Instruction Structure Analysis Appling Fuzzy Number

Seiji Saito[1] and Takenobu Takizawa[2]
[1]*Graduate School of Education, Waseda University, Shinjuku-ku, Tokyo, Japan*
[2]*Faculty of Political Science and Economics, Waseda University, Shinjuku-ku, Tokyo, Japan*

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Abstract: Applying fuzzy clustering method to the instruction structure analysis, we can investigate whether the order of teaching item is suitable or not. However, when the teacher gives learners partial points, it is difficult to judge whether the leaner solve the problem correctly or not. In this paper, the authors regard the score of the test as the fuzzy number, and present a new analysis method using fuzzy number. We show some graphs required for analysis based on the results of examination for high school students and represent the effectivity of the method.

1 INTRODUCTION

When we teach a learning unit, we need to consider that what problems should be taught and in what order we teach items. There is a method to investigate the similarity and the connectivity among the problems. We call this method "Instruction Structure Analysis". Applying the analysis based on the score of the test, we can obtain some graphs. From the graphs, we can verify and improve the teacher's instruction structure. The following figure shows the process of the analysis.

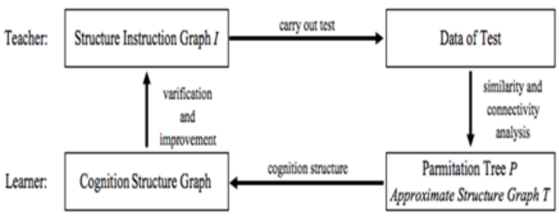

Figure 1: Process of analysis.

In this analysis, we assumed that we give learner 1 on correct or 1 on incorrect as the score. But, we sometimes have to give a learner partial point depending on the leaner's answer. So, we improved the method to use partial points. Consequently, we examined to obtain the similar result using only binary points. However, a new problem has occurred. If a leaner gets 0.5 point, it is difficult to judge whether the leaner solved the problem correctly. So, we propose new method to regard the point of the test as fuzzy number. From the method, we obtain some indexes to figure whether reliable the problem is in the analysis.

In section 2, we introduce the conventional method of the instruction structure analysis. In section 3, we propose anew method with fuzzy number. In section 4, we apply the method to the real data and show the effectivity of the method.

2 CONVENTIONAL METHOD

First, we present the conventional method of the instruction structure analysis. If we execute test of m questions $\{P_i|1 \leq i \leq m\}$ to n students $\{S_k|1 \leq k \leq n\}$, we have the score matrix $X = (x_{ki})$, where $x_{ki} = 1$ if student S_k gives a correct answer for P_i, else we give $0 \leq x_{ki} < 1$ for incorrect answer.

Next, from the score matrix X, we obtain the contingency table C_{ij} in Figure 2.

P_i \ P_j	Correct	Incorrect	Sum
Correct	a	b	$a+b$
Incorrect	c	d	$c+d$
Sum	$a+c$	$b+d$	n

Figure 2: Contingency table C_{ij}.

Definition 1. Elements of the contingency table

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$$a=\sum_{k=1}^{n}\min\{x_{ki},x_{kj}\},\qquad b=\sum_{k=1}^{n}\min\{x_{ki},1-x_{kj}\}$$

$$c=\sum_{k=1}^{n}\min\{1-x_{ki},x_{kj}\},\qquad d=\sum_{k=1}^{n}\min\{1-x_{ki},1-x_{kj}\}$$

According to the contingency table C_{ij}, we have similarity index s_{ij} and connectivity index t_{ij}.

Definition 2. Similarity Index

$$s_{ij}=\frac{a+d}{n}\ \in[0,1]$$

From the similarity index s_{ij}, we have the similarity matrix $S=(s_{ij})$. We can evaluate the similsrity among the questions.

Definition 3. Connectivity Index

$$t_{ij}=\frac{a+d}{(a+c)+(c+d)}\in[0,1]$$

From the Connectivity index t_{ij}, we have the connectivity matrix $T=(t_{ij})$. We can evaluate the connectivity among the questions.

From the similarity matrix *S*, we obtain partition tree P which presents the clustering situation. Also, from the connectivity matrix *T*, we obtain an approximate ternary graph *T** which presents the relational flow among items. From the partition tree *P* and the approximate ternary graph *T**, we obtain a cognition structure graph ϕ^z.

3 PROPOSAL METHOD

We propose the method to create membership function to regard the score as fuzzy number.

Definition 4. Membership Function of the Score

(i) $x_{ki}=0\ \ \mu(x)=\begin{cases}1 & (x=0)\\ 0 & (x\neq 0)\end{cases}$

(ii) $x_{ki}=1\ \ \mu(x)=\begin{cases}1 & (x=1)\\ 0 & (x\neq 1)\end{cases}$

(iii) $0<x_{ki}\le\frac{1}{2}\ \ \mu(x)=\max\left\{0,1-\frac{1}{x_{ki}}|x-x_{ki}|\right\}$

(iv) $\frac{1}{2}<x_{ki}<1\ \ \mu(x)=\max\left\{0,1-\frac{1}{1-x_{ki}}|x-x_{ki}|\right\}$

The narrower the shape of membership function is, the more accurately the problem represents learner's feature.

Next, we define some operations of fuzzy number because we extend similarity index by operating fuzzy number.

Here, α^* in the following definitions is the fuzzy set defined by follows.

$$\mu_{\alpha^*}(x)=\alpha\quad(x\in\mathbb{R})$$

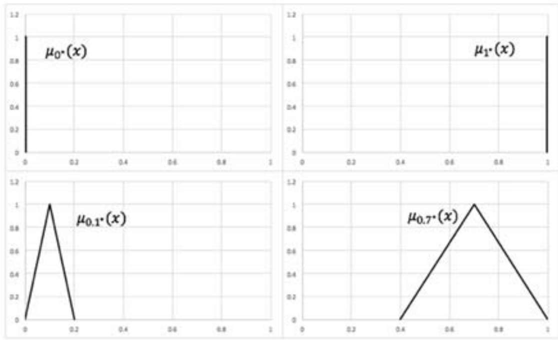

Figure 3: Example of membership function.

Definition 5. Addition of Fuzzy Numbers
Let x_1^*,x_2^* be fuzzy numbers with α-cuts

$$C_\alpha(x_i^*)=[\,a_{\alpha,i},b_{\alpha,i}](\alpha\in\ \mathbb{R},0\le\alpha\le1)$$

then the mean value $x_1^*+x_2^*$ is;

$$x_1^*+x_2^*=\bigcup_{\alpha\in[0,1]}\alpha^*\cap\ C_\alpha(x_1^*+x_2^*)$$

$$C_\alpha(x_1^*+x_2^*)=[a_{\alpha,1}+a_{\alpha,2},b_{\alpha,1}+b_{\alpha,2}]$$

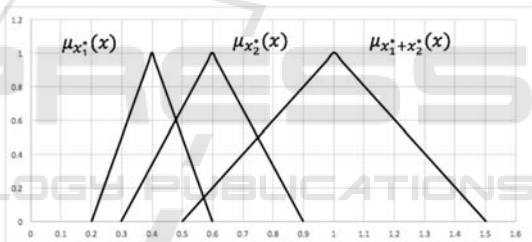

Figure 4: Example of membership function of addition of fuzzy number.

Definition 6. Minimum Value of Fuzzy Numbers

Let x_1^*,x_2^* be fuzzy numbers with α-cuts

$$C_\alpha(x_i^*)=[\,a_{\alpha,i},b_{\alpha,i}](\alpha\in\ \mathbb{R},0\le\alpha\le1)$$

then the minimum value $min(x_1^*,x_2^*)$ is;

$$min(x_1^*,x_2^*)=\bigcup_{\alpha\in[0,1]}\alpha^*\cap\ C_\alpha(\min(x_1^*,x_2^*))$$

$$C_\alpha(\min(x_1^*,x_2^*))=[\min(a_{\alpha,1},a_{\alpha,2}),\min(b_{\alpha,1},b_{\alpha,2})]$$

Definition 7. Scalar Multiple of Fuzzy Number

Let x^* be fuzzy number with α-cuts

$$C_\alpha(x^*)=[\,a_\alpha,b_\alpha](\alpha\in\ \mathbb{R},0\le\alpha\le1)$$

then the scalar multiple kx^* $(k\in\mathbb{R})$ is;

$$kx^* = \bigcup_{\alpha \in [0,1]} \alpha^* \cap C_\alpha(kx^*)$$
$$C_\alpha(kx^*) = [ka_\alpha, kb_\alpha]$$

Then, we extend the similarity index s_{ij}, and similarity matrix $S = (s_{ij})$.

Definition 8. Fuzzy Elements of Contingency Table

$$a^* = \sum_{k=1}^{n} min(x^*_{ki}, x^*_{kj})$$
$$b^* = \sum_{k=1}^{n} min(x_{ki^*}, (1 - x_{kj})^*)$$
$$c^* = \sum_{k=1}^{n} min((1 - x_{ki})^*, x^*_{kj})$$
$$d^* = \sum_{k=1}^{n} min((1 - x_{ki})^*, (1 - x_{kj})^*)$$

Definition 9. Fuzzy Similarity Index

$$s^*_{ij} = \frac{1}{n}(a^* + d^*)$$

From the fuzzy similarity index s^*_{ij}, we obtain fuzzy similarity matrix $S^* = (s^*_{ij})$.

We'd like to know the reliability of each problem. We define the width index using the width of membership function of similarity index. Then, we define the reliability index normalized value of width index.

Definition 10. Width Index h_{ij}

Let s^*_{ij} be fuzzy similarity index with α-cuts

$$C_\alpha(s^*_{ij}) = [a_{\alpha,i,j}, b_{\alpha,i,j}]$$

then the reliability index h_{ij} is;

$$h_{ij} = b_{0,i,j} - a_{0,i,j} \;\; \in \; [0, 2]$$

Definition 11. Reliability Index r_{ij}

$$r_{ij} = \frac{2 - h_{ij}}{2} \; \in \; [0, 1]$$

From the relativity matrix and reliability matrix, we obtain fuzzy relativity index and fuzzy relativity matrix as follows.

Definition 12. Fuzzy Relativity Index $\widetilde{r_{ij}}$

$$\widetilde{r_{ij}} = \min\{t_{ij}, r_{ij}\}$$

From fuzzy relativity index, we can obtain fuzzy relativity matrix. Finally, we make fuzzy cognition graph ϕ^{z^*} from cognition graph ϕ^z, and fuzzy reliability index. We alter the gridlines of the items of cognition graph ϕ^z depending on each reliability index $\tilde{r}_{ij}$ $(i = j)$ as follows.

If $\frac{3}{4} \leq \widetilde{r_{ii}} < 1$, then the gridline is bold line.
If $\frac{1}{2} \leq \widetilde{r_{ii}} < \frac{3}{4}$, then the gridline is normal line.
If $\frac{1}{4} \leq \widetilde{r_{ii}} < \frac{1}{2}$, then the gridline is narrow line.
If $0 \leq \widetilde{r_{ii}} < \frac{1}{4}$, then the gridline is dotted line.

4 CASE STUDY

As the case study of the instruction structure analysis, we carried out test subject to 43 tenth grade students in a high school attached to a university. The contents of the test are Logic and Propositions.

We gave students an examination as shown in table 1. Then we got score matrix X from the result of the test in figure 5.

Table 1: Questions.

1	*I. Let $A = \{1,2,3\}$. List all subsets of A.*
	II .Let $U = \{n \mid n \in \mathbb{R},\ n \leq 20\}$, $A = \{n \mid n \in U \text{ and } n \text{ is even number}\}$, $B = \{n \mid n \in U \text{ and } n \text{ is multiple of } 3\}$, $C = \{n \mid n \in U \text{ and } n \text{ is multiple of } 5\}$. Find:
2	*(i)* $\overline{A \cap B}$
3	*(ii)* $\overline{A \cup B}$
4	*(iii)* $A \cap B \cap C$
5	*(iv)* $(\overline{A} \cup \overline{B}) \cap C$
	III. Let condition p, q be the follows. Write that it means a necessary condition, sufficient condition or necessary and sufficient condition.
6	*(i)* $p: x = y,\ q: x^2 = y^2$.
7	*(ii)* $p: x + y > 2, q: x > 1 \text{ and } y > 1$
8	*IV. Write the converse, inverse and contrapositive of the following statement.* $P: a + b > 0 \Rightarrow a > 0 \text{ and } b > 0\ \ (a, b \in \mathbb{R})$
	V.40 students are in a classroom. We asked them whether they like Mathematics and they are good at Mathematics. 35 students answered I like Mathematics. 29 students answered I'm good at Mathematics. 35 students answered I don't like and am not good at Mathematics.
9	*(i) Find number of students who answer I like and am good at Mathematics.*
10	*(ii) Find number of students who answer I like Mathematics but I am not good at Mathematics.*
11	*VI. Proof the following proposition.($n \in \mathbb{Z}$)* $If\ n^2 \text{ is multiple of } 2 \text{ then } n \text{ is multiple of } 2$.
12	*VII. Proof the following proposition.* $\sqrt{2}$ *is irrational number*.
13	*VIII. Proof the following proposition.* $\sqrt{2} + \sqrt{3}$ *is irrational number*.

From the score matrix X, we obtained similarity matrix S in figure 6 and connectivity structure matrix T in figure 7.

From similarity structure matrix S, we obtain partition tree P in figure 8.

$S_k \backslash P_i$	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	1	1	1	1	1	1	1	1	1	1	0.2
2	1	1	1	1	0	0	1	0.2	1	1	1	0.8667	0
3	1	1	1	1	1	1	1	1	1	1	1	1	0.8
41	1	1	1	1	1	1	1	1	1	1	1	0.8667	0.4
42	1	0.8	0.8	1	0	1	0	0.6667	1	1	0	1	0.2
43	1	1	0	1	0	1	1	1	1	1	0.8	1	0.7

Figure 5: Score matrix X.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	0.772	0.772	0.781	0.74	0.73	0.73	0.719	0.772	0.772	0.755	0.777	0.481
2	0.772	1	0.781	0.772	0.74	0.73	0.73	0.719	0.847	0.847	0.755	0.772	0.481
3	0.772	0.781	1	0.772	0.74	0.73	0.73	0.719	0.781	0.781	0.755	0.772	0.481
4	0.781	0.772	0.772	1	0.74	0.73	0.73	0.719	0.772	0.772	0.755	0.777	0.481
5	0.74	0.74	0.74	0.74	1	0.73	0.73	0.719	0.74	0.74	0.74	0.74	0.481
6	0.73	0.73	0.73	0.73	0.73	1	0.93	0.719	0.73	0.73	0.73	0.73	0.481
7	0.73	0.73	0.73	0.73	0.73	0.93	1	0.719	0.73	0.73	0.73	0.73	0.481
8	0.719	0.719	0.719	0.719	0.719	0.719	0.719	1	0.719	0.719	0.719	0.719	0.481
9	0.772	0.847	0.781	0.772	0.74	0.73	0.73	0.719	1	0.977	0.755	0.772	0.481
10	0.772	0.847	0.781	0.772	0.74	0.73	0.73	0.719	0.977	1	0.755	0.772	0.481
11	0.755	0.755	0.755	0.755	0.74	0.73	0.73	0.719	0.755	0.755	1	0.755	0.481
12	0.777	0.772	0.772	0.777	0.74	0.73	0.73	0.719	0.772	0.772	0.755	1	0.481
13	0.481	0.481	0.481	0.481	0.481	0.481	0.481	0.481	0.481	0.481	0.481	0.481	1

Figure 6: Similarity matrix S.

	1	2	3	4	5	6	7	8	9	0	11	12	13
1	1	0.544	0.558	1	0.523	0.553	0.542	0.642	0.529	0.54	0.56	0.649	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1
5	0.548	0.644	0.557	0.565	1	0.565	0.574	0.561	0.6	0.592	0.598	0.538	1
6	0.667	0.629	0.651	1	0.65	1	1	1	0.66	0.652	0.664	0.665	1
7	1	1	1	1	1	1	1	1	0.652	0.644	1	1	1
8	0.658	0.564	0.559	0.594	0.549	0.607	0.553	1	0.598	0.617	0.669	0.639	1
9	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1
11	0.668	1	0.661	0.644	1	0.657	0.665	1	0.669	0.662	1	1	1
12	1	1	1	1	1	1	1	1	1	1	1	1	1
13	0	0	0	0	0	0	0	0	0	0	0	0	1

Figure 7: Connectivity matrix T.

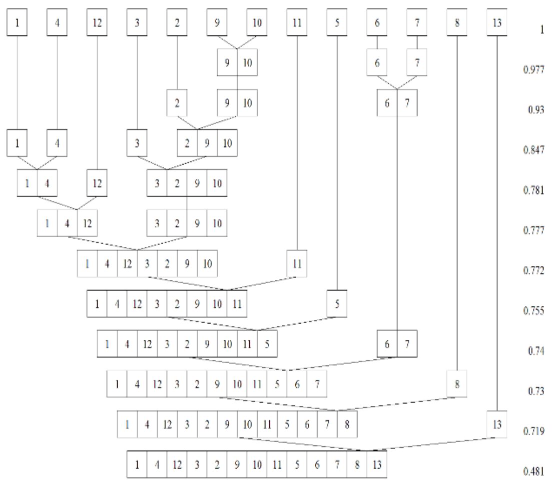

Figure 8: Partition tree P.

On the other hand, from the connectivity structure graph T, we obtained approximate ternary graph T^* in figure 9.

Summarizing the partition tree P and the approximate ternary graph T^*, we have obtained the cognition structure graph ϕ^z in figure 10.

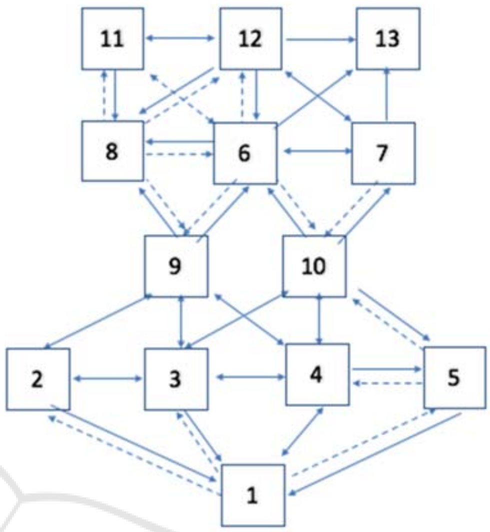

Figure 9: Approximate ternary graph T^*.

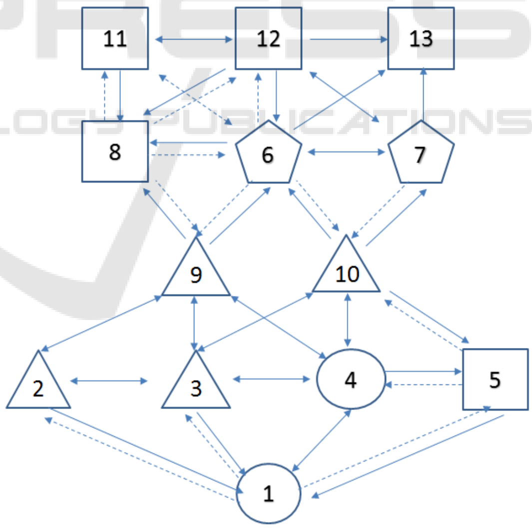

Figure 10: Cognition structure graph ϕ^z.

To compute the reliability index, we obtained reliability matrix R in figure 11.

From connectivity structure matrix T and reliability matrix R, we obtained fuzzy connectivity structure matrix in figure 12.

Finally, we obtained fuzzy cognition graph ϕ^{z*} in figure 13.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.53	0.75	0.76	0.65	0.77	0.77	0.77	0.62	0.77	0.77	0.72	0.64	0.57
2	0.75	0.97	0.98	0.87	0.99	0.99	0.99	0.84	0.99	0.99	0.93	0.86	0.79
3	0.76	0.98	0.99	0.87	1	1	1	0.85	1	1	0.94	0.87	0.8
4	0.65	0.87	0.87	0.76	0.88	0.88	0.88	0.73	0.88	0.88	0.83	0.76	0.69
5	0.77	0.99	1	0.88	1	1	1	0.85	1	1	0.95	0.88	0.81
6	0.77	0.99	1	0.88	1	1	1	0.85	1	1	0.95	0.88	0.81
7	0.77	0.99	1	0.88	1	1	1	0.85	1	1	0.95	0.88	0.81
8	0.62	0.84	0.85	0.73	0.85	0.85	0.85	0.71	0.85	0.85	0.8	0.73	0.66
9	0.77	0.99	1	0.88	1	1	1	0.85	1	1	0.95	0.88	0.81
10	0.77	0.99	1	0.88	1	1	1	0.85	1	1	0.95	0.88	0.81
11	0.72	0.93	0.94	0.83	0.95	0.95	0.95	0.8	0.95	0.95	0.9	0.83	0.76
12	0.64	0.86	0.87	0.76	0.88	0.88	0.88	0.73	0.88	0.88	0.83	0.76	0.68
13	0.57	0.79	0.8	0.69	0.81	0.81	0.81	0.66	0.81	0.81	0.76	0.68	0.61

Figure 11: Reliability matrix R.

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0.535	0.544	0.558	0.647	0.523	0.553	0.542	0.62	0.529	0.54	0.56	0.645	0.574
2	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	0.719	0.713	1	1	0.703
4	0.647	0.713	1	1	0.707	1	1	1	0.681	0.719	0.707	1	0.686
5	0.548	0.644	0.557	0.565	1	0.565	0.574	0.561	0.6	0.592	0.598	0.538	1
6	0.667	0.629	0.651	0.679	0.65	1	1	0.713	0.66	0.652	0.664	0.665	1
7	0.684	0.674	0.696	0.717	0.692	1	1	0.682	0.652	0.644	0.703	0.69	1
8	0.62	0.564	0.559	0.594	0.549	0.607	0.553	0.705	0.598	0.617	0.669	0.639	0.66
9	1	1	1	1	1	1	1	1	1	1	1	1	1
10	1	1	0.72	1	1	1	0.707	1	1	1	1	1	1
11	0.668	0.691	0.661	0.644	0.682	0.657	0.665	1	0.669	0.662	1	0.719	1
12	0.645	0.721	1	1	0.678	0.727	0.721	1	0.71	0.71	1	1	0.684
13	0.325	0	0	0	0	0	0	0.32	0	0	0	0	0.614

Figure 12: Fuzzy connectivity structure matrix.

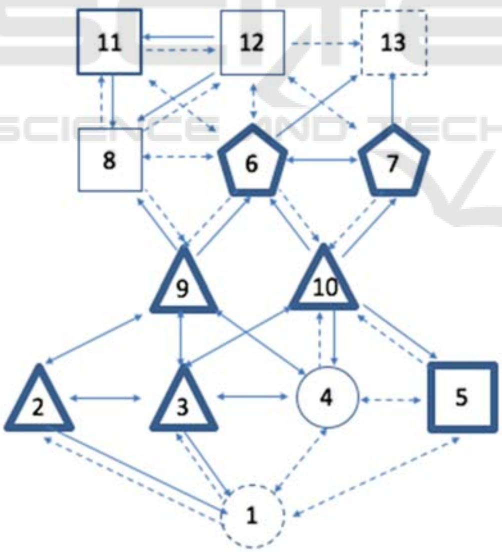

Figure 13: Fuzzy cognition graph ϕ^{z^*}.

According to the fuzzy cognition structure graph ϕ^{z^*}, we found following results:

(1) We classified four groups {2,3,9,10}, {6,7}, {5,8,11,12,13},{1,4} from fuzzy cognition structure graph ϕ^{z^*}.

(2) 1 wasn't suitable for analysis because many students forgot empty set therefore we gave them partial points.

(3) 13 wasn't suitable for analysis because it was proof question therefore many students couldn't solve correctly.

(4) Many students found it easier to solve the problem of Set than Proposition.

5 CONCLUSIONS

The authors have discussed the analysis method to use partial points, and have also illustrated its example of the high school mathematics. Using the fuzzy cognition structure graph, we have been able to judge whether the leaner solve the problem correctly or not. The graph is complicated therefore we would like to improve analytical methods in the future.

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