The Modularity Matrix as a Source of Software Conceptual Integrity

Iaakov Exman
Software Engineering Dept., The Jerusalem College of Engineering – JCE - Azrieli, POB 3566, Jerusalem, Israel

Keywords: Conceptual Integrity, Modularity Matrix, Conceptual Lattice, Linear Software Models, Liskov Substitution Principle, Abstract Mathematical Concepts, Standard Modularity Matrix, Software System Design.

Abstract: Conceptual Integrity has been declared the most important consideration for software system design. However, the very concept of Conceptual Integrity remained quite vague, lacking a precise formal definition. This paper offers a path to a novel definition of Conceptual Integrity in terms of the Modularity Matrix, the basic structure of Linear Software Models. We provide arguments for the plausibility of the Modularity Matrix as the suggested source of software system Conceptual Integrity, viz. the orthogonality and propriety of the Matrix modules. Furthermore, the paper also reveals some new characteristic properties of Software Conceptual Integrity.

1 INTRODUCTION

We can trace back the idea of conceptual integrity, in the context of software, to Brooks in his well-known book “The Mythical Man-Month” – in Chapter 4, page 42 of the anniversary edition – (Brooks, 1995). Already there, it is said that conceptual integrity is the most important idea for software system design. The idea has been proposed and praised, but not exactly defined.

This paper offers a path to a formal definition of conceptual integrity in terms of the Modularity Matrix. This matrix is the basic algebraic structure of Linear Software Models.

In this Introduction section we clarify the idea of software conceptual integrity, as far as it has been done since its initial presentation by Brooks, and concisely review the basics of the Modularity Matrix.

1.1 Software Conceptual Integrity

The idea of conceptual integrity, in the context of software, has been reiterated in a more recent book by Brooks “The Design of Design: Essays of a computer scientist” – in Chapter 6, pages 69-70 – (Brooks, 2010).

There, conceptual integrity is said to consist of three principles referring to system functions. These principles have been verbally formulated in a paper by De Rosso and Jackson (De Rosso and Jackson, 2013) as follows:

- Orthogonality – individual functions should be independent of one another;
- Propriety – the system should have only the functions essential to its purpose and no more;
- Generality – a single function should be usable in many ways.

There are a few problems with these formulations, but the most important one is that there is no “algorithm” or “protocol” to make concrete usage of these principles.

This work has as its aim to offer a formal path to conceptual integrity, gaining in this process both a deeper understanding of this idea and the basis for a practical application of the above principles. Thus, among other things, if the above principles are indeed the essence behind conceptual integrity, then they must follow as consequences of the offered formal path.

1.2 Modularity Matrix

The Modularity Matrix – see e.g. (Exman, 2014) is a representation of a hierarchical software system in its several abstraction levels, through sub-systems, down to indivisible basic modules. The matrix columns, the structors, stand for architectural structure units, generalizing classes. The matrix
rows, the functionals, stand for architectural behavioural units, generalizing class methods.

Algebraic manipulations lead in the optimal case to a standard square and block diagonal matrix, in which the blocks along the diagonal are the modules of the current matrix level. This is seen in the abstract Modularity Matrix displayed in Fig. 1.

<table>
<thead>
<tr>
<th>Structure</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F2</td>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F3</td>
<td></td>
<td></td>
<td>1</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F4</td>
<td></td>
<td></td>
<td>1</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>F5</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>F6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 1: An Abstract Standard Modularity Matrix – The matrix is standard as it is strictly square and block-diagonal. It has 6 structors (columns) and 6 functionals (rows). Its 3 modules are the three blocks in the diagonal, having 1 valued matrix elements (with light blue background). Outside the modules (blank areas) there are only zero-valued matrix elements. These values are omitted for simplicity.

In case there remain some non-zero outlier matrix elements, i.e. outside the diagonal modules, these elements point out to undesirable couplings among the modules. These couplings should be resolved by moving structors/functionals among modules or by adding/removing structors and/or functionals, in the columns/rows containing the outliers.

1.3 Related Work

In this concise review of the related literature we mention works referring to Conceptual Integrity.

We also mention tools which support this notion, particular discussions of orthogonality in this context and different kinds of matrices used to analyse software design, besides the Modularity Matrix.

The origin of Conceptual Integrity ideas, as already mentioned in the beginning of this Introduction, is Brooks’ book originally published in 1975, with an extended edition 20 years later (Brooks, 1995).

Jackson and co-workers have further elaborated on the Brooks’ notions, say by means of cases studies, see e.g. (De Rosso and Jackson, 2013). Jackson also formulated a relevant Research Agenda (Jackson, 2013). In a recent essay Jackson stresses the importance of concepts for software systems, and gives examples of arrangements of concepts in a dependence graph, from which coherent subsets can be extracted and analysed. But, these graphs have not been formalized (Jackson, 2015).

Documents that explicitly refer to Conceptual Integrity occasionally mention it, and often formulate some vague statement about what this means, see e.g. Beynon et al. in (Beynon, 2008).

Kazman and Carriere in page 31 of a Technical Report (Kazman, 1997) describe the problem of reconstructing the software architecture of a system. Their guide to a good and meaningful architecture is conceptual integrity. It should be built from a small number of components connected in regular ways, with consistent allocation of functionality to the architecture’s components.

Clements et al. in their book (Clements, 2001) refer to conceptual integrity as the underlying theme that unifies the design of the system at all levels. The architecture should do similar things in similar ways, having a small number of data and control mechanisms, and patterns throughout the system. There are two points to be stressed in this statement: a- they refer to the system at all levels; b- a possible approach to a more precise definition would be counting mechanisms and patterns.

The concept of orthogonality also appears occasionally in the software development literature. For instance, Krone and Snelting refer to it in a paper using conceptual lattices inferred from source code (Krone, 1994).

Another kind of works refers to software tools to support software systems analysis and design. For instance, (Kazman, 1996) describes a so-called SAAtool, with visualization capability. Conceptual Integrity is estimated by the number of primitive patterns that a system uses.

Finally, there are papers dealing with other matrices, besides the Modularity Matrix, for software systems design. The DSM (Design Structure Matrix) is part of the Design Rules approach (Baldwin and Clark, 2000) and adopted by many works, appeared and has been applied outside the software engineering context. For a set of references to this approach see e.g. (Exman, 2014).
1.4 Paper Organization

The remaining of the paper describes a formal path to conceptual integrity (section 2); shows how conceptual integrity principles follow from the Modularity Matrix (section 3); characterize properties of conceptual integrity (section 4); and conclude with a discussion (section 5).

2 FORMAL PATH TO SOFTWARE CONCEPTUAL INTEGRITY

We propose a formal path from an Abstract Domain Conceptualization to Software Conceptual Integrity. We claim that Conceptual Integrity exists, say in abstract mathematics, before being formally defined.

We first overview the formal path, leading through the Modularity Matrix to the main goal of Software Conceptual Integrity. We then look in more detail at each of the steps made in this path.

2.1 Overview of the Formal Path

A formal path from Abstract Conceptual Integrity to Software Conceptual Integrity, passing through the Modularity Matrix has 5 states with 3 intermediate states in between. These are shown in Fig. 2.

![Figure 2: From An Abstract Domain To Software Conceptual Integrity](image)

The meaning of the five states in the formal path is as follows:

1. Abstract Mathematics Conceptualization – due to its long history, concepts in Mathematics were classified in fields and sub-fields, within hierarchies obeying conceptual integrity;
2. Liskov Substitution – it is an attempt to translate abstract mathematics notions to equivalent software notions; the central idea here is to link “structure” to “behavior”;
3. Modularity Matrix – the basic algebraic structure of Linear Software Models, used to guide software system design; it restricts whole domains to a modularized class of software systems;
4. Conceptual Modularity Lattice – this is a particular case of conceptual lattices (defined in FCA = Formal Concept Analysis) derived from the Modularity Matrix; it obtains the concepts of the software system modules;
5. Software Conceptual Integrity – the desired goal of the whole path, will assure software system orthogonality and propriety.

One can summarize the above states by their roles, as shown in Fig. 3.

![Figure 3: FORMAL PATH: TOOLS, GOALS, AND ROLES](image)

2.2 Preliminary Definitions

Here we provide some preliminary definitions needed to discuss in detail each of the above states.

Despite starting with an abstract mathematics domain, the ultimate goal of the formal path refers to software systems. Therefore, when talking about...
structure and behavior we think in terms of software. Here are the relevant definitions.

**Definition 1 – Software Structure**

*Software Structure* is a relation among software architectural units (classes and their generalization “structors”) involving the following operators: sub-classing (sometimes called “inheritance”) and composition.

It is not by chance that we are using the same operators for software systems and for abstract ontologies. We are just following common practice, which emphasizes the analogies between abstract concepts and their respective software classes.

**Definition 2 – Software Behavior**

*Software Behavior* is the performance of the computation of a function (sometimes called a method). The result of the function computation is a change of state of a software system. We call functions (and their generalization “functionals”) software architectural units of behavior.

Functionals are provided by structors, but are not necessarily invoked. Thus we often, by linguistic license, refer to the functions themselves – without the performance of a computation – as software behavior.

**2.3 Conceptual Integrity in Abstract Mathematics**

Mathematical concepts are classified by properties’ similarity in a hierarchical fashion. The hierarchy is determined by which concepts are particular cases of other ones. We clarify the idea with some examples.

A square is a subclass (particular case) of a rectangle, which is a subclass of a parallelogram, which in turn is a subclass of a quadrilateral. A quadrilateral, the most general case in this small hierarchy (in Fig. 4), is a polygon with four sides. A parallelogram is a subclass of quadrilateral with opposite sides parallel. A rectangle is a subclass of a parallelogram with four right angles. A square is a subclass of a rectangle with all four sides equal.

Each lower hierarchy class has all the properties of the upper classes. A square has 4 sides (as the quadrilateral), which are parallel (as in the parallelogram), and 4 right angles (as the rectangle).

This is also true with respect of behavior, i.e. the outcome of what the respective functions calculate for each concept (or each class). For instance, the perimeter of any of the classes in this hierarchy is calculated in general by summing the length of the four sides (which may be all different, partially different or all equal).

A different hierarchy would have a circle as a subclass of an ellipse. A third different hierarchy would deal with 3-dimensional objects such as a sphere as a subclass of an ellipsoid.

Each of the three referred hierarchies (quadrilaterals, ellipses, 3-D ellipsoids) display conceptual integrity, both intuitively and by some specific well-defined characteristic. For example, all quadrilaterals in Fig. 4 have linear segments as sides of a polygon (literally meaning “multiple angles”), while the ellipses have no linear segments and no angles in between at all in their perimeters.

These hierarchies, such as that the quadrilaterals in Fig. 4, are in fact small fragments of an ontology of geometric figures, e.g. (Rovetto, 2011), which may encompass the three referred hierarchies.

We summarize conceptual integrity in an abstract domain such as mathematics by means of the following theorem:

**Theorem 1 – Conceptual Integrity in Abstract Domain Hierarchy of Concepts**

In a class hierarchy determined by sub-classing, in an abstract domain, all the concepts of the hierarchy have at least one common concept, and one common function defined in the most general member of the hierarchy. The common concept and the common function stand for the conceptual integrity.
2.4 Liskov Substitution

The Liskov Substitution Principle (LSP) attempts to translate, as precisely as possible, notions found in Abstract Mathematics, as discussed above (in subsection 2.3), to the realm of software. This is possible since the ontology fragments (hierarchies) in abstract mathematics are built upon the subclassing operator, while it can be said that LSP is an effort to define “inheritance” which is the software term for subclassing.

The basic idea of Liskov Substitution which is relevant to conceptual integrity is to link “structure” to “behavior”, effectively transforming concepts in an abstract (e.g. “mathematics”) domain into generic software.

A formulation of the Liskov Substitution Principle (Liskov, 1988) is shown in Fig. 5. A corresponding class diagram illustrates it in Fig. 6.

The main principle conditions are marked in bold: a- ‘all programs P’ assures that the principle still is generic in terms of software, i.e. refers neither to a specific software system nor to a specific class of software systems, but is not clear how to test it in real systems; b- 'behavior of P is unchanged' is particularly interesting, as a “structural” class diagram (type T and its subtype S) is being linked to a “behavioural” condition, which is precisely what transforms an abstract domain to software concepts!

We summarize conceptual integrity within Software – according to the Liskov Substitution Principle – by the following theorem.

**Theorem 2 – Conceptual Integrity in Liskov Substitution**

If by the Liskov Substitution Principle, a sub-class object substitution by a parent class object, causes no change of behavior of a system, Conceptual Integrity is preserved, when one passes from an abstract domain to a whole domain of software systems.

Figure 5: Liskov Substitution Principle Formulation – The significant terms of the principle are stressed in bold face: “all programs P” for generality; “behavior is unchanged” linking structure to behavior.

2.5 Modularity Matrix

The Modularity Matrix of a software system is built using structors (a class generalization) preserving the notion of subclassing. Thus, the Modularity Matrix implicitly conveys the central ideas for which the Liskov Substitution Principle was formulated.

The important contribution of the Modularity Matrix, from the viewpoint of the argumentation along the current Formal Path is to restrict the complete generality of Liskov Substitution – i.e. for all programs P, translated to for all software systems P – into a limited set of software systems defined by the Modularity Matrix structors and functionals. This is summarized by the following theorem:

**Theorem 3 – Conceptual Integrity in the Modularity Matrix**

If the Modularity Matrix is standard (square and block-diagonal), then specific structors provide related functionals within modules, and the modules conceptual integrity is preserved for the restricted set of software systems represented by the Matrix.

2.6 Conceptual Modularity Lattice

The Conceptual Modularity Lattice – the last link towards Conceptual Integrity – has been shown (Exman and Speicher, 2015) to be equivalent to the Modularity Matrix, in terms of information conveyed about its software system modularity.

On the other hand, by its very definition - from FCA (Ganter and Wille, 1998) – the Conceptual Modularity Lattice is an algebraic structure restricted to the concepts relevant to its software system.

Summarizing, the role of the Conceptual Modularity Lattice in our formal path, is by its
equivalence to the Modularity Matrix, and the conceptual relevance, to enable to extract from the Modularity Matrix the module concepts which may be tested for Conceptual Integrity.

This is shown by the following theorem.

**Theorem 4 – Conceptual Integrity in the Modularity Lattice**

Since the Modularity Lattice is software design equivalent to its corresponding Modularity Matrix, the concepts fitting to the Matrix modules preserve conceptual integrity and this can be explicitly tested for the restricted set of software systems represented by the Modularity Lattice.

### 3 CONCEPTUAL INTEGRITY PRINCIPLES FROM THE MODULARITY MATRIX

In this section we finally assign a formal definition to two of the principles – referred to in sub-section 1.1 – behind Conceptual Integrity, viz. Orthogonality and Propriety. The proposed definitions are based on the Modularity Matrix properties.

#### 3.1 Orthogonality

Orthogonality, as already stated (DeRosso and Jackson, 2013) in section 1.1 is "individual functions should be independent of one another". Based upon the Modularity Matrix, this definition has two associated meanings:

- **Linear independence** – among structors and among functionals;
- **Strict orthogonality** – among modules, which is also a consequence of linear independence, is easily visually recognized in the diagonal blocks of the Modularity Matrix.

So, orthogonality, from the Modularity Matrix, is totally consistent with the earlier intuitive formulation.

#### 3.2 Propriety

Propriety reflects the fact that the Modularity Matrix is the result of optimization of the number of linear independent structors and their provided functionals. In other words, linear independence of structors (and linear independence of the corresponding functionals), means that there are no superfluous structors, just the strictly necessary minimal number.

This is also consistent with the earlier intuitive formulation, viz. "a system should only have the functions essential to its purpose and no more".

See the Discussion sub-section 5.2 for considerations on “Generality”.

### 4 CONCEPTUAL INTEGRITY CHARACTERISTICS

In this section we go beyond the definitions based upon the Modularity Matrix, suggesting additional conceptual integrity characteristics that could sharpen the understanding the nature of conceptual integrity.

#### 4.1 Conceptual Integrity Is Intensive

Here we suggest that conceptual integrity, besides a property of a whole hierarchical software system, it should be a recursive property of each of its subsystems down to basic blocks. It is plausible that if any subsystem does not have conceptual integrity, the whole system cannot display it either.

Let us explain what are intensive versus extensive quantities by an example. Suppose that our system is a vehicle – a car or a truck. A family car typically has 4 wheels. A truck may have a bigger number of wheels.

The weight of a vehicle is an extensive quantity, since the weight of the system is the sum of the weights of its components. For instance, additional wheels increase the weight of the vehicle.

On the other hand, the speed of a vehicle is an intensive quantity. The speed of the system is not the sum of the speeds of its components. For instance, additional wheels increase the weight of the vehicle.

We claim that Conceptual Integrity is an intensive quantity. It is not the sum of the conceptual integrities of the components of a system.

#### 4.2 Increasing Conceptual Integrity by Components Exchange among Modules

Let us use another physical metaphor as a further illustration for the idea of Conceptual Integrity being intensive.
Assume a system composed of 4 sub-systems as in Fig. 7:

1. glass container;
2. water contained by the glass;
3. sphere mostly filled with air partially floating in the water;
4. small solid metal cube inside the sphere.

Heating the glass container by an external heat bath, despite the different thermal conducting properties of the sub-system materials (glass, water, air, metal), heat energy will flow between the different sub-systems from those with higher temperatures to those with lower temperatures, until the whole system reaches a uniform temperature.

The metaphor suggests that conceptual integrity is not an extensive property, like heat energy, but an intensive property, like temperature.

In a software system, each sub-system may have different computation characteristics – one dealing with data, another one with business logic, and so on. But, moving some concepts (classes) from one sub-system to another may increase conceptual integrity in both sub-systems. As a consequence, one could say that Conceptual Integrity in the whole system is optimized by flow of concepts (classes) among sub-systems. One should note, however, that such flow and the hypotheses of conceptual integrity being intensive, do not imply a single value of conceptual integrity throughout a whole software system.

5 DISCUSSION

We summarize the basic claim of this paper, discuss fundamental issues, consider future work on open issues and conclude with the main paper contribution.

5.1 Basic Claim

Conceptual Integrity has been up to now, on one hand been considered of fundamental importance for software system design, on the other hand, only has been vaguely defined.

The basic claim of this paper is that the Modularity Matrix is a source of a formally defined Conceptual Integrity. To this end we have provided two lines of argumentation:

a. Formal Path from Abstract Domains through the Modularity Matrix to Conceptual Integrity – We started from the accepted conceptual integrity of abstract mathematics, made a transition to generic software with the help of Liskov Substitution, whose meanings are conveyed by the Modularity Matrix to a restricted set of software systems. Using the equivalence to the Modularity Conceptual Lattice, we returned to "conceptual" aspects, to finally reach Conceptual Integrity.

b. Plausibility supported by the intuitive Principles behind Conceptual Integrity – We directly used the Modularity Matrix to obtain the intuitive principles in a formal way, viz. orthogonality and propriety.

Both these lines of argumentation deserve further considerations.

The "formal path" transitions were formulated into a series of reasonable theorems. But, in order to have a really formal path, these theorems demand rigorous demonstrations, or eventual reformulation of the theorems.

The intuitive principles – at least the two first ones, viz. orthogonality and propriety – have a very neat definition by the Modularity Matrix properties. We suggest inverting the situation: instead of trying to derive the principles from the Matrix, take the Modularity Matrix is the actual source of Conceptual Integrity.

We may summarize the current situation, stating that some promising progress has been achieved, but additional investigation is needed to further clarify the issues, as detailed in the next sub-section.
5.2 Fundamental Issues

a- Are hierarchies with conceptual integrity really independent?

We have referred in sub-section 2.3 to two independent hierarchies, one of polygons and another one of ellipses, say a circle. However, one may think of a circle as a regular polygon in the limit of an infinite number of sides, enabling a transition between two of the above hierarchies. One can easily estimate the value of $\pi$ in the perimeter of a circle $2\pi r\text{Radius}$ by taking the limit of the perimeter of a polygon inscribed in the circle, when the number of polygon sides goes to infinity.

b- Are conceptual hierarchies stable along time?

The situation is more complex than the naïve view of Fig. 4 would suggest. One could say that concepts evolve – see e.g. (Lakatos, 1976) in his book on “Proofs and Refutations” which discusses the empirical contribution to the concept evolution of regular polyhedrons (from the initial five of Euler). Concepts also can be said to expand along time – see e.g. (Buzaglo, 2002) according to the terminology of his book “The Logic of Concept Expansion”.

c- What is the origin of the conceptual integrity of major software systems?

Brooks in his books has defended the position that only a single brilliant mind, can provide conceptual integrity to a major work of art, say an architect of a cathedral, or similarly to a major engineering enterprise such as a very large software system.

Gabriel challenges Brooks’ position that a single mind is the best originator of Conceptual Integrity (Gabriel, 2007).

In our opinion, Brooks’ position is difficult to rationally prove for real systems. But its main drawback is that it leaves us depending on the existence and the opportunistic presence of a single brilliant mind. We obviously prefer a systematic construction of formal tools, based upon conceptual integrity ideas, as proposed in this paper.

d- Are the Liskov Substitution Principle specific problems detrimental to our argumentation?

We can mention two problems of the Liskov Substitution Principle. First, how to measure the lack of change of behavior for all programs $P$? Second, the well-known problem of setter functions for subclasses: for instance, if a 'Square' class has inherited from the 'Rectangle' class setter functions (like 'SetWidth' and 'SetHeigth'), independent application of these functions may distort a Square object into a Rectangle object. Thus, software inheritance has subtleties in addition to those within abstract mathematical sub-typing.

We claim that from the point of view of conceptual integrity we can ignore these subtle problems, and our argumentation remains valid.

e- What are the difficulties to formally interpret the Generality property of Conceptual Integrity?

Generality, has been described as the quality that "a single function should be usable in many ways" in the same system. This intuitive formulation seems so vague that its more formal interpretation is not so clear-cut.

One possible interpretation could be the repeated provision of the same functional by two different structors. This should not be allowed for the same functional in different modules. But, such interpretation, besides being obvious in case of inheritance among classes, is not an interesting contribution to the overall understanding of conceptual integrity.

5.3 Future Work

Open issues for future work include, more strictly formalization and more extensive investigation of the implications of the formalization, providing case studies, to exemplify the claims.

5.4 Main Contribution

The main contribution of this work is to propose a path to a formal definition of Conceptual Integrity, pointing to the Modularity Matrix as a possible source of such definition.

REFERENCES


