

Evolution of Generic Square Calculations in Cellular Automata

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Abstract: The paper deals with the design of uniform multi-state one-dimensional cellular automata using an evolutionary algorithm and their application to solve the problem of generic square calculations. The key idea is based on the representation of the transition functions for the automata, which utilises the concept of conditionally matching rules. This technique allows us to design complex cellular automata for which the conventional representations have failed. A study is proposed with various settings of the experimental system, which concerns the way of evaluating the candidate solutions, the number of cell states and the number of conditional rules of the transition functions. It is shown that various generic solutions for the square calculation can be obtained in one-dimensional cellular automata using local interactions of cells only. The results presented demonstrates an ability of the evolution to discover innovative solutions both from the view of complexity of the cellular automaton and the number of steps needed to calculate the results in comparison with the known solution.

1 INTRODUCTION

The problem of performing computations represents one of the typical tasks often investigated in relation with cellular automata (CAs). The concept of cellular automata was introduced by von Neumann in (von Neumann, 1966). One of the aspects widely studied in his work was the problem of (universal) computational machines and the question about their ability to make copies of themselves (i.e. to self-reproduce). Von Neumann proposed a model with 29 cell states to perform this task. Later Codd proposed another approach and showed that the problem of computation and construction can be performed by means of a simplified model working with 8 states only (Codd, 1968).

Several other researchers have dealt with this issue and studied cellular automata usually by means of various rigorous techniques. Sipper studied computational properties of *binary* cellular automata (i.e. those working with 2 cell states only) and proposed a concept of universal computing platform using a two-dimensional (2D) CA with non-uniform transition function (i.e. each cell can, in general, be controlled by a different set of transition rules) (Sipper, 1995). Sipper showed that, by introducing the non-uniform concept to the binary CAs, universal computation can be realised, which was not possible using the Codd's model. In fact, Sipper's work significantly reduced

the complexity of the CA in comparison with the models published earlier. Nevertheless even the binary uniform 2D CAs can be computationally universal if 9-cell neighbourhood is considered. Such CA was implemented using the famous rules of the Game of Life (Berlekamp et al., 2004) (original proof of the concept was published in 1982 and several times revisited – e.g. see (Durand and Rka, 1999)(Ilachinski, 2001)(Rendell, 2011)(Rendell, 2013)).

Although binary CAs may be advantageous due to simple elementary rules and hardware implementations in particular, many operations and real-world problems can effectively be solved rather by *multi-state* cellular automata (i.e. those working with more than 2 cell states). A technique for the construction of computing systems in a 2D CA was demonstrated in (Stefano and Navarra, 2012) using rules of a simple game called Scintillae working with 6 cell states. Computational universality was also studied with respect to one-dimensional (1D) CA, e.g. in (Lindgren and Nordahl, 1990)(Yuns, 2010).

However, in some cases application specific operations (algorithms) may be more suitable than programming a universal system, allowing to better optimize various aspects of the design (e.g. resources, efficiency, data encoding etc.). For example, Tempesti (Tempesti, 1995) and Perrier et al. (Jean-Yves Perrier, 1996) showed that specific arrangements of cell states can encode sequences of instructions (programs) to

denotes a state value. Each part ($cond_{\star} s_{\star}$) on the left of the arrow is evaluated with respect to the state of a specific cell in the neighbourhood (in this case c_W , c_C and c_E respectively). In this paper the relation operators $=$, \neq , \geq and \leq are considered as the condition functions. A finite sequence of CMRs represents a transition function. In order to determine the new state of a cell, the CMRs are evaluated sequentially. If a rule is found in which all conditions are true (with respect to the states in the cell neighbourhood), $s_{C_{new}}$ from this rule is the new state of the central cell. Otherwise the cell state does not change. For example, consider a transition function that contains a CMR $(\neq 1)(\neq 2)(\leq 1) \rightarrow 1$. Let c_W, c_C, c_E be states of cells in a neighbourhood with values 2, 3, 0 respectively, and a new state of the central cell ought to be calculated. According to the aforementioned rule, $c_W \neq s_W$ is true as $2 \neq 1$, similarly $c_C \neq s_C$ is true ($3 \neq 2$) and $c_E \leq s_E$ ($0 \leq 1$). Therefore, this CMR is said to match, i.e. $s_{C_{new}} = 1$ on its right side will update the state of the central cell.

Note that the evolved CMRs can be transformed to the conventional table rules (Bidlo, 2016). In this work the transformation is performed as follows: (1) For every possible combination of states $c_W c_C c_E$ in cellular neighborhood a new state $s_{C_{new}}$ is calculated using the CMR-based transition function. (2) If $c_C \neq s_{C_{new}}$ (i.e. the cell state ought to be modified), then a table rule of the form $c_W c_C c_E \rightarrow s_{C_{new}}$ is generated. Note that the combinations of states not included amongst the table rules do not change the state of the central cell, which is treated implicitly during the CA simulation. The number of such *generated rules* will represent a metrics indicating the complexity of the transition function.

In order to determine the complexity of the transition function with respect to a specific square calculation in CA, a set of *used rules* is created using the aforementioned principle whereas the combinations of states $c_W c_C c_E$ are considered just occurring during the given square calculation in the CA. These metrics (together with the number of states and CA steps) will allow us to compare the solutions obtained by the evolution and to identify the best results with respect to their complexity and efficiency.

An evolutionary algorithm will be applied to search for suitable CMR-based transition functions as described in the following section.

3 EVOLUTIONARY SYSTEM SETUP

In this paper a custom evolutionary algorithm (EA)

was utilised, which is a result of our long-term experimentation in this area. Note, however, that neither tuning of the EA nor in-depth analysis of the evolutionary process is a subject of this work. The EA is based on a simple genetic algorithm (Holland, 1975) with a tournament selection of base 4 and a custom mutation operator. Crossover is not used as it has not shown any improvement in success rate or efficiency of our experiments.

The EA utilises the following fixed-length representation of the conditionally matching rules in the genomes. For the purpose of encoding the condition functions $=$, \neq , \geq and \leq , integer values 0, 1, 2 and 3 will be used respectively. Each part ($cond_{\star} s_{\star}$) of the CMR is encoded as a single integer P_{\star} in the range from 0 to M where $M = 4 * S - 1$ (4 is the fixed number of condition functions considered and S is the number of cell states) and the part $\rightarrow s_{C_{new}}$ is represented by an integer in the range from 0 to $S - 1$. In order to decode a specific condition and state value, the following operations are performed: $cond_{\star} = P_{\star} / S$, $s_{\star} = P_{\star} \bmod S$ (note that $/$ is the integer division and \bmod is the modulo-division). This means that a CMR $(cond_W s_W)(cond_C s_C)(cond_E s_E) \rightarrow s_{C_{new}}$ can be represented by 4 integers; if 20 CMRs ought to be encoded in the genome, then $4 * 20 = 80$ integers are needed. For example, consider $S = 3$ for which $M = 4 * 3 - 1 = 11$. If a 4-tuple of integers (2 9 11 2) representing a CMR in the genome ought to be decoded, then the integers are processed respectively as:

- $cond_W = 2/3 = 0$ which corresponds to the operator $=$, $s_W = 2 \bmod 3 = 2$,
- $cond_C = 9/3 = 3$ which corresponds to the operator \leq , $s_C = 9 \bmod 3 = 0$,
- $cond_E = 11/3 = 3$ which corresponds to the operator \leq , $s_E = 11 \bmod 3 = 2$,
- $s_{C_{new}} = 2$ is directly represented by the 4th integer.

Therefore, a CMR of the form $(= 2)(\leq 0)(\leq 2) \rightarrow 2$ has been decoded.

The following variants of the fitness functions are treated (note that the input x is set to the middle of the cellular array):

1. **RESULT ANYWHERE (RA-fitness):** The fitness is calculated with respect to any valid arrangement (position) of the result sequence in the CA. For example, $y = 4$ in an 8-cell CA may be $rrrr0000$, $0rrrr000$, $00rrrr00$, $000rrrr0$ or $0000rrrr$, where $r \neq 0$ represent the result states that may be generally different within the result sequence. A partial fitness value is calculated for every possible arrangement of the result sequence as the sum of the number of cells in the expected state for the

given values of x . The final fitness is the highest of the partial fitness values.

2. **SYMMETRIC RESULT (SR-fitness):** The result is expected symmetrically with respect to the input. For example, if 0000011100000 corresponds to initial CA state for $x = 3$, then the result $y = 3^2$ is expected as a specific CA state 00rrrrrrrr00 (each r may be represented by any non-zero state). The fitness is the number of cells in the expected state.

The fitness evaluation of each genome is performed by simulating the CA for initial states with the values of x from 2 to 6. The result of the x^2 calculation is inspected after the 99th and 100th step of the CA, which allows to involve the state stability check into the evaluation. This approach was chosen on the basis of the maximal x evaluated during the fitness calculation and on the basis of the number of steps needed for the square calculation using the existing solution (Wolfram, 2002). In particular, the fitness of a fully working solution evaluated for x from 2 to 6 in a 100-cell CA is given by $F_{max} = 5 * 2 * 100 = 1000$ (there are 5 different values of x for which the result x^2 is investigated in 2 successive CA states, each consisting of 100 cells). The evolved transition functions, satisfying the maximal fitness for the given range of x , are checked for the ability to work in larger CAs for up to $x = 25$. For the purposes of this paper, the solutions which pass the check are considered as generic.

The EA works with a population of 8 genomes initialised randomly at the beginning of evolution. After evaluating the genomes, four candidates are selected randomly, the candidate with the highest fitness becomes a parent. An offspring is created by mutating 2 randomly selected integers in the parent. The selection and mutation continue until a new population of the same size is created and the evolutionary process is repeated until 2 million generations are performed. If a solution with the maximal fitness is found, then the evolutionary run is considered as successful. If no such solution is found within the given generation limit, then the evolutionary run is terminated.

4 EXPERIMENTAL RESULTS

The evolutionary design of CAs for the generic square calculation has been investigated for the following settings: the number of states 4, 6, 8 and 10, the transition functions consisting of 20, 30, 40 and 50 CMRs and two ways of the fitness calculation described in Section 3. For each setup, 100 independent evolutionary runs have been executed. The success rate and average number of generations needed to find a working

solution were observed with respect to the evolutionary process. As regards the parameters of the CA, the minimal number of rules and steps needed to calculate the square of x were determined.

4.1 Results for the RA-fitness

For the RA-fitness, the statistical results are summarised in Table 1. The table also contains the total numbers of generic solutions discovered for the given state setups and parameters determined for these solutions. For every number of states considered, at least one generic solution was identified. For example, a transition function was discovered for the 4-state CA, which consists of 36 table rules (transformed from the CMR representation evolved). This solution can be optimised to 26 rules (by eliminating the rules not used during the square calculation) which represents the simplest CA for generic square calculations known so far (note that Wolfram's CA works with 8 states and 51 rules (Wolfram, 2002)). Moreover, for example, our solution needs 74 steps to calculate 6^2 whilst Wolfram's CA needs 112 steps, which also represents a substantial innovation discovered by the EA. The CA development corresponding to this solution is shown in Figure 2.

Another result obtained using the RA-fitness is illustrated by the CA development in Figure 3. In this case the CA works with 6 states and its transition function consists of 52 effective rules. The number of steps needed, for example, to calculate 6^2 , is 46 (and compared to 112 steps of Wolfram's CA, it is an improvement of the CA efficiency by more than 50%) which represents the best CA known so far for this operation and the best result obtained in this paper.

One more example of evolved CA is shown in Figure 4. This generic solution was obtained in the setup with 8-state CA, however, the transition function works with 6 different states only. There are 49 transition rules, the CA needs 68 steps to calculate 6^2 . This means that the EA discovered a simpler solution (regarding the the number of states and table rules) which is a part of the solution space of the 8-state CA. Again, this result exhibits generally better parameters compared to the known solution from (Wolfram, 2002). The CA development, that was not observed in any other solution, is also interesting visually - as Fig. 4 shows, the CA generates a pattern with some "dead areas" (cells in state 0) within the cells that subsequently form the result sequence. The size of these areas is gradually reduced, which finally lead to derive the number of steps after which a stable state containing the correct result for the given x has emerged (illustrated by the right part of Figure 4 for

Table 1: Statistics of the evolutionary experiments conducted using the RA-fitness (the upper part of the table) and the parameters of the generic solutions (in the lower part of the table). The parameters of the best results obtained are marked **bold**. Note that # denotes “the number of”, the meaning of “generated rules”, “used rules” and ”steps” of the CA is defined in Section 2.

| the num. of CMRs | the number of states | | | | | | | | | | | | | | | |
|------------------|----------------------|-----------|------------|------------|------------|-----------|------------|------------|------------|-----------|------------|------------|------------|-----------|------------|------------|
| | 4 | | | | 6 | | | | 8 | | | | 10 | | | |
| | succ. rate | avg. gen. | min. steps | min. rules | succ. rate | avg. gen. | min. steps | min. rules | succ. rate | avg. gen. | min. steps | min. rules | succ. rate | avg. gen. | min. steps | min. rules |
| 20 | 3 | 844364 | 54 | 35 | 30 | 769440 | 45 | 120 | 45 | 570939 | 39 | 232 | 35 | 328210 | 47 | 569 |
| 30 | 3 | 620998 | 52 | 36 | 24 | 749837 | 40 | 120 | 38 | 595467 | 42 | 340 | 33 | 363360 | 45 | 663 |
| 40 | 2 | 1344286 | 77 | 46 | 19 | 629122 | 37 | 136 | 30 | 701612 | 41 | 365 | 29 | 244566 | 46 | 662 |
| 50 | 2 | 959689 | 73 | 43 | 20 | 813803 | 41 | 134 | 35 | 582342 | 39 | 348 | 38 | 373490 | 40 | 762 |

| #generic, parameters | the number of generic solutions (#generic) obtained for the given number of states and parameters of the generic solutions: #generated rules/#used rules/#steps for 6 ²) | | | |
|----------------------|--|---|--|------------------------------------|
| | 1 | 5 | 6 | 3 |
| | 36/26/74 | 176/52/46 , 164/33/87, 152/49/78, 185/66/70, 175/52/69 | 435/49/68, 403/51/79, 422/39/65, 392/62/76, 423/41/68, 429/94/76 | 934/64/56, 835/61/79, 916/35/76 |

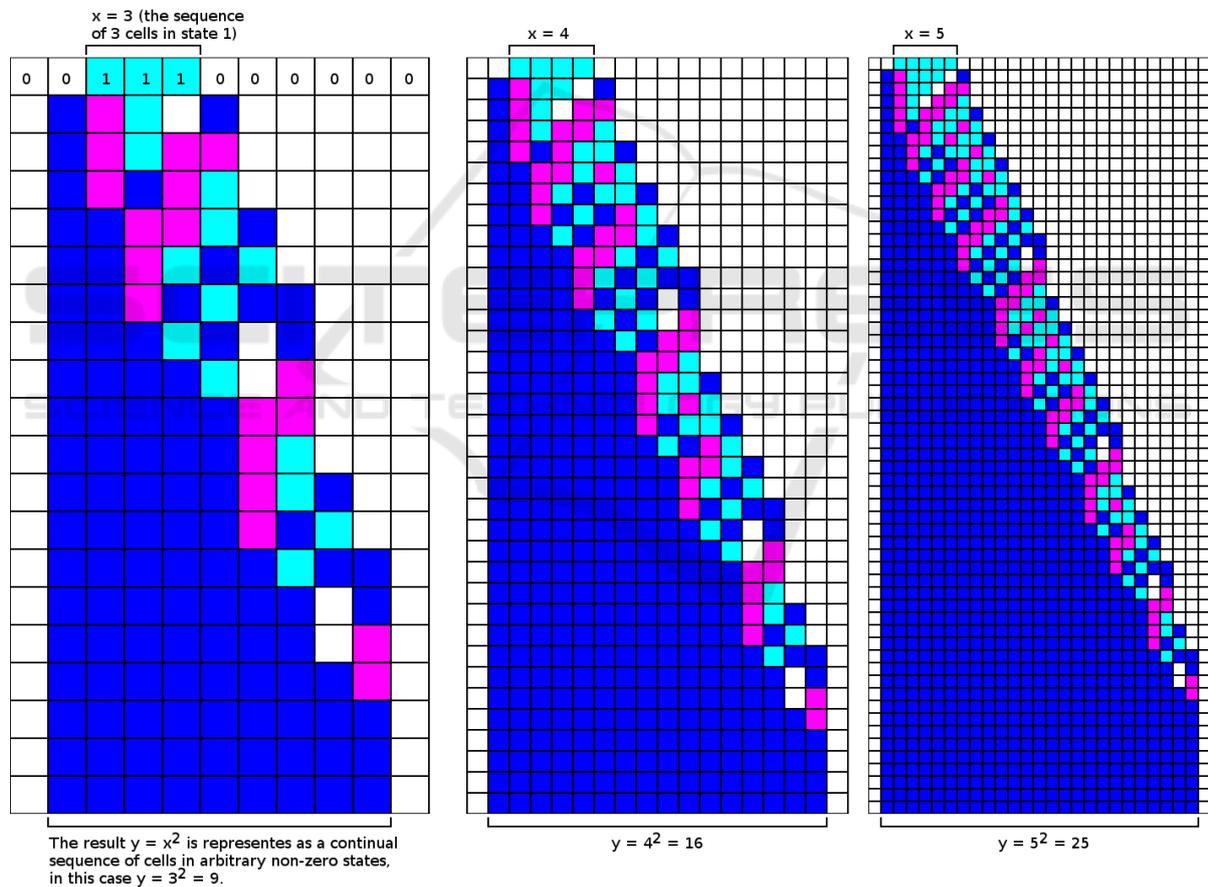


Figure 2: Example of a 4-state squaring CA development for $x = 3, 4$ and 5 using our most compact transition function. The rules are: $001 \rightarrow 3, 011 \rightarrow 2, 030 \rightarrow 2, 100 \rightarrow 3, 102 \rightarrow 2, 103 \rightarrow 2, 110 \rightarrow 0, 112 \rightarrow 2, 113 \rightarrow 2, 121 \rightarrow 1, 130 \rightarrow 1, 131 \rightarrow 1, 132 \rightarrow 2, 133 \rightarrow 0, 212 \rightarrow 3, 213 \rightarrow 3, 220 \rightarrow 1, 221 \rightarrow 1, 231 \rightarrow 1, 232 \rightarrow 2, 302 \rightarrow 3, 310 \rightarrow 3, 311 \rightarrow 3, 313 \rightarrow 3, 320 \rightarrow 3, 323 \rightarrow 3$.

$x = 8$ whereas the CA needs 122 steps to produce the result).

4.2 Results for the SR-fitness

Table 2 shows the statistics for the SR-fitness together with the total numbers of generic solutions discovered

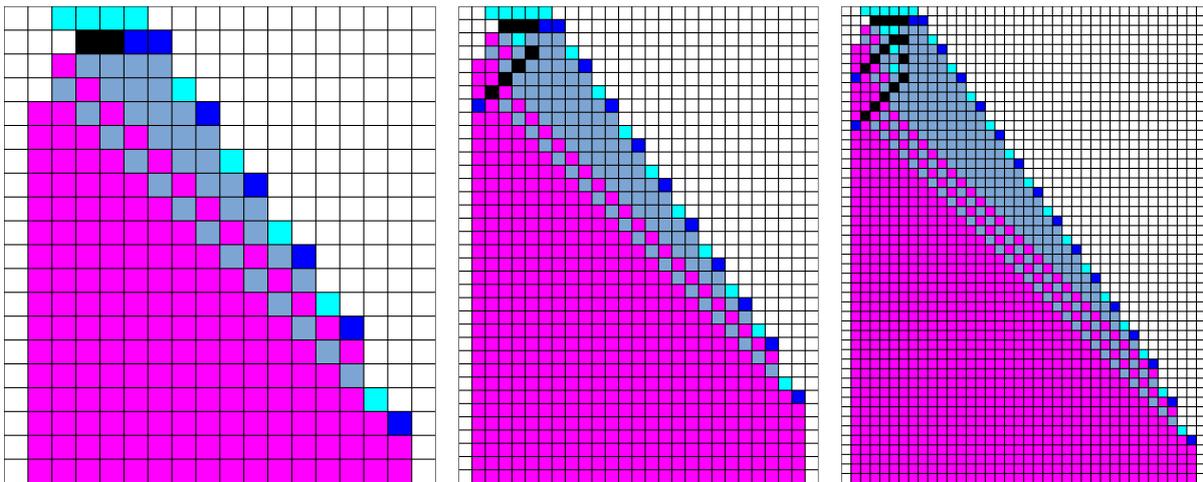


Figure 3: Example of a 6-state CA development for $x = 4, 5$ and 6 . This is the fastest CA-based (3-neighbourhood) solution known so far and the best result obtained in this paper. The rules are: $004 \rightarrow 2, 005 \rightarrow 2, 011 \rightarrow 0, 023 \rightarrow 0, 024 \rightarrow 4, 025 \rightarrow 3, 032 \rightarrow 2, 033 \rightarrow 0, 040 \rightarrow 2, 042 \rightarrow 2, 053 \rightarrow 0, 055 \rightarrow 4, 100 \rightarrow 3, 110 \rightarrow 3, 111 \rightarrow 5, 114 \rightarrow 5, 144 \rightarrow 5, 204 \rightarrow 3, 210 \rightarrow 2, 225 \rightarrow 5, 230 \rightarrow 2, 234 \rightarrow 2, 240 \rightarrow 2, 241 \rightarrow 2, 242 \rightarrow 2, 243 \rightarrow 2, 244 \rightarrow 2, 245 \rightarrow 5, 252 \rightarrow 2, 254 \rightarrow 2, 330 \rightarrow 4, 344 \rightarrow 2, 400 \rightarrow 1, 410 \rightarrow 4, 411 \rightarrow 4, 414 \rightarrow 4, 420 \rightarrow 4, 421 \rightarrow 4, 423 \rightarrow 4, 424 \rightarrow 4, 430 \rightarrow 4, 445 \rightarrow 1, 451 \rightarrow 4, 454 \rightarrow 4, 455 \rightarrow 1, 511 \rightarrow 4, 514 \rightarrow 4, 524 \rightarrow 4, 533 \rightarrow 4, 553 \rightarrow 4, 554 \rightarrow 4, 555 \rightarrow 1$.

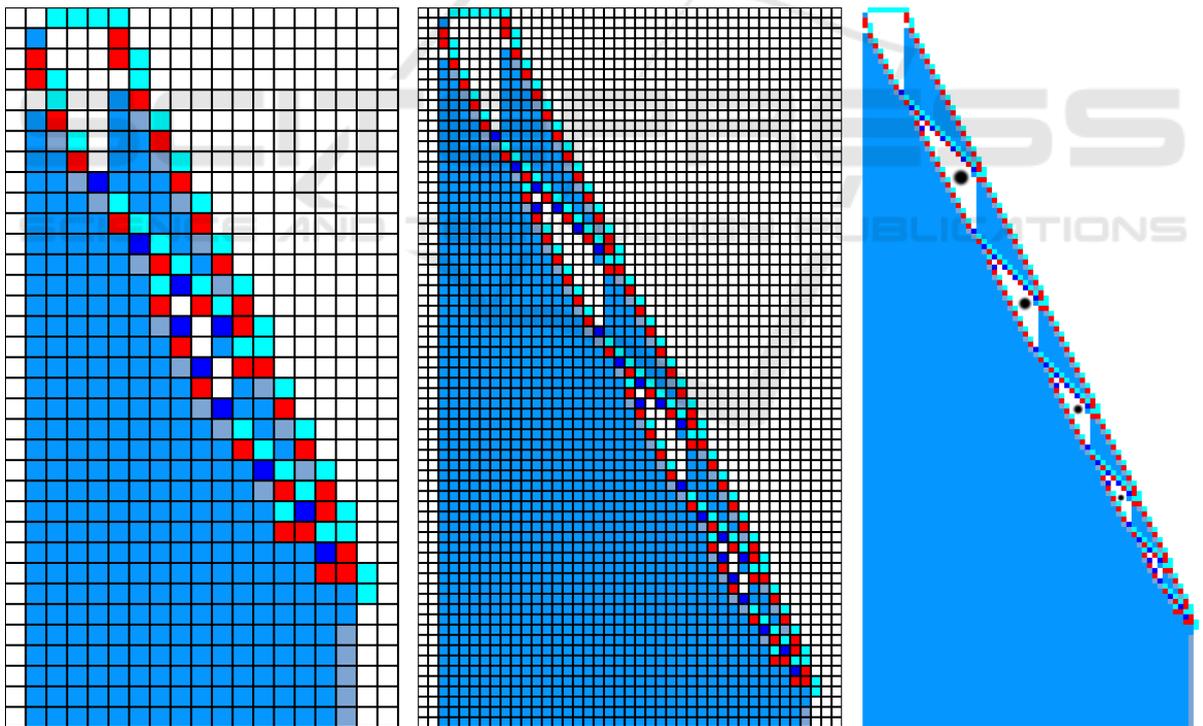


Figure 4: Example of a 6-state squaring CA development (originally designed using 8-state setup) for $x = 4, 6$ and 8 . The development shows a specific pattern evolved to derive the result of x^2 , which was not observed in any other solution. The part on the right shows a complete global behaviour of this CA for $x = 8$ with some “dead areas” (marked by black spots) which lead to the correct stable result by progressively reducing the size of these areas (in this case the result of 8^2 is achieved after 122 steps).

for the given state setups and parameters determined for these solutions. As evident, the success rates are

generally lower compared to the RA-fitness which is expectable because the SR-fitness allows a single

arrangement only of the result sequence in the CA. Moreover, just two generic CAs have been identified out of all the runs executed for this setup. However, the goal of this experiment was rather to determine whether solutions of this type ever exist for cellular automata and evaluate the ability of the EA to find them. As regards both generic solutions, their numbers of used rules and CA steps are significantly better in comparison with Wolfram's solution (Wolfram, 2002). Specifically, Wolfram's solution uses 51 rules and the calculation of 6^2 takes 112 steps, whilst the proposed results use 33, respective 36 rules and calculate 6^2 in 71, respective 78 steps. Moreover, one of them was discovered using a 4-state CA (Wolfram used 8 states), which belongs to the most compact solutions obtained in this paper and known so far.

Figure 5 shows examples of a CA (identified as generic) evolved using the SR-fitness. The transition function, originally obtained in 8-state CA setup, is represented by 36 used rules and works with 7 states only. Although this result cannot be considered as very efficient (for 6^2 the CA needs 78 steps), it exhibits one of the most complex emergent process obtained for the square calculation, the result of which is represented by a non-homogeneous state. The sample on the right of Fig. 5 shows a catout of development for $x = 11$ in which the global behaviour can be observed. This result demonstrates that the EA can produce generic solutions to a non-trivial problem even for a single specific position of the result sequence required by the SR-fitness evaluation.

5 DISCUSSION

In most cases of the experimental settings the EA was able to produce at least one generic solution for the CA-based square calculation. Despite the 2 million generation limit, the results from Table 1 and 2 show that the average number of generations is mostly below 1 million, which indicates a potential of the EA to efficiently explore the search space. In comparison with the initial study of this problem proposed in (Bidlo, 2016), where 200,000 generations were performed, the significant increase of this parameter herein is important with respect to achieving a reasonable success rate and producing generic solutions (note that an initial comparison of various ranges for x evaluated in the fitness was proposed in (Bidlo, 2016), the result of which was considered in this paper).

As regards the RA-fitness, which can be considered as the main technique proposed in this paper for the evolution of cellular automata, a more detailed analysis was performed with various multi-state CA.

As the results in Table 1 show that the number of generic solutions increases for the number of states from 4 to 8, then for 10-state CAs a significant reduction can be observed. This is probably caused by the exponential increase of the search space depending on the number of states. The results indicate that the 8-state setup represents a very feasible value that may be considered as sufficient for this kind of problem (note that 6 generic solutions were obtained for this setup).

In both sets of experiments with the RA-fitness and SR-fitness, a phenomenon of a reduction of the number of states was observed. This is possible due to the identification of just the rules that are needed for the CA development to calculate the square out of all the rules generated from the evolved CMR-based transition function for every valid combination of states in the cellular neighbourhood. It was determined that the CAs in some cases do not need all the available cell states to perform the given operation.

6 CONCLUSIONS

The goal of this paper was the evolutionary design of uniform cellular automata for the generic square calculation of integer numbers. The representation of the transition functions for the automata, which utilised conditionally matching rules, in combination with the evolutionary algorithm showed that it is possible to design various non-trivial transition functions for CAs (published herein for the first time) to perform the generic square calculations in the cellular space using just local interactions between cells. The analysis of the results showed a significant improvements of the evolved CA, which overcomes the existing solution regarding both the complexity of the transition functions and the number of steps (i.e. speed of the CA) needed to calculate the given operation. For example, a 4-state CA was discovered whose transition function consists of 26 rules only, which represents the most compact solution of this problem in the CA working with a 3-cell neighbourhood known so far. Another solution presented exhibits a reduction in the number of CA steps by more than 50% against the existing approach.

The method applied showed a potential to design complex CAs that exhibit the given generic behaviour. Although some different emergent processes in the resulting CAs were discovered, the detailed analysis of their behaviour and principles of functioning has not yet been done (long it was also not the goal of this paper). Moreover, the evolutionary process itself (regarding the details about exploring the solu-

Table 2: Statistics of the evolutionary experiments conducted using the SR-fitness (the upper part of the table) and the parameters of the generic solutions (in the lower part of the table). The parameters of the best result obtained are marked **bold**. Note that # denotes “the number of”, the meaning of “generated rules”, “used rules” and ”steps” of the CAs is defined in Section 2.

| the num. of CMRs | the number of states | | | | | | | | | | | | | | | |
|---|----------------------|--------------|---------------|---------------|---------------|--------------|---------------|---------------|----------------|--------------|---------------|---------------|---------------|--------------|---------------|---------------|
| | 4 | | | | 6 | | | | 8 | | | | 10 | | | |
| | succ. rate | avg. gen. | min. steps | min. rules | succ. rate | avg. gen. | min. steps | min. rules | succ. rate | avg. gen. | min. steps | min. rules | succ. rate | avg. gen. | min. steps | min. rules |
| 20 | 2 | 634948 | 71 | 38 | 4 | 734200 | 38 | 126 | 11 | 982446 | 34 | 234 | 18 | 855791 | 53 | 542 |
| 30 | 0 | - | - | - | 5 | 905278 | 48 | 150 | 17 | 934123 | 51 | 327 | 15 | 910269 | 35 | 742 |
| 40 | 1 | 1546681 | 79 | 45 | 4 | 928170 | 33 | 147 | 11 | 1033059 | 53 | 317 | 15 | 898314 | 52 | 748 |
| 50 | 0 | - | - | - | 3 | 989039 | 44 | 138 | 12 | 811686 | 32 | 380 | 17 | 861850 | 52 | 796 |
| the number of generic solutions (#generic) obtained for the given number of states and parameters of the generic solutions: #generated rules/#used rules/#steps for 6 ²) | | | | | | | | | | | | | | | | |
| #generic, parameters | 1 38/33/71 | | | | 0 | | | | 1 234/36/78 | | | | 0 | | | |

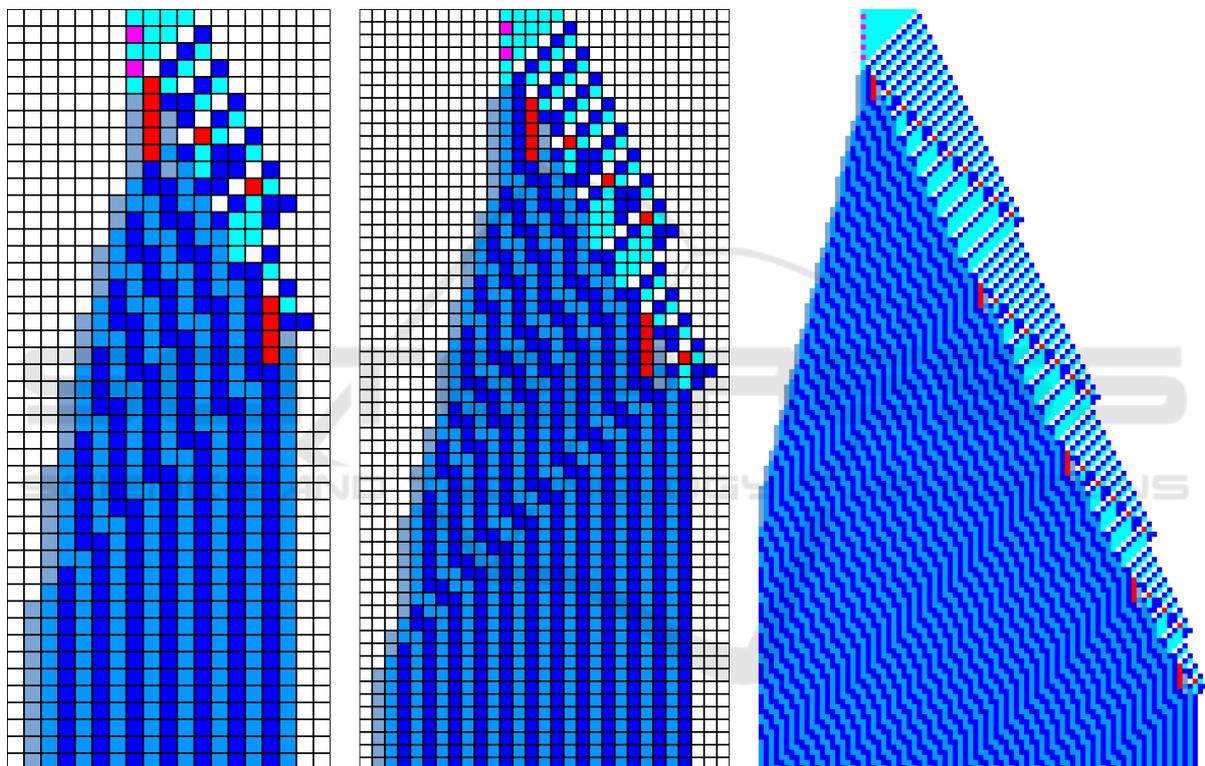


Figure 5: Example of a 7-state CA controlled by a transition function evolved using the SR-fitness. A complete development is shown for $x = 4$ and 5 (the left and middle sample respectively), the part on the right demonstrates a cutout of global behaviour of the CA for $x = 11$.

tion space using the CMR approach and optimal evolutionary setup) still represents a topic with the work in progress. Therefore these issues are considered in our future research.

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REFERENCES

Berlekamp, E. R., Conway, J. H., and Guy, R. K. (2004). *Winning Ways for Your Mathematical Plays, 2nd Ed., Volume 4*. A K Peters/CRC Press.
 Bidlo, M. (2015). Investigation of replicating tiles in cel-

- lular automata designed by evolution using conditionally matching rules. In *2015 IEEE International Conference on Evolvable Systems (ICES)*, Proceedings of the 2015 IEEE Symposium Series on Computational Intelligence (SSCI), pages 1506–1513. IEEE Computational Intelligence Society.
- Bidlo, M. (2016). On routine evolution of complex cellular automata. *IEEE Transactions on Evolutionary Computation*, PP(99):1–13, URL: <http://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=7377086&isnumber=4358751>.
- Bidlo, M. and Vasicek, Z. (2013). Evolution of cellular automata with conditionally matching rules. In *2013 IEEE Congress on Evolutionary Computation (CEC 2013)*, pages 1178–1185. IEEE Computer Society.
- Codd, E. F. (1968). *Cellular Automata*. Academic Press, New York.
- Durand, B. and Rka, Z. (1999). The game of life: Universality revisited. In *Mathematics and Its Applications, Volume 460 Cellular Automata*, pages 51–74. Springer Netherlands.
- Holland, J. H. (1975). *Adaptation in Natural and Artificial Systems*. University of Michigan Press, Ann Arbor.
- Ilachinski, A. (2001). *Cellular Automata: A Discrete Universe*. World Scientific.
- Jean-Yves Perrier, Moshe Sipper, J. Z. (1996). Toward a viable, self-reproducing universal computer. *Physica D*, 97(4):335–352.
- Lindgren, K. and Nordahl, M. G. (1990). Universal computation in simple one-dimensional cellular automata. *Complex Systems*, 4(3):299–318.
- Mardiris, V., Sirakoulis, G., and Karafyllidis, I. (2015). Automated design architecture for 1-d cellular automata using quantum cellular automata. *Computers, IEEE Transactions on*, 64(9):2476–2489.
- Ninagawa, S. (2013). Solving the parity problem with rule 60 in array size of the power of two. *Journal of Cellular Automata*, 8(3–4):189–203.
- Rendell, P. (2011). A universal turing machine in Conway’s game of life. In *2011 International Conference on High Performance Computing and Simulation (HPCS)*, pages 764–772.
- Rendell, P. (2013). A fully universal turing machine in Conway’s game of life. *Journal of Cellular Automata*, 9(1–2):19–358.
- Sahoo, S., Choudhury, P. P., Pal, A., and Nayak, B. K. (2014). Solutions on 1-d and 2-d density classification problem using programmable cellular automata. *Journal of Cellular Automata*, 9(1):59–88.
- Sahu, S., Oono, H., Ghosh, S., Bandyopadhyay, A., Fujita, D., Peper, F., Isokawa, T., and Pati, R. (2010). Molecular implementations of cellular automata. In *Cellular Automata for Research and Industry*, Lecture Notes in Computer Science, Vol. 6350, pages 650–659. Springer.
- Sipper, M. (1995). Quasi-uniform computation-universal cellular automata. In *Advances in Artificial Life, ECAL 1995, Lecture Notes in Computer Science, Vol. 929*, pages 544–554. Springer Berlin Heidelberg.
- Sridharan, K. and Pudi, V. (2015). *Design of Arithmetic Circuits in Quantum Dot Cellular Automata Nanotechnology*. Springer International Publishing Switzerland.
- Stefano, G. D. and Navarra, A. (2012). Scintillae: How to approach computing systems by means of cellular automata. In *Cellular Automata for Research and Industry*, Lecture Notes in Computer Science, Vol. 7495, pages 534–543. Springer.
- Tempesti, G. (1995). A new self-reproducing cellular automaton capable of construction and computation. In *Advances in Artificial Life, Proc. 3rd European Conference on Artificial Life*, Lecture Notes in Artificial Intelligence, Vol. 929, pages 555–563. Springer.
- von Neumann, J. (1966). *The Theory of Self-Reproducing Automata*. A. W. Burks (ed.), University of Illinois Press.
- Wolfram, S. (2002). *A New Kind of Science*. Wolfram Media, Champaign IL.
- Yuns, J.-B. (2010). Achieving universal computations on one-dimensional cellular automata. In *Cellular Automata for Research and Industry*, Lecture Notes in Computer Science Volume 6350, pages 660–669. Springer.