Verifying and Mapping the Mereotopology of Upper-level Ontologies

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Keywords: DOLCE, DOLCE-CORE, SUMO, Ontology Mapping, Ontology Verification, Upper-level Ontology, Mereology, Topology, Mereotopology.

Abstract: Upper-level ontologies provide an account of the most basic, domain-independent, existing entities, such as time, space, objects, and processes. *Ontology verification* is the process by which a theory is checked to rule out unintended models, and possibly characterize missing intended ones. In this paper, we verify the core characterization of mereotopology of the Suggested Upper Merged Ontology (SUMO), and the mereology of the Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE), while relating their axiomatizations via *ontology mapping*. As a result, we propose the correction and addition of some axioms to the analyzed theories which eliminate unintended models and characterize missing ones. In addition, we show by formal means which is the relation existing between the axiomatization of mereology in both upper-level ontologies, and make available a modular representation in first-order logic of the SUMO characterization of mereotopology.

1 INTRODUCTION

Automatic applications appealing to ontologies for interoperation are unambiguously integrated only when the models of their shared features are equivalent. However, ontologies admitting unintended models ambiguously characterize their vocabularies, which can generate misunderstandings that hinder interoperability.

Upper-level ontologies, also called *foundational* ontologies, provide an account of the most basic, domain independent, existing entities, such as time, space, objects and processes. As ontologies are crucial for the Semantic Web, upper level ontologies are essential for the ontology engineering cycle in activities such as ontology building and integration. Upper level ontologies can be used as the foundational substratum on which new ontologies are developed, because they provide some fundamental ontological distinctions, which can help the designer in her task of conceptual analysis, (Guarino, 1998). They can be used as a backbone on top of which more specific concepts can be characterized while reusing their root vocabulary and their general knowledge. In ontology integration, they can be used as oracles for meaning clarification (Euzenat and Shvaiko, 2013).

Various upper level ontologies have been developed in languages with higher or equivalent expressivity to first-order logic, such as SUMO (Niles and Pease, 2001) and DOLCE (Gangemi et al., 2002)(Borgo and Masolo, 2009), and translations of them, with loss, to lightweight language OWL[1], made available. Therefore, semantic mappings connecting their axiomatizations are necessary to facilitate interoperability among applications that commit to the characterizations provided by different upper level ontologies. Those mappings need to be formal, which guarantees their interpretability by automatic agents, and also need to be represented in an expressive language such as standard first-order logic.[2]

Ontology verification (Grüninger et al., 2010) is the process by which a theory is checked to rule out unintended models, and possibly characterize missing intended ones. Therefore, ontology verification

[1]https://www.w3.org/2001/sw/wiki/OWL

[2]The expressive power of first-order logic makes its use necessary for the representation of mappings that characterize features that are not representable in lightweight languages, such as Description Logics. In addition, checking the correctness of those mappings results facilitated by the fact that first-order theorem proving in standard first-order logic is a mature field, and, although semi-decidable, first-order reasoning on small modules results in an acceptable trade-off among expressivity and efficiency.

Muñoz, L. and Grüninger, M.
Verifying and Mapping the Mereotopology of Upper-Level Ontologies.
DOI: 10.5220/0006052100310042
In *Proceedings of the 8th International Joint Conference on Knowledge Discovery, Knowledge Engineering and Knowledge Management (IC3K 2016) - Volume 2: KEOD*, pages 31-42
ISBN: 978-989-758-203-5

reduces semantic ambiguity. Since foundational ontologies are expected to be broadly reused, their verification results necessary.

In this paper, we verify the subtheory of core mereotopological concepts of the SUMO foundational ontology and the mereology of the DOLCE-CORE, the fragment of DOLCE focused on entities that exist on time. In addition, we formally relate their respective axiomatizations via first-order logic mappings. As a result, we propose the correction, and addition, of some axioms which rule out unintended models or characterize missing ones. As an additional outcome of our work, we have produced a modular representation stated in standard first-order logic of the complete SUMO subtheory of mereotopology. We have used automatic theorem prover Prover9 and model finder Mace4 (McCune, 2010) for the automatic tasks involved in the work described in this paper.

2 ONTOLOGY MAPPING

Ontology mapping, also called *ontology matching*, and *ontology alignment*, is concerned with the explicit representation of the existing semantic correspondences among the axiomatizations of different ontologies[3] via *bridge axioms* (Euzenat and Shvaiko, 2013), which are called *translations definitions* in the context of first-order logic.

Building a map between two first-order logic ontologies T_1 and T_2 that interprets the first into the second involves translating every symbol of theory T_1 into the language of T_2, translating every sentence of T_1 into the language of T_2, and checking the ability of T_2 to entail every axiom of T_1. The following definition formalizes the notion of relative interpretation between first-order logic theories.

Definition 1. *A map π interprets a theory T_1 into a theory T_2 iff for every sentence α in the language of T_1, $T_1 \models \alpha \Rightarrow T_2 \models \alpha^\pi$; being α^π the syntactic translation of α into the language of T_2.*

The following theorem that follows fom (Enderton, 1972), introduces a fundamental relation between the models of a theory and the models of the theories that it interprets. Given such a relation, in order to demonstrate that a given theory T_2 can represent every feature that another theory T_1 represents, it suffices to demonstrate that theory T_2 is able to interpret theory T_1.

Theorem 1. *If a theory T_1 is interpreted by a theory T_2 by means of a given map π, there is another map δ that sends every model of T_2 into a model of T_1.*

3 ONTOLOGY VERIFICATION

An ontology admits unintended models when it is possible to find features of its underlying conceptualization which are not characterized by its axiomatization. *Ontology Verification* in first-order logic (Grüninger et al., 2010) is based on the fact that theories with different vocabularies unambiguously characterize the same concepts only if their sets of models are equivalent. Verifying an ontology T ideally consists of classifying the actual models $\mathfrak{M}$ of T by means of a *representation theorem*,[4] which relates the models of T with the models $\mathfrak{M}^{intended}$ of an alternative axiomatization of T built with well understood theories. Such a representation theorem must be either proved or disproved. The following definition from (Pearce and Valverde, 2012) relates the notion of *ontology mapping* with the fundamentals of *ontology verification*:

Definition 2. *Two theories T_1 and T_2 are **synonymous** iff there exist two sets of translation definitions Δ and Π, respectively from T_1 to T_2 and from T_2 to T_1, such that $T_1 \cup \Pi$ is logically equivalent to $T_2 \cup \Delta$.*

Given Definition 2, from Theorem 1 follows that the models of synonymous theories are equivalent, and therefore *ontology mapping* can be used for classifying the sets of models of two ontologies as equivalent.

4 DOLCE

The Descriptive Ontology for Linguistic and Cognitive Engineering DOLCE (Gangemi et al., 2002) (Masolo et al., 2003) is a freely available upper ontology that is part of the WonderWeb project[5], which is aimed to provide the infrastructure required for a large-scale deployment of ontologies intended to be

[3]We assume that an ontology is a set of sentences called *axioms* closed under logical entailment that state the properties that characterize the behaviour of a set of symbols representing constants, relations and functions, called the *signature* of the ontology.

[4]A *representation theorem* is a theorem that formally classifies a given class of structures as equivalent to another class of structures whose properties are better understood. The stated equivalence makes possible the extrapolation of those properties to the classified structures, facilitating their understanding.

[5]http://wonderweb.semanticweb.org

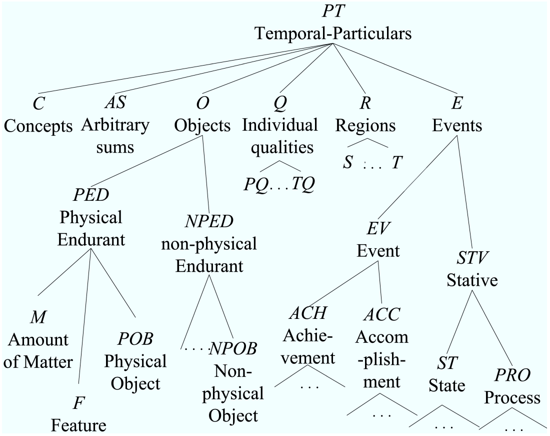

Figure 1: Top categories of DOLCE-CORE.

the foundation for the Semantic Web. DOLCE has a cognitive approach, i.e., it presents the world as it is grasped by humans, based on human knowledge and culture, in opposition to *ontological realism* (Grenon and Smith, 2004), which intends to present the world as it is, independently of the bias of human perception. The development of DOLCE has followed the principles of the OntoClean methodology (Guarino and Welty, 2002). The first version of DOLCE had a representation in Modal Logic, a translation with loss into standard first-order logic, a translation with further loss into OWL, and also an alignment with WordNet (Gangemi et al., 2003). A new version of the fragment of the original ontology that focuses on entities that exist on time, called *temporal particulars*, was presented in (Borgo and Masolo, 2009), called DOLCE-CORE, whose main categories are shown in Figure 1. We will circumscribe our work to the axiomatization of DOLCE-CORE.

At the top of DOLCE-CORE the category of temporal-particulars PT is partitioned into six basic categories: objects O, events E, individual qualities Q, regions R, concepts C, and arbitrary sums AS. Categories ED $(endurant)$ and PD $(perdurant)$ of DOLCE were, respectively, renamed O $(object)$ and E $(event)$ in DOLCE-CORE. The axiomatization of mereology in DOLCE-CORE is as follows,[6] where predicate P represents parthood, and (1)-(3) respectively stand for the reflexivity, transitivity, and antisymmetry of relation P. Overlap of parts and mereological sum representing binary fusion of parts are respectively defined in (4) and (5), while (7)-(11) characterize the dissectivity of P across categories, and (12)-(17) close the sum of parts inside each category.

$$(\forall x)P(x,x) \tag{1}$$

$$(\forall x,y)P(x,y)\wedge P(y,z)\rightarrow P(x,z) \tag{2}$$

$$(\forall x,y)P(x,y)\wedge P(y,x)\rightarrow (x=y) \tag{3}$$

$$(\forall x,y)Ov(x,y)\equiv(\exists z)(P(z,x)\wedge P(z,y)) \tag{4}$$

$$\begin{gathered}(\forall x,y,z)SUM(z,x,y)\equiv\\(\forall v)Ov(v,z)\leftrightarrow Ov(v,x)\vee Ov(v,y)\end{gathered} \tag{5}$$

$$(\forall x,y)\neg P(x,y)\rightarrow(\exists z)P(z,x)\wedge\neg Ov(z,y) \tag{6}$$

$$(\forall x,y)O(y)\wedge P(x,y)\rightarrow O(x) \tag{7}$$

$$(\forall x,y)E(y)\wedge P(x,y)\rightarrow E(x) \tag{8}$$

$$(\forall x,y)T(y)\wedge P(x,y)\rightarrow T(x) \tag{9}$$

$$(\forall x,y)TQ(y)\wedge P(x,y)\rightarrow TQ(x) \tag{10}$$

$$(\forall x,y)C(y)\wedge P(x,y)\rightarrow C(x) \tag{11}$$

$$(\forall x,y,z)O(x)\wedge O(y)\wedge SUM(z,x,y)\rightarrow O(z) \tag{12}$$

$$(\forall x,y,z)E(x)\wedge E(y)\wedge SUM(z,x,y)\rightarrow E(z) \tag{13}$$

$$(\forall x,y,z)T(x)\wedge T(y)\wedge SUM(z,x,y)\rightarrow T(z) \tag{14}$$

$$(\forall x,y,z)TQ(x)\wedge TQ(y)\wedge SUM(z,x,y)\rightarrow TQ(z) \tag{15}$$

$$(\forall x,y,z)C(x)\wedge C(y)\wedge SUM(z,x,y)\rightarrow C(z) \tag{16}$$

$$(\forall x,y,z)AS(x)\wedge AS(y)\wedge SUM(z,x,y)\rightarrow AS(z) \tag{17}$$

Due to the ontological commitment represented by axiom (6), the mereology characterized in DOLCE-CORE is an *extensional mereology*[7] according to (Casati and Varzi, 1999) (Varzi, 2007).

5 SUMO

SUMO (Niles and Pease, 2001) is a freely available upper level ontology whose top categories are shown in Figure 2. Like DOLCE, SUMO has a cognitive bias. In addition to the main ontology, which contains about 4000 axioms, SUMO has been extended with a mid-level ontology and a number of domain specific ontologies, all of which account for 20,000 terms and 70,000 axioms. SUMO has been translated into OWL and WordNet (Niles and Pease, 2003). The representation language of SUMO is SUO-KIF[8], a very expressive dialect of KIF[9] with many-sorted features, whose syntax permits higher-order constructions such as predicates that have other predicates, or formulas, as their arguments, and the existence of predicates

[6] Axioms (9), (10), (14), and (15) are the instantiation of DOLCE higher-order axiom schemas for the subcategories of main categories Q and R which are relevant for our work. A complete version of DOLCE-CORE mereology represented in first-order logic is available at colore.oor.net/ontologies/dolce-core/mereology.in

[7] It can be proved that in an *extensional mereology* non-atomic entities whose proper parts are the same, are identical, i.e., every entity is exhaustively defined by its parts.

[8] http://suo.ieee.org/SUO/KIF/suo-kif.html

[9] http://logic.stanford.edu/kif/kif.html

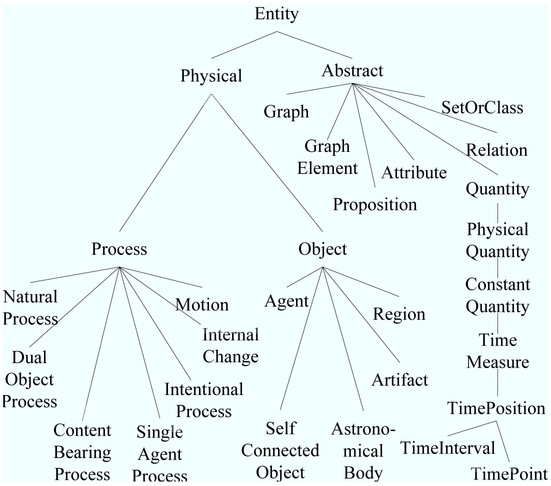

Figure 2: Top categories of SUMO.

and functions of variable arity (Benzmüller and Pease, 2012).

We have translated (with loss) into standard first-order logic, and modularized, the subset of SUMO that characterizes the notion of mereotopology, which resulted in the hierarchy of subtheories shown in Figure 3, where each theory conservatively extends[10] its related theories below. Due to space limitations, we only address in this work the study of modules PART, SUM, PRODUCT, DECOMPOSITION, TOPOLOGY, and MEREOTOPOLOGY. The first-order logic axiomatization of all the modules shown in Figure 3 can be found at colore.oor.net/ontologies/sumo/modules.

Differently from DOLCE-CORE, which defines parthood by means of unique relation P across every category representing entities that exist on time and space, SUMO adopts various partial orderings to address the part-whole relationship in different categories. Regarding entities that are in space and time, classified as *Physical* in SUMO, relations *part* and *subProcess* respectively characterize part-whole relations for members of *Object* and *Process*, while relation *temporalPart* represents part-whole for members of *TimePosition*, which extends to points and intervals of time.

5.1 Module PART

Module *PART* represents the relation among a whole and its parts by characterizing relation *part* as a partial order, and defines the overlapping of parts, partial overlapping, and relation *properPart*. Given the axiomatization of *part*, relation *properPart* results to be a strict partial order.

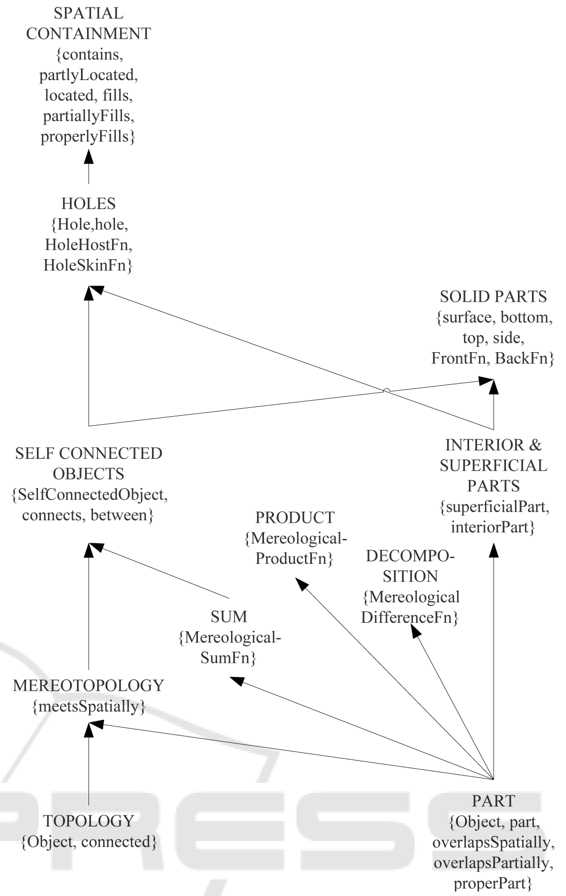

Figure 3: Modular decomposition of the SUMO axiomatization of concepts related to mereotopology. Arrows point to conservative extensions[10] among modules. Signature members are shown in the module that first introduces them.

Definition 3. *Module PART is the subtheory composed by axioms (18) to (24).*

$$(\forall x,y) part(x,y) \rightarrow Object(x) \land Object(y) \quad (18)$$

$$(\forall x) Object(x) \rightarrow part(x,x) \quad (19)$$

$$(\forall x,y) part(x,y) \land part(y,x) \rightarrow (x = y) \quad (20)$$

$$(\forall x,y,z) part(x,y) \land part(y,z) \rightarrow part(x,z) \quad (21)$$

$$(\forall x,y) overlapsSpatially(x,y) \leftrightarrow (\exists z (part(z,x) \land part(z,y))) \quad (22)$$

$$(\forall x,y) overlapsPartially(x,y) \leftrightarrow \neg part(x,y) \land \neg part(y,x) \land (\exists z) part(z,x) \land part(z,y) \quad (23)$$

$$(\forall x,y) properPart(x,y) \leftrightarrow part(x,y) \land \neg part(y,x) \quad (24)$$

5.2 Module SUM

The mereological sum of two parts to conform a whole is represented in module *SUM* by function symbol $MereologicalSumFn$.

[10]A theory T' is a *conservative extension* of a theory T if every theorem of T is a theorem of T', and every theorem of T' in the signature of T is also a theorem of T.

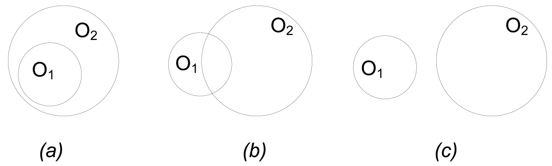

Figure 4: With the original characterization of mereological sum, every two objects in every model of SUMO must be in relation *part*, such as objects O_1 and O_2 in *(a)*. Models corresponding to *(b)* and *(c)* with overlapping objects without being one part of the other, or with disjoint objects, are not admitted by SUMO submodule SUM.

Definition 4. *Module SUM is the subtheory that extends module PART by means of axioms (25) and (26).*

$$\begin{aligned}&(\forall x,y,z) Object(x) \land Object(y) \rightarrow \\ &((z = MereologicalSumFn(x,y)) \rightarrow \\ &(\forall p)(part(p,z) \leftrightarrow (part(p,x) \lor part(p,y)))\end{aligned} \tag{25}$$

$$\begin{aligned}&(\forall x,y) Object(x) \land Object(y) \rightarrow \\ &Object(MereologicalSumFn(x,y))\end{aligned} \tag{26}$$

Given two objects, the existence of their mereological sum is vacuously guaranteed in this theory due to the use of a function symbol to represent such an operation.

We have found that due to the reflexivity property of relation *part*, there exists always a part p of the object indicated by variable z in axiom (25), which is z itself, for which $(part(p,x) \lor part(p,y)$ holds. Therefore, the following is a theorem of SUMO:[11]

$$\begin{aligned}&(\forall x,y,z) Object(x) \land Object(y) \land \\ &(z = MereologicalSumFn(x,y)) \rightarrow \\ &(part(z,x) \lor part(z,y)\end{aligned} \tag{27}$$

Also due to the reflexivity of relation *part* and axiom (25), both arguments x and y must be a part of their mereological sum z, and the following is also a theorem of SUMO:[11]

$$\begin{aligned}&(\forall x,y,z) Object(x) \land Object(y) \land \\ &(z = MereologicalSumFn(x,y)) \rightarrow \\ &(part(x,z) \land part(y,z)\end{aligned} \tag{28}$$

Given theorems (27) and (28), and due to the antisymmetry of relation *part*, it holds that z must be x or y, this fact entails the inconvenient consequence that every pair of objects in the universe of every interpretation of SUMO must be in relation *part*, which shows that SUMO misses intended models where there exist objects that are disjoint, or that overlap without being one part of the other, as depicted in parts *(b)* and *(c)* of Figure 4. The following proposition proves our claim:

[11]Proof available at: colore.oor.net/ontologies/sumo/mereotopology/proofs.

Proposition 1. $SUM \models (\forall x,y) Object(x) \land Object(y) \rightarrow (part(x,y) \lor part(y,x)$.

Proof. By using Prover9, we have produced a proof for this proposition.[11] □

In order to characterize those missing models that Proposition 1 identifies, we propose the substitution of axiom (25) by sentence (29) in module *SUM*, which corresponds to the representation of the sumpremum, or *join* of lattices (Davey and Priestley, 2002), where the partial order is given by the relation *part*. We call *EXTENDED SUM* to the resulting theory, and prove that it does not rule out intended models where objects exist that overlap or are disjoint, and also that the characterization of mereological sum satisfies the commutative and idempotence laws.

$$\begin{aligned}&(\forall x,y,z) Object(x) \land Object(y) \rightarrow \\ &((z = MereologicalSumFn(x,y) \rightarrow \\ &(\forall p)(part(z,p) \leftrightarrow part(x,p) \land part(y,p)))\end{aligned} \tag{29}$$

Proposition 2. *Let $EXTENDED\ SUM$ be the theory that results from substituting axiom (25) in module SUM by axiom (29). Then,*
(a) EXTENDED SUM $\not\models (\forall x,y) Object(x) \land Object(y) \rightarrow (part(x,y) \lor part(y,x)$
(b) EXTENDED SUM $\models (\forall x,y,z) Object(x) \land Object(y) \land Object(z) \land (MereologicalSumFn(x,y) = z) \rightarrow (MereologicalSumFn(y,x) = z)$
(c) EXTENDED SUM $\models (\forall x,y,z) part(x,y) \rightarrow (MereologicalSumFn(x,y) = y)$

Proof. (a): Let S_1 be the theory that results from adding sentence $(\exists x,y) Object(x) \land Object(y) \land \neg(part(x,y) \land \neg part(y,x)$ to module $EXTENDED\ SUM$. By using Mace4, we have created a model of S_1.[11]
(b);(c): By using Prover9 we have demonstrated that sentences $(\forall x,y,z) Object(x) \land Object(y) \land Object(z) \land (MereologicalSumFn(x,y) = z) \rightarrow (MereologicalSumFn(y,x) = z)$ and $(\forall x,y,z) part(x,y) \rightarrow (MereologicalSumFn(x,y) = y)$ are theorems of theory $EXTENDED\ SUM$.[11] □

5.3 Module PRODUCT

Given two objects, its mereological product intuitively corresponds to the intersection of both objects. SUMO represents the notion of mereological product by means of function symbol $MereologicalProductFn$.

Definition 5. *Module PRODUCT is the subtheory that extends module PART by means of axioms (30) and (31).*

$$\begin{aligned}&(\forall x,y,z) Object(x) \land Object(y) \rightarrow \\ &((z = MereologicalProductFn(x,y)) \rightarrow \\ &(\forall p)(part(p,z) \leftrightarrow part(p,x) \land part(p,y)))\end{aligned} \tag{30}$$

$$(\forall x,y)Object(x) \wedge Object(y) \rightarrow Object(MereologicalProductFn(x,y)) \tag{31}$$

Given two objects, the existence of their mereological product is vacuously guaranteed in SUMO due to the use of a function symbol to represent such an operation.

The characterization of mereological product in SUMO corresponds to the *infimum* or *meet* of the corresponding arguments on the lattice that relation *part* defines. We have found that from the characterization of mereological product of SUMO follows that every pair of objects (x,y) must overlap, which indicates that SUMO misses those intended models where there exist objects that do not overlap. The following proposition proves our claim:

Proposition 3. $PRODUCT \models (\forall x,y)Object(x) \wedge Object(y) \rightarrow (overlapsSpatially(x,y))$.

Proof. By using Prover9, we have produced a proof for this proposition.[11] □

In order to make possible the admission of those missing models that Proposition 3 identifies, we propose substitutig axiom (30) by sentence (32), and call *EXTENDED PRODUCT* to the resulting theory:

$$\begin{aligned}&(\forall x,y,z)overlapsSpatially(x,y) \rightarrow\\ &((z = MereologicalProductFn(x,y)) \rightarrow\\ &(\forall p)(part(p,z) \leftrightarrow part(p,x) \wedge part(p,y)))\end{aligned} \tag{32}$$

Proposition 4. *Let EXTENDED PRODUCT be the theory that results from substituting axiom (30) in module PRODUCT by axiom (32). Then,* $EXTENDED\ PRODUCT \not\models (\forall x,y)Object(x) \wedge Object(y) \rightarrow overlapsSpatially(x,y)$.

Proof. Let P_1 be the theory that results from adding sentence $(\exists x,y)Object(x) \wedge Object(y) \wedge (\neg(overlapsSpatially(x,y))$ to module $EXTENDED\ PRODUCT$. By using Mace4, we have created a model of theory P_1.[11] □

5.4 Module DECOMPOSITION

The remainder between a whole and its proper parts is represented by function symbol *MereologicalDifferenceFn* in module *DECOMPOSITION*:

Definition 6. *Module DECOMPOSITION is the subtheory that extends module PART by means of axioms (33) and (34).*

$$\begin{aligned}&(\forall x,y,z)Object(x) \wedge Object(y) \rightarrow\\ &((z = MereologicalDifferenceFn(x,y)) \rightarrow\\ &(\forall p)properPart(p,z) \leftrightarrow\\ &properPart(p,x) \wedge \neg properPart(p,y))\end{aligned} \tag{33}$$

$$(\forall x,y)Object(x) \wedge Object(y) \rightarrow Object(MereologicalDifferenceFn(x,y)) \tag{34}$$

Because the mereological difference, or remainder, between a whole and one of its parts is represented in SUMO by a function symbol, its existence is vacuously guaranteed in every case at the expenses of having spurious evaluations of the symbol $MereologicalDifferenceFn$. However, regarding the supplementation principles (35) to (38), respectively named in (Varzi, 2007) as *weak company, strong company, supplementation*, and *strong supplementation*, Proposition 5 shows that those principles are not theorems of SUMO. These principles contribute to classify the degree of ontological commitment of the ontology with the existence of the remainder between a whole and one of its proper parts.

Proposition 5. *Axioms (35), (36), (37), and (38) are not theorems of theory* $PART \cup DECOMPOSITION$.

$$(\forall x,y)properPart(x,y) \rightarrow \exists z(properPart(z,y) \wedge -(z = x)) \tag{35}$$

$$(\forall x,y)properPart(x,y) \rightarrow (\exists z)(properPart(z,y) \wedge \neg part(z,x)) \tag{36}$$

$$(\forall x,y)properPart(x,y) \rightarrow (\exists z)(Part(z,y) \wedge \neg overlapsSpatially(z,x)) \tag{37}$$

$$(\forall x,y)\neg part(y,x) \rightarrow (\exists z)(Part(z,y) \wedge \neg overlapsSpatially(z,x)) \tag{38}$$

Proof. Let P_1 be the union of theories *PART* and *DECOMPOSITION* with the respective negation of axioms (35), (36), (37), and (38). By using Mace4, we have built a model of P_1.[11] □

We have found that the characterization of symbol *MereologicalDifferenceFn* given by (33) and (34) introduces unintended models where the remainder overlaps with the subtrahend:

Proposition 6. $DECOMPOSITION \models (\forall x,y,z)\ Object(x) \wedge Object(y) \wedge (z = MereologicalDifferenceFn(x,y)) \wedge properPart(y,x) \rightarrow properPart(y,z))$.

Proof. Let us assume that $Object(x) \wedge Object(y) \wedge z = MereologicalDifferenceFn(x,y)$ holds, and let p be such that $(p = y)$ in (33), then, it results $properPart(y, Mereological\ DifferenceFn(x,y))$ □

In order to eliminate such a class of unintended models, we propose the addition of definitions (39) to (42), and the substitution of axiom (33) by sentence (43) in module *DECOMPOSITION*, and call *EXTENDED DECOMPOSITION* to the resulting theory. We following demonstrate that this theory does not admit the unintended models that Proposition 6 identifies

$$(\forall x,y)weak\_disjoint(x,y) \leftrightarrow (\forall z)(part(z,x) \wedge part(z,y) \rightarrow N(z)) \quad (39)$$

$$(\forall x)N(x) \leftrightarrow (\forall z)part(x,z) \quad (40)$$

$$(\forall x)U(x) \leftrightarrow (\forall z)part(z,x) \quad (41)$$

$$(\forall x,z)comp(x,z) \leftrightarrow (\forall y)(part(y,z) \leftrightarrow weak\_disjoint(y,x)) \quad (42)$$

$$(\forall x,y,z)Object(x) \wedge Object(y) \rightarrow ((MereologicalDifferenceFn(x,y) = z) \rightarrow (\forall p)(part(p,z) \leftrightarrow part(p,x) \wedge weak\_disjoint(p,y))) \quad (43)$$

Proposition 7. *EXTENDED DECOMPOSITION* $\not\models$ $(\forall x,y,z)Object(x) \wedge Object(y) \wedge (z = MereologicalDifferenceFn(x,y)) \wedge properPart(y,x) \rightarrow properPart(y,z))$.

Proof. Let D_1 be the theory that results from adding sentence $(\exists x,y,z)Object(x) \wedge Object(y) \wedge (z = MereologicalDifferenceFn(x,y)) \wedge properPart(y,x) \wedge \neg properPart(y,z))$ to module $EXTENDED\ DECOMPOSITION$. By using Mace4, we have created a model of theory D_1.[11] □

Finally, We prove that the resulting mereology after all our proposed changes is satisfiable.

Proposition 8. *The theory PART ∪ EXTENDED SUM ∪ EXTENDED PRODUCT ∪ EXTENDED DECOMPOSITION is satisfiable.*

Proof. By using Mace4 we have constructed a model for the proposed union of theories.[11] □

5.5 Module TOPOLOGY

Since mereology can only represent the relation of parts with their respective wholes, predicate *connected* is characterized in this module to represent a more general symmetric and reflexive spatial relationship among objects which are not necessarily in a part-whole relation.

Definition 7. *Module TOPOLOGY is the subtheory composed by axioms (44) to (46).*

$$(\forall x)Object(x) \rightarrow connected(x,x) \quad (44)$$

$$(\forall x,y)connected(x,y) \rightarrow Object(x) \wedge Object(y) \quad (45)$$

$$(\forall x,y)connected(x,y) \rightarrow connected(y,x) \quad (46)$$

5.6 Module MEREOTOPOLOGY

This module is intended to characterize the relationship between the notions of mereology and topology. In it, both predicates, *meetsSpatially*, which represents external connection among objects, and *overlapsSpatially*, are declared disjoint specializations of predicate *connected*. However, the axiomatization of this theory is logically equivalent to conservative definitions (47) and (48). The module *MEREOTOPOLOGY* is therefore a definitional extension of modules *TOPOLOGY* and *PART*.

Definition 8. *Module MEREOTOPOLOGY is the theory that extends modules TOPOLOGY and PART by means of definitions (47) and (48).*

$$(\forall x,y)overlapsSpatially(x,y) \leftrightarrow connected(x,y) \wedge (\exists z)\, part(z,x) \wedge part(z,y) \quad (47)$$

$$(\forall x,y)meetsSpatially(x,y) \leftrightarrow connected(x,y) \wedge \neg(\exists z)\, part(z,x) \wedge part(z,y) \quad (48)$$

We have found that the monotony of relation *connected* with respect to parthood was not characterized in SUMO, which introduces unintended models as the one represented in Figure 5 where all parts share one point, but only shaded ones result to be connected.

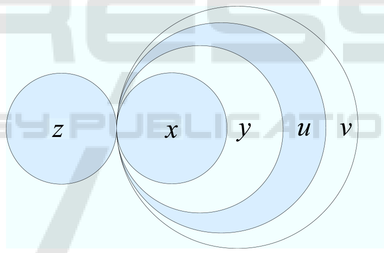

Figure 5: Model of SUMO where the monotony of relation *connected* with respect to parthood was not characterized. Even though $connected(z,x)$, $part(x,y)$, $part(y,u)$, and $part(u,v)$ hold, $connected(z,y)$ and $connected(z,v)$ do not hold, while $connected(z,u)$ does hold.

Proposition 9. MEREOTOPOLOGY $\not\models$ $(\forall x,y)\ part(x,y) \rightarrow \forall z(connected(z,x) \rightarrow connected(z,y))$

Proof. Let M_1 = be the theory that results from adding sentence $(\exists x,y)(part(x,y) \wedge (\exists z)(connected(z,x) \wedge \neg(connected(z,y))$ to module MEREOTOPOLOGY, using Mace4 we have built a model of M_1.[11] □

In order to rule out those unintended models that proposition 9 identifies, we propose the addition of axiom (49) to this module and call EXTENDED MEREOTOPOLOGY to the resulting theory.

$$(\forall x,y)part(x,y) \rightarrow \forall z(connected(z,x) \rightarrow connected(z,y)) \quad (49)$$

Table 1: Mapping of SUMO and DOLCE main categories.

$$(\forall x)Object(x) \equiv O(x) \tag{50}$$
$$(\forall x)Process(x) \equiv E(x) \tag{51}$$
$$(\forall x)TimePosition(x) \equiv T(x) \tag{52}$$
$$(\forall x)Region(x) \equiv S(x) \tag{53}$$

Table 2: Translations DOLCE PART-T into SUMO TIME.

$$(\forall x)T(x) \equiv TimePosition(x) \tag{54}$$
$$(\forall x,y)P(x,y) \equiv temporalPart(x,y) \tag{55}$$
$$\begin{aligned}(\forall x,y)Ov(x,y) \leftrightarrow (&TimeInterval(x) \\ &\wedge TimeInterval(y) \\ &\wedge overlapsTemporally(x,y)) \vee \\ &(TimePoint(x) \wedge TimeInterval(y) \\ &\wedge temporalPart(x,y)) \vee \\ &(TimeInterval(x) \wedge TimePoint(y) \\ &\wedge temporalPart(y,x)) \vee \\ &(TimePoint(x) \wedge TimePoint(y) \wedge x = y)))\end{aligned} \tag{56}$$

6 MAPPING SUMO AND DOLCE

In order to relate SUMO and DOLCE we assume that the changes that we have proposed in section 5 for eliminating unintended models and characterizing missing intended ones have been performed in SUMO. There is no axiomatization in DOLCE-CORE, neither in DOLCE, that corresponds to the notion of topology, therefore our mappings are circumscribed to the axiomatization of mereology in both theories.

Analyzing the axiomatizations of SUMO and DOLCE-CORE, we have found that the concept of *time*, as a region where objects exists and events occur, is represented in SUMO by category *TimePosition*, and in DOLCE-CORE by category T. By examining the predicates that characterize the participation of objects in events in both ontologies, and also by the type of relation that the main categories of SUMO and DOLCE-CORE have with time and space, we have built the translation definitions of Table 1 for the main categories shown in Figures 1 and 2.

6.1 Mapping Time

The subtheory SUMO TIME, whose modular structure is shown in Figure 7, characterizes the behaviour of time in SUMO. This theory, which was verified in (Silva Muñoz and Grüninger, 2016), includes 3 submodules[12] TIME POINT, TIME MEREOLOGY, and TIME INTERVAL, such that each module is a conservative extension of each connected subtheory below it in Figure 7. These 3 subtheories respectively characterize a linear ordering between instants of time, a part-whole relation among intervals of time, and an account of Allen's interval relations *starts, finishes, during, earlier*, and *meetsTemporally* (Hayes, 1996). Finally, theory SUMO TIME characterizes a part-whole relationship that includes intervals and instants of time.

Table 3: Translations TIME MEREOLOGY into DOLCE PART-T.

$$(\forall x)TimeInterval(x) \equiv T(x) \tag{57}$$
$$\begin{aligned}(\forall x,y)temporalPart(x,y) \equiv P(x,y) \\ \wedge T(x) \wedge T(y)\end{aligned} \tag{58}$$
$$\begin{aligned}(\forall x,y)overlapsTemporally(x,y) \equiv Ov(x,y) \\ \wedge T(x) \wedge T(y)\end{aligned} \tag{59}$$

On the other hand, DOLCE-CORE characterizes parthood by unique predicate P across every category, including T. By means of the following definition and theorem we classify the relationship that exists among DOLCE PART-T and SUMO TIME:

Definition 9. SUMO TIME *is the theory given by the axioms in colore.oor.net/ontologies/sumo/modules/sumo-time,* TIME MEREOLOGY *is the theory given by SUMO axioms (60) to (65), and* DOLCE PART-T *is the theory given by axioms (1)-(4) and (9).*

$$(\forall x)TimeInterval(x) \rightarrow temporalPart(x,x). \tag{60}$$

$$\begin{aligned}(\forall x,y)temporalPart(x,y) \wedge \\ temporalPart(y,x) \rightarrow (x = y).\end{aligned} \tag{61}$$

$$\begin{aligned}(\forall x,y,z)temporalPart(x,y) \wedge \\ temporalPart(y,z) \rightarrow temporalPart(x,z).\end{aligned} \tag{62}$$

$$\begin{aligned}(\forall x,y)overlapsTemporally(x,y) \rightarrow \\ TimeInterval(x) \wedge TimeInterval(y)\end{aligned} \tag{63}$$

$$\begin{aligned}(\forall x)TimeInterval(x) \rightarrow \\ overlapsTemporally(x,x)).\end{aligned} \tag{64}$$

$$\begin{aligned}(\forall x,y)TimeInterval(x) \wedge TimeInterval(y) \rightarrow \\ (overlapsTemporally(x,y) \leftrightarrow \\ ((\exists z)(TimeInterval(z) \wedge \\ temporalPart(z,x) \wedge temporalPart(z,y))))\end{aligned} \tag{65}$$

Theorem 2. *Theory* SUMO TIME *interprets theory* DOLCE PART-T.

[12]Available at colore.oor.net/ontologies/sumo/modules

Table 4: Translations DOLCE PART-E into SUMO SUBPROCESS.

$$(\forall x)E(x) \equiv Process(x) \tag{66}$$
$$(\forall x,y)P(x,y) \equiv subProcess(x,y) \tag{67}$$
$$(\forall x,y)Ov(x,y) \equiv (\exists z)(subProcess(z,x) \wedge subProcess(z,y)) \tag{68}$$

Table 5: Translations SUMO SUBPROCESS into DOLCE PART-E.

$$(\forall x)Process(x) \equiv E(x) \tag{69}$$
$$(\forall x,y)subProcess(x,y) \equiv E(x) \wedge E(y) \wedge P(x,y) \tag{70}$$

Proof. Let us call Δ to the set of translations definitions shown in Table 2. Using Prover9 we have shown that SUMO TIME $\cup\, \Delta \models$ DOLCE PART-T. □

Theorem 3. *Theory* DOLCE PART-T *interprets theory* TIME MEREOLOGY.

Proof. Let us call Δ to the set of translations definitions shown in Table 3. Using Prover9 we have shown that DOLCE PART-T $\cup\, \Delta \models$ TIME MEREOLOGY. □

6.2 Mapping Events

Regarding the representation of events in SUMO and DOLCE, by means of the following definition and theorem we classify the relationship that their respective part-whole axiomatizations have as *synonymy*.

Definition 10. SUMO SUBPROCESS *is the theory given by axioms (71)-(74), and* DOLCE PART-E *is the theory given by axioms (1)-(3) and (8).*

$$(\forall x,y)subProcess(x,y) \rightarrow Process(x) \wedge Process(y) \tag{71}$$
$$(\forall x)Process(x) \rightarrow subProcess(x,x) \tag{72}$$
$$(\forall x,y)subProcess(x,y) \wedge subProcess(y,z) \rightarrow subProcess(x,z) \tag{73}$$
$$(\forall x,y)subProcess(x,y) \wedge subProcess(y,x) \rightarrow (x = y) \tag{74}$$

Theorem 4. SUMO SUBPROCESS *is synonymous with* DOLCE PART-E.

Proof. Let Δ be the set of translations definitions shown in Table 4. Using Prover9 we have shown that SUMO SUBPROCESS $\cup\, \Delta \models$ DOLCE PART-E. Let Γ be the set of translations definitions shown in Table 5. Using Prover9 we have shown that DOLCE PART-E $\cup\, \Gamma \models$ SUMO SUBPROCESS.[13]. □

[13]Proof available at: colore.oor.net/ontologies/sumo/mereotopology/proofs

Table 6: Translations DOLCE PART-O into SUMO PART.

$$(\forall x,y)P(x,y) \equiv part(x,y)) \tag{75}$$
$$(\forall x,y)Ov(x,y) \equiv overlapsSpatially(x,y)) \tag{76}$$

Table 7: Translations SUMO PART into DOLCE PART-O.

$$(\forall x,y)part(x,y) \equiv O(x) \wedge O(y) \wedge P(x,y) \tag{77}$$
$$(\forall x,y)properPart(x,y) \equiv O(x) \wedge O(y) \wedge P(x,y) \wedge \neg P(y,x) \tag{78}$$
$$(\forall x,y)overlapsSpatially(x,y) \equiv O(x) \wedge O(y) \wedge Ov(x,y)) \tag{79}$$
$$(\forall x,y)overlapsPartially(x,y) \equiv Ov(x,y) \wedge \neg P(x,y))) \wedge \neg P(y,x)))) \tag{80}$$

We have verified theories DOLCE PART-E, and SUMO SUBPROCESS by demonstrating their synonymy with already verified theory colore.oor.net/ontologies/mereology/m_mereology.clif.

6.3 Mapping Objects

Regarding the representation of objects in SUMO and DOLCE-CORE, by means of the following definition and theorem we classify the relationship among their respective part-whole axiomatizations as *synonymy*.

Definition 11. SUMO PART *is the theory given by axioms (18)-(24), and* DOLCE PART-T *is the theory given by axioms (1)-(4) and (7).*

Theorem 5. SUMO PART *is synonymous with* DOLCE PART-O.

Proof. Let us call Δ to the set of translations definitions shown in Table 6. Using Prover9 we have shown that SUMO PART $\cup\, \Delta \models$ DOLCE PART-O.[13]

Let us call Π to the set of translations definitions shown in Table 7. Using Prover9 we have shown that DOLCE PART-O $\cup\, \Pi \models$ SUMO PART.[13] □

We have verified theories DOLCE PART-O, and SUMO PART by demonstrating their synonymy with already verified theory colore.oor.net/ontologies/mereology/m_mereology.clif.

6.4 Mapping SUM

We observe that theories DOLCE SUM, and SUMO SUM axiomatize the same intended conceptualization regarding fusion of parts, and they are respective extensions of synonymous and verified theories DOLCE PART-O and SUMO PART. Based on that, we verify theories DOLCE SUM, and SUMO SUM by defining

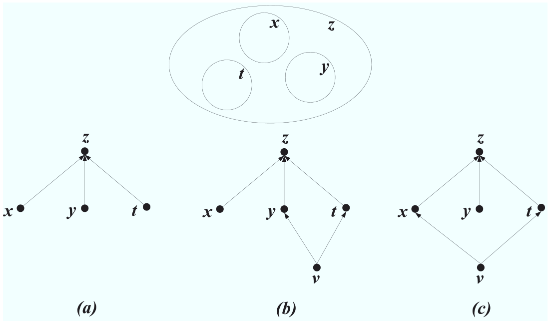

Figure 6: Objects x, y, z, t, which do not hold $SUM(z,x,y)$ but hold $MereologicalSumFn(x,y) = z$. Arrows represent relation *part* of theory SUMO SUM, and relation P of theory DOLCE SUM.

Table 8: Translations DOLCE SUM into SUMO SUM.

$$(\forall x,y,z)SUM(z,x,y) \equiv Object(x) \wedge Object(y) \wedge (MereologicalSumFn(x,y) = z) \tag{81}$$

mappings among their signatures, then, finding which axioms of each ontology are not theorems of the other. By analyzing the obtained results, we identify missing axioms and corresponding unintended models. Based on such a verification we have identified the changes proposed in section 5 to the SUMO ontology.

Definition 12. DOLCE SUM *is the theory given by axioms (1)-(5), (7), and (12).*

The axiomatization of SUMO SUM is weaker than the axiomatization of DOLCE SUM. In fact, let us consider objects x, y, z, t of Figure 6, such that $properPart(x,z)$, $properPart(y,z)$, and $properPart(t,z)$ hold, while none of $overlapsSpatially(x,y)$, $overlapsSpatially(t,y)$, or $overlapsSpatially(x,t)$ hold. In parts (a), (b), and (c) of the bottom of Figure 6 parthood is indicated with arrows from the part to the whole. According to the characterization of mereological sum on SUMO SUM $(MereologicalSumFn(x,y) = z)$ hold. However, Parts (b) and (c) of Figure 6 depict alternative additional conditions that the characterization of mereological sum on module DOLCE SUM has. In DOLCE, any other object t which overlaps with the sum z must overlap with at least one of the addends x or y, therefore $SUM(z,x,y)$ does not hold in DOLCE. The following theorem formalizes our claim.

Theorem 6. SUMO SUM *can not interpret* DOLCE SUM.

Proof. Let us call Δ to the translations shown in Table 6, and Π to the translation shown in Table 8, and let S_1 be the theory that results from adding sentence (82) to theory SUM. Using Mace4, we have built a model of $S_1 \cup \Delta \cup \Pi$.[13]

$$(\exists x,y,z)SUM(z,x,y) \wedge \neg(\forall w)(Ov(w,z) \leftrightarrow Ov(w,x) \vee Ov(w,y)) \tag{82}$$

□

Table 9: Characterization of predicate MSum in SUMO.

$$(\forall x,y,z)MSum(z,x,y) \rightarrow (\forall w)(part(z,w) \leftrightarrow (part(x,w) \wedge part(y,w)))) \tag{83}$$

$$(\forall x,y,z)MSum(z,x,y) \rightarrow Object(x) \wedge Object(y) \wedge Object(z) \tag{84}$$

$$(\forall x,y)Object(x) \wedge Object(y) \rightarrow \exists z(Object(z) \wedge MSum(z,x,y)) \tag{85}$$

$$(\forall x,y,z,t)MSum(z,x,y) \wedge MSum(t,x,y) \rightarrow (z = t) \tag{86}$$

Table 10: Translation SUMO SUM into DOLCE SUM.

$$(\forall x,y,z)MSum(z,x,y) \equiv Object(x) \wedge Object(y) \wedge SUM(z,x,y) \tag{87}$$

In order to translate the symbol $MereologicalSumFn$ of theory SUM into the language of DOLCE-CORE, we have represented the graph[14] of function $MereologicalSumFn$ by means of predicate *MSum*, as shown in Table 9.

Theorem 7. DOLCE SUM *can not interpret* SUMO SUM.

Proof. Let us call Δ to the translations shown in Table 1, Π to the translations in Table 7, and Υ to the translation in Table 10, and let DOLCE SUM_1 be the theory that results from adding sentence (88) to DOLCE SUM. Using Mace4, we have built a model of DOLCE $\text{SUM}_1 \cup \Delta \cup \Pi \cup \Upsilon$.[13]

$$(\exists x,y)Object(x) \wedge Object(y) \wedge (\forall z)(\neg Object(z) \vee \neg MSum(z,x,y)) \tag{88}$$

□

Figure 7 shows conservative extensions by means of thin black arrows and relative interpretations (mappings), by thick gray arrows from interpreted to interpreting theories. Because every theorem of a

[14]A n-ary function f from A^n to B is representable by a relation ρ with arity (n+1), called *the graph of f*, such that:
(a) Every tuple of ρ is a tuple $\langle \bar{x}, f(\bar{x}) \rangle$ with $\bar{x} \in A^n$ and $f(\bar{x}) \in range(f)$.
(b) If $f(\bar{x}) = b$ and $f(\bar{z}) = c$, then $b = c$.

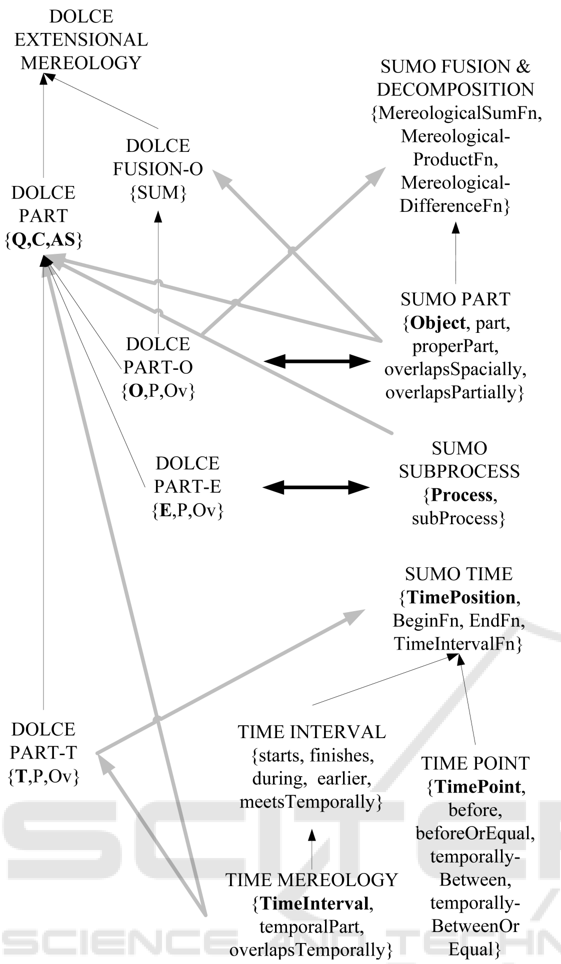

Figure 7: Mappings between modules of DOLCE-CORE and SUMO. Black thin arrows point to conservative extensions, thick grey arrows are directed from interpreted theories to interpreting theories, and thick black arrows connect synonymous theories. Signature members are situated in the subtheories that introduce them.

theory is also a theorem of its conservative extensions, each conservative extension is capable of interpreting every theory that the modules that it extends interpret. In particular, module DOLCE PART, shown in Figure 7, is the theory resulting from the union of DOLCE PART-T, DOLCE PART-E, and DOLCE PART-O, plus axioms (10), (11), while module DOLCE EXTENSIONAL MEREOLOGY is the union of DOLCE PART, DOLCE SUM, and axioms (6), (13), (14), (15), (16), and (17). As indicated by oriented grey arrows, the axiomatization of part-whole relations in categories *Object*, *Process*, and *TimeInterval* of SUMO are mappable to DOLCE minimal axiomatization of mereology represented by module DOLCE PART. Although not represented in Figure 7, it holds that because DOLCE EXTENSIONAL MEREOLOGY extends DOLCE PART, it also interprets SUMO PART, SUMO SUBPROCESS, and TIME MEREOLOGY. In turn, SUMO SUM interprets DOLCE PART-O. The strongest subtheories of SUMO and DOLCE-CORE that are synonymous, and therefore have equivalent models, are the pairs indicated by double black arrows, i.e, DOLCE-PART-O with SUMO PART and DOLCE-PART-E with SUMO SUBPROCESS.

7 CONCLUSIONS

We have verified the representation of mereology of the DOLCE-CORE and the core axiomatization of mereotopology of SUMO. In the process, we have identified a series of unintended and missing models on the analysed subtheories, and have isolated the strongest subtheories of each ontology in which they both agree to the extent of having equivalent models. As a result, we have proposed a set of corrections and some axioms to be added to the analyzed theories. We have built a series of formal maps stated in standard first-order logic which unambiguously relate the axiomatizations of both upper-level ontologies. Finally, we have produced a modular representation in standard first-order logic of the complete SUMO subtheory of mereotopology originally stated in higher order language SUO-KIF.

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