delta Q_{i,j} - \tau \bar{I}_\alpha) \bar{x}_{i,j} \quad (6)$$

## 5 SIMULATIONS

We produced a road network in MATLAB consisting of 7 junctions that we assume they are traffic lights equipped with traffic cameras. Furthermore, we have 10 roads connecting the junctions in a fashion that is given in figure 2 (a). We also show the distance of the roads between the junctions. This is since we wish to compare our algorithm with Dijkstra's shortest path algorithm (Dijkstra, 1959).

The parameters that the backpressure algorithm uses are given in table 1. More specifically, we provide the times that a car requires to pass from a road

and the queue backpressure for each junction. Moreover, the rate of each road is assumed to be equal, notably 4 cars per round (traffic light green to red). Finally, we set the parameter  $\tau = 2$ . We assume that there are certain roads that require more time to be driven even though they are of longer distance, due to more cars selecting these routes or by an occurrence of accidents.

Table 1: Time and Queue Backpressure per Junction.

Road	Time	Queues (i,j)	Queue Backpressure
A-B	4	15-10	5
A-C	6	12-5	7
A-D	10	13-6	7
B-C	10	10-5	5
B-F	7	10-3	7
C-E	5	5-2	3
C-F	5	5-3	2
F-S	4	3-1	2
E-S	5	2-1	1
D-S	12	6-1	5

Initially, we run the algorithm computing the shortest path for a car starting from junction  $A$  to the destination  $S$ . We observe in figure 2 (b) that the route computed is  $A \rightarrow C \rightarrow E \rightarrow S$ . This can be easily seen from the distances provides for each road.

Thereafter, we applied the backpressure algorithm, where we see that the route selected by a car starting from the origin  $A$ , in order to reach the destination  $S$  is different from the shortest path algorithm. In figure 2 (c) we see that the car drives to junctions  $A \rightarrow C \rightarrow F \rightarrow S$ . Furthermore, the driving time of the route using the backpressure as opposed to the shortest path algorithm is shorter by 1 minute. We are positive that in other scenarios the time difference may be longer.

In figure 3 (a) - (c) we present the steps of the backpressure algorithm by showing the routes selected by cars in different junctions. Each vertex includes the letter of the junction and its respective queue size i.e.,  $A - 14$ . Furthermore, we provide the backpressure weight in each edge of the graphs, in order to make the route selections clear. Finally, we converted the driving time from minutes to hours to calculate the backpressure weights. Note that we assume that the incoming cars to junction  $A = 3$  and that the junctions' traffic lights work simultaneously, for the sake of simplicity.

During the first round that the traffic lights become green we have the following configuration, which appears in figure 3 (a). As we can see, the backpressure weights that are given in the edges of the figure, promote the cars to route from  $A - C$ ,  $B - C$ ,  $C - E$ ,  $D - S$  and  $F - S$ . Note that the routes to  $S$  occur, since there are no other routes from the respective junctions.

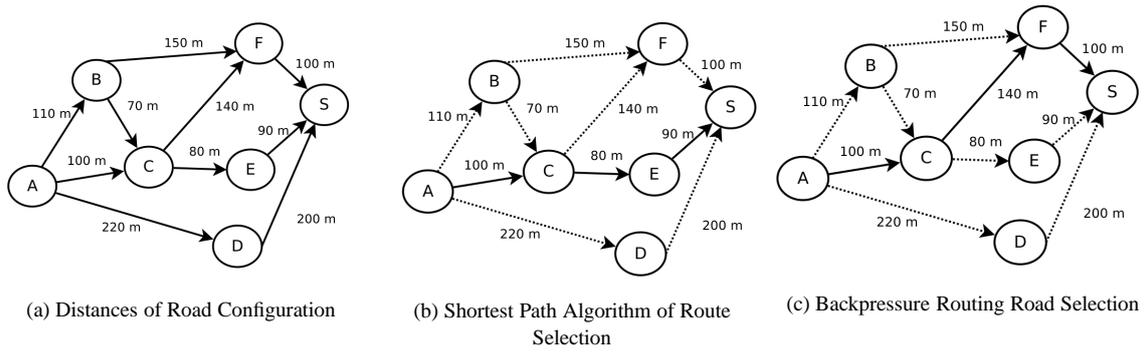


Figure 2: Routes of Backpressure, Shortest Path and initial configuration.

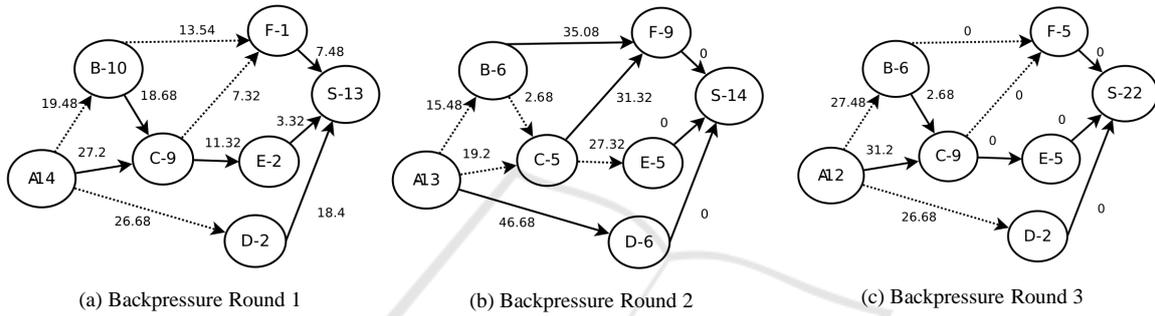


Figure 3: Backpressure Routing Rounds.

Furthermore, we assume that cars routed to junction C from junction A arrive first, since the driving time of the road is less than than the road originated from junction B.

In round 2 of the backpressure algorithm in figure 3 (b), we observe that the car that interests us moves from junction C to junction F, since the backpressure weight is higher than the one from C to E. Furthermore, the other routing decisions in this round consist of B – F, A – D and F, E, D to the destination S, since there are no other routes to select from. Moreover, we observe that these routes have a weight of 0, since it is the smallest value our algorithm may accept (the weight has a negative value in our calculations).

In figure 3 (c), the car that we are routing, reaches the destination S from junction F. We also see that other routing decisions include A – C, B – C, C – E and all the one hop junctions to the destination. Note that the routing decision for the routing of the cars of junction C to either F or E has been done using the shortest path, since both the respective backpressure weights are 0. The end of this round routes our car from junction A to the destination S.

## 6 CONCLUSIONS

In this paper we applied a state-of-the-art routing protocol for wireless networks to electric vehicle routing. The backpressure routing is essentially routing without routes. It calculates the next route of the car dynamically based on the calculation of the backpressure weight and the strong stability of the formed queues on the junctions. We utilised the driving time of each road as a penalty to be minimised by every car. The penalty is selected based on the necessity of a company’s fleet to reach the clients’ addresses within a given time window and in a sigle depot.

Our simulations showed that the backpressure algorithm creates faster routes than Dijkstra’s shortest path algorithm. This may be very useful, since roads may have several problems such as traffic accidents, road works or traffic jams, which may prolong their driving times even if the distance between two junctions is small. Hence the backpressure algorithm establishes an efficient routing mechanism.

Our future work includes the integration of the electric vehicle battery to the penalty minimisation process. This might provide a useful insight as to the route selection problem, since the fastest route may not be the most energy-efficient one. It is crucial

to compare our approach with state-of-the-art routing protocols for EVs that take into consideration charging station locations and charging times. The backpressure framework provides us with the flexibility to employ the penalty function of our liking, in order to produce the backpressure weight. Hence we are optimistic that it will perform well comparing other routing schemes. Moreover, our approach is not restricted to EVs only and it may be adjusted to operate in conventional vehicle routing as well. Finally, we aim to put the backpressure algorithm to a more complex road network, and perhaps to a real test.

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