Neighborhood Strategies for QPSO Algorithms to Solve Benchmark Electromagnetic Problems

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Abstract: Several neighborhood strategies for QPSO algorithms are proposed and analyzed in order to improve the performances of the original methods. The proposed strategies are applied to some of the most well known QPSO algorithms such as the QPSO with random mean, the QPSO with Gaussian attractor and of course the basic QPSO. To prevent the premature convergence and to avoid being trapped in local minima the neighborhoods are dynamically changed during the optimization process. For testing the efficiency of the neighborhood techniques two benchmark optimization problems from the electromagnetic field computation have been chosen, Loney's solenoid and TEAM22.

1 INTRODUCTION

The PSO (Particle Swarm Optimization) algorithms are part of stochastic optimization methods which use a population of candidate solutions that evolve over time. Comparable in terms of performance with the genetic algorithms, these algorithms are problem independent and are suitable for solving difficult optimization problems where the analytical expression of the objective function is not known.

First proposed by Kennedy and (Kennedy, Eberhart, 1995), the original PSO (classic) has its root in biology and is inspired by social behavior within fish schools or bird flocks. Each particle in the swarm (population) is characterized by its current position and velocity. The position encapsulates the potential solution of the optimization problem, while the velocity influences how that position will be changed at the next iteration.

The main issues of the classical PSO are the high probability to get stuck in a local minimum and the large number of iterations required to find the global solution. Over time, to improve the performance of the PSO algorithm several solutions have been proposed in the literature (Ciuprina et al., 2002), but the most efficient options currently available are based on SPSO (Standard PSO) (Bratton, Kennedy, 2007) (Clerc, 2012) and QPSO (Quantum-behaved PSO) (Sun et al., 2004).

The SPSO and QPSO algorithms have been successfully used to solve a variety of problems, such as (Li et al., 2007) (Zhang, Zuo, 2013), but also to solve electromagnetic optimization problems (Mikki, Kishk, 2006) (Coelho, 2007) (Coelho, Alotto, 2008). In (Mikki, Kishk, 2006) authors solve the LAA problem (Linear Antenna Array) proposing a method for the optimal control of the QPSO algorithm parameters. In (Coelho, 2007) and (Coelho, Alotto, 2008) the author makes a comparison between the PSO and QPSO algorithms for the TEAM22 problem and the Loney's solenoid problem. Even if the solutions mentioned in (Coelho, Alotto, 2008) do not mention/verify the quench condition, the conclusions of both articles highlight the superiority of the QPSO algorithms over the PSO.

Although the latest QPSO versions proposed by Sun&others (Xi et al., 2008) (Sun et al., 2011) (Sun et al., 2012) are better than the SPSO algorithm when optimizing CEC benchmark functions, in case of statistical studies conducted on electromagnetic optimization problems the SPSO algorithm is more stable (Duca et al., 2014) (Duca et al., 2014, 2). In some cases the performance offered by the QPSO based algorithms provide better solutions (smaller values of the objective function) but statistically the QPSO based methods are outperformed by the SPSO algorithm, which provides a smaller mean and a

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smaller standard deviation.

To improve the QPSO algorithms performances when solving electromagnetic problems this paper proposes and studies several neighborhood strategies.

Starting from the idea used in SPSO, in the current paper different neighborhood strategies are applied to enhance the performances of the best QPSO algorithms available at present time, QPSO-WM (weighted mean) (Xi et al., 2008), QPSO-GA (Gaussian attractor) (Sun et al., 2011), QPSO-RO (ranking operator) (Sun et al., 2012), QPSO-RM (random mean) (Sun et al., 2012), and basic (Sun et al., 2004). To avoid the premature convergence and local minima the neighborhoods are dynamically changed during the optimization process. The influence of the change frequency over the performances is analyzed for each neighborhood scheme and each QPSO algorithm. The tests are carried for two benchmark electromagnetic optimization problems, Loney's solenoid and TEAM22.

2 QPSO ALGORITHMS

Unlike PSO and SPSO algorithms, where the particle trajectories are according to Newton's mechanic laws, QPSO is a quantum system proposed by Sun & others (Sun et al., 2004). In QPSO the behavior of each particle is described by a wave function Ψ (Schrödinger's equation) $|\Psi|^2$ being the probability density function for the particle position. On the other hand, while in the PSO algorithm the particles converge to the solution through the global best position, in QPSO the particles exert a greater influence on each other through an average of the personal best positions, so the probability to get stuck in a local minimum is smaller.

The coordinate j for a particle i at the step $(t + 1)$ is given by:

$$x_{i,j}(t+1) = p_{i,j}(t) \pm \\ \pm \beta \cdot \left| m_j(t) - x_{i,j}(t) \right| \cdot \ln(1/u_{i,j}), \quad (1)$$

$$p_{i,j}(t) = \alpha_{i,j} \cdot x_{PB\ i,j}(t) + \\ + (1-\alpha_{i,j}) \cdot x_{GB\ j}(t), \quad (2)$$

where p is called attractor, u and α are random generated numbers uniformly distributed in the interval [0, 1) (for each component of a particle), and β is a contraction–expansion factor linearly decreased at each iteration with values between 1 and 0.5. The particles in a QPSO algorithm exert great influence on each other through a mean best (m) calculated using the formula:

$$m(t) = [m_1(t), m_2(t), \ldots m_n(t)] = \\ = \left[\frac{1}{M}\sum_{i=1}^{M} x_{PBi,1}(t), \ldots \frac{1}{M}\sum_{i=1}^{M} x_{PBi,n}(t) \right], \quad (3)$$

where M is the swarm size (the number of particles) and n is the number of coordinates for the position of a particle.

During time, to enhance the performances of the QPSO several variants have been proposed. The most effective QPSO based algorithms available today are: QPSO with weighted mean (QPSO-WM), QPSO with Gaussian attractor (QPSO-GA), QPSO with ranking operator (QPSO-RO), and QPSO with random mean (QPSO-RM).

In QPSO-WM (Xi et al., 2008), at each iteration, the particles are sorted according to their fitness values. Each particles is assigned a weight (α) related to its ranking position (better solution means larger weight), to enforce an elitism behavior. The mean is calculated as a weighted sum as follows:

$$m(t) = \frac{1}{M}\left[\sum_{i=1}^{M} \alpha_{i,1} \cdot x_{PBi,1}(t), \ldots \sum_{i=1}^{M} \alpha_{i,n} \cdot x_{PBi,n}(t) \right]. \quad (4)$$

For improving the convergence, diversify the swarm, and escaping the local minima, in (Sun et al., 2011) the authors propose a new QPSO-GA algorithm which changes the attractor (p) from the position formula to a Gaussian probability distribution. The new attractor (np) is evaluated with the formula:

$$np_{i,j}(t) = N(p_{i,j}(t),\ m_j - p_{i,j}(t)), \quad (5)$$

$$x_{i,j}(t+1) = np_{i,j}(t) \pm \\ \pm \beta \cdot \left| m_j(t) - x_{i,j}(t) \right| \cdot \ln(1/u_{i,j}), \quad (6)$$

where N is a function with Gaussian distribution. In the beginning the particles are scattered in a wider space, and the standard deviation is larger, while in the late stage of the search the particles converge toward the mean, and the standard deviation decreases toward zero.

In (Sun et al., 2012), the authors demonstrate using the probabilistic metric spaces theory that QPSO can converge to a global optimum, and propose two new improvements QPSO-RM and QPSO-RO. The QPSO-RM algorithm replaces the mean best position with a personal best position of a random selected particle. This change diversifies the swarm and enhances the ability of the global search.

Unlike the previous algorithms, where the particles move toward global best, in the QPSO-RO each particle is guided by its personal best and the personal best of a random selected particle. The selected particle is chosen from the particles with a better fitness value, and its selection is based on a ranking operator. The formula for the attractor becomes:

$$p_{i,j}(t) = \alpha_{i,j} \cdot x_{PB\ i,j}(t) + \\ + (1-\alpha_{i,j}) \cdot x_{PB\ q,j}(t), \quad (7)$$

where q is the random selected particle with a better fitness value.

3 QPSO NEIGHBORHOOD STRATEGIES

Three different neighborhood strategies are proposed and analyzed for combining with the QPSO algorithms, one with unidirectional links and two with bidirectional links between particles.

For the first strategy, which is inspired from Clerc's SPSO (Clerc, 2012), the particles of the swarm are connected, each connection representing a unidirectional link between two particles. A unidirectional link has an informed and an informing particle, the informed particle knowing the position and the personal best of the second particle. Thus, each informed particle has a set of informing particles called neighborhood (Figure 1). This strategy will be referred as INF (from informed/informing).

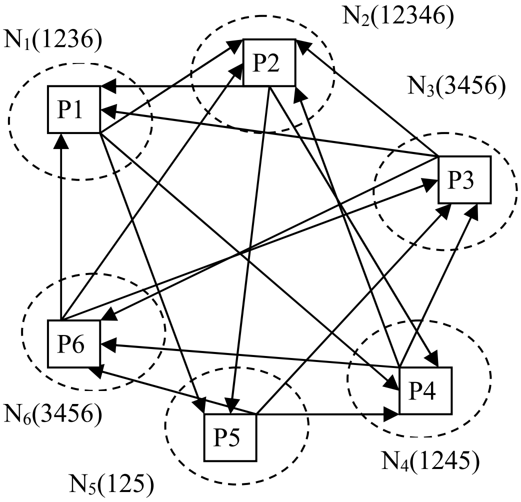

Figure 1: INF neighborhood strategy for a swarm with 6 particles and 3 informants.

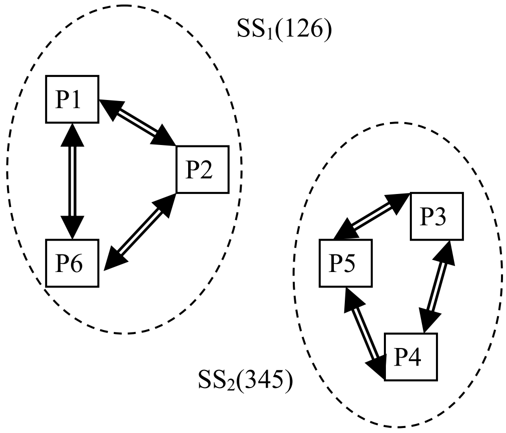

Figure 2: Subswarm neighborhood strategy for a swarm with 6 particles and 2 subswarms.

When adapted to our QPSO algorithms, the INF strategy will compute for each particle the mean only using particle's neighborhood, and the attractor using the local best a from the same neighborhood. In the case of QPSO-RO/RM the random chosen particles for mean and attractor components will only be from the neighborhood.

If for a given number of iterations the current structure of the INF swarm does not improve the global best the structure is regenerated randomly. The pseudocode for this strategy is the following:

```
;generate links between particles
noImprovmentSteps=0
foreach iteration
    foreach particle i
        ;calculate localbest for Ni
        ;calculate medium for Ni
        ;calculate attractor for Ni
        ;calculate new position
        ;evaluate
    foreach particle i
        ;update position
           ;update personal best
    ;calculate global best
    if (global best was improved)
        noImprovmentSteps=0
    else
        noImprovmentSteps ++
        if (noImprovmentSteps==MAX)
            ;generate new links
                 noImprovmentSteps=0
```

For the second and third strategies the swarm is divided into subswarms. The subswarms are disjoint meaning there are no connections between particles belonging to different subswarms. Inside a subswarm the particles are fully connected, one particle has as neighbors all the other particles (Figure 2).

Just as for the first strategy, the subswarms are dynamically changed during the optimization process if the global best is not improved for several iterations. The particles are assigned to subswarms randomly, each subswarm having a fixed number of particles.

The second strategy will compute the attractor of each particle using the local best of the belonging subswarm (or particles inside it, in case of QPSO-RO), while the third strategy will use the global best. For both strategies the mean will be evaluated (generated in case of QPSO-RM) only using particles inside the corresponding subswarm. The second strategy will be reffered as SS-LB while the third strategy will be reffered as SS-GB (from subswarm with local/global best).

The pseudocode for the second strategy (similar for third strategy) is the following:

```
;generate subswarms
noImprovmentSteps=0
foreach iteration
    foreach subswarm SSj
        ;calculate localbest
        ;calculate medium
        foreach particle i in SSj
            ;calculate attractor
            ;calculate new position
            ;evaluate
      foreach particle i
        ;update position
            ;update personal best
    ;calculate global best
    if (global best was improved)
        noImprovmentSteps=0
    else
        noImprovmentSteps ++
        if (noImprovmentSteps==MAX)
            ;generate subswarms
                 noImprovmentSteps=0
```

4 ELECTROMAGNETIC PROBLEMS

The QPSO based algorithms have been tested on two electromagnetic benchmark problems defined by the COMPUMAG community.

4.1 The TEAM22 Problem

Two coaxial coils carry current with opposite directions (Figure 3), operate under superconducting conditions and offer the opportunity to store a significant amount of energy in their magnetic fields, while keeping within certain limits the stray field (Ioan et al., 1999). An optimal design should couple the energy to be stored by the system with a minimum stray field into one objective function.

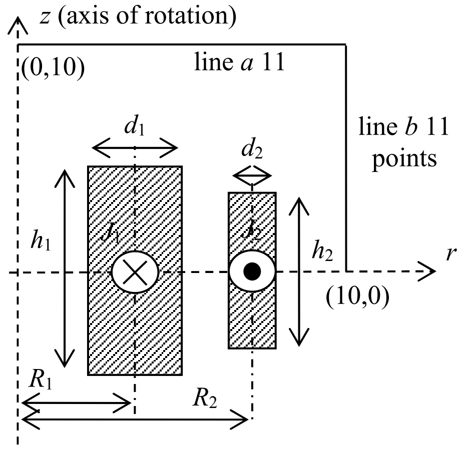

Figure 3: TEAM22 problem configuration.

The objective function has as parameters, the radii (R), the heights (h) , the thicknesses (d) and the current densities (J). Besides domain restrictions, the problem must take into account that the solenoids do not overlap each other, and the superconducting material should not violate the quench condition that links together the value of the current density and the maximum value of magnetic flux density.

The evaluation method of the objective function is based on the Biot-Savart-Laplace formula in which the elliptic integrals are computed by using the King algorithm and numerical integration as in (TEAM22, 2015).

4.2 The Loney's Solenoid

The Loney's solenoid benchmark problem, formulated in (Di Barba et al., 1995) consists of a main coil and two identical correction coils, having fixed dimensions (Figure 4).

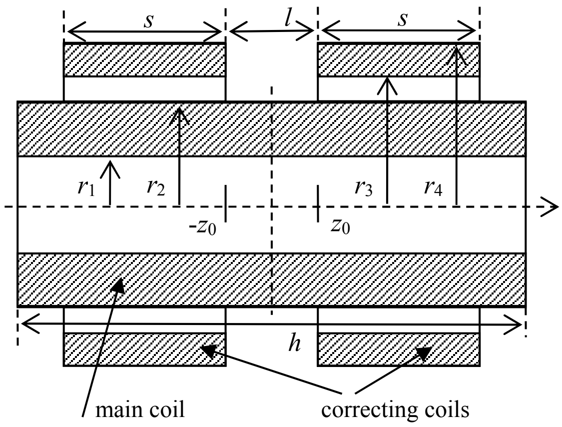

Figure 4: Loney's solenoid problem configuration.

A constant current flows through the coils such that their current density is the same. The aim is to produce a constant magnetic flux density in the middle of the main coil. The parameters to be optimized are the length of the correction coils (s) and the axial distance between them (l).

The objective function is of minmax type, i.e. minimize the maximum difference between the values of the magnetic flux density along a straight segment in the middle of the main solenoid, i.e. minimize $(B_{max} - B_{min})/B_0$, where B_0 is the magnetic field density in the middle of the main coil. The maximum and minimum values are sought along the segment $[-z_0,z_0]$. Tests done by the authors of this benchmark revealed that the problem is non convex and ill conditioned (Di Barba, Savini, 1995). The electromagnetic field problem is easily solved, in a magnetostatic regime, by discretizing the coils in elementary coils without thickness and by applying well known analytical formulas for the field along the solenoid axis.

5 RESULTS

To solve the electromagnetic optimization problems four QPSO based algorithms have been considered QPSO-WM, QPSO-Gauss, QPSO-RM and QPSO-RO. After a preliminary study, QPSO-Gauss and QPSO-RM, together with the basic QPSO, have been chosen for further testing. Each of the three mentioned QPSO algorithms have been adapted and combined with each of the described neighborhood strategies, INF, SS-LB and SS-GB. Further more, for each combination have been analyzed the influence over the performances of the structure change frequency.

Tables 1 and 2 (see Appendix section) present the solution fitness values for 30 independent tests (runs), each run having different random values for the initial population. For each test the swarm size was 32, and the stop criteria was the maximum number of iterations equivalent to 2560 objective function evaluations. Mean-best is the average of the best solutions (minimum values) obtained at each of the 30 runs, while Min-best (Max-best) is the minimum (maximum) of the minimum values obtained at each run. The number of informants for INF strategy was 3, and the number of subswarms for SS-LB(GB) was 4. Two different frequencies were tested, a low frequency (LF) of 10 iterations, and a high frequency (HF) which meant change at each iteration if the global best was not improved.

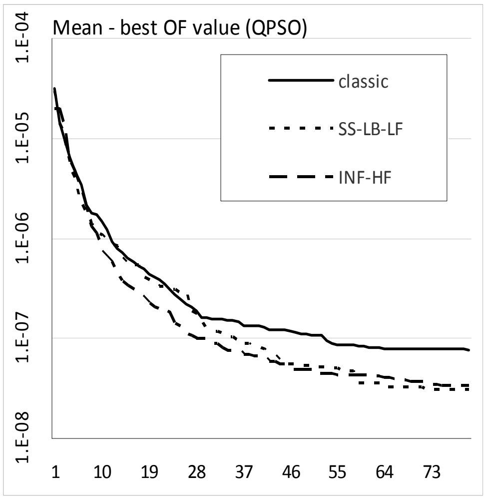

Figure 5: QPSO mean for Loney.

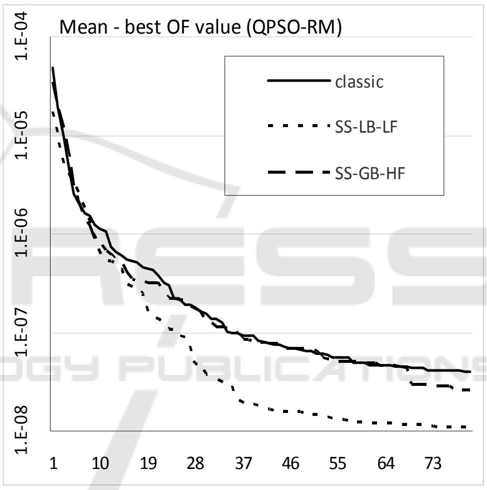

Figure 6: QPSO-RM mean for Loney.

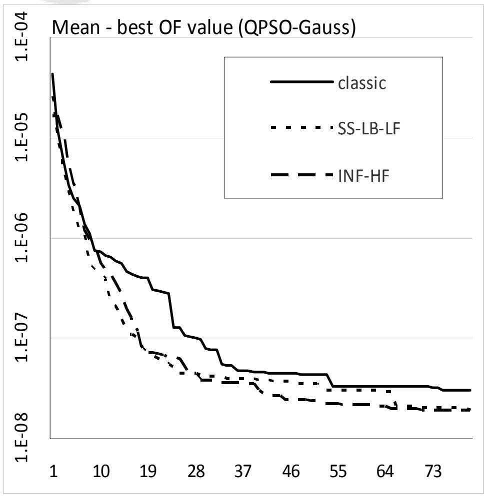

Figure 7: QPSO-Gauss mean for Loney.

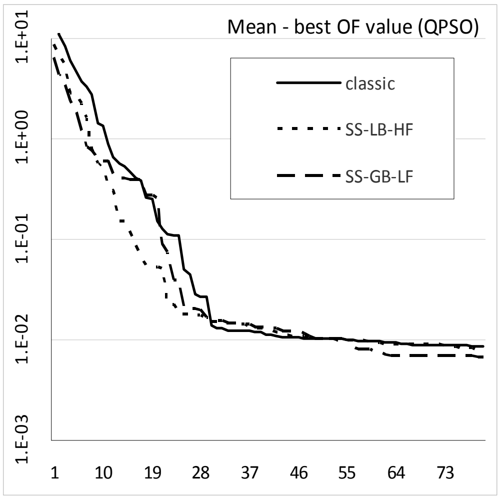

Figure 8: QPSO mean for TEAM22.

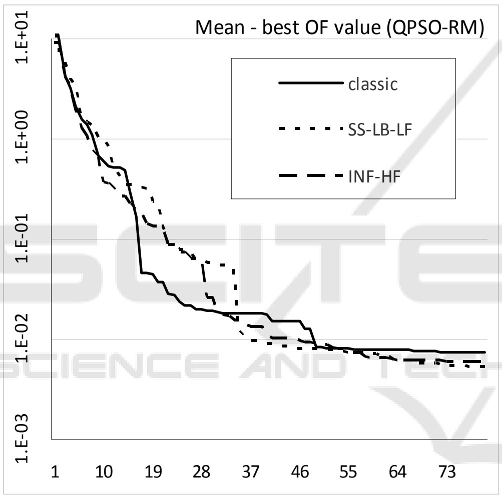

Figure 9: QPSO-RM mean for TEAM22.

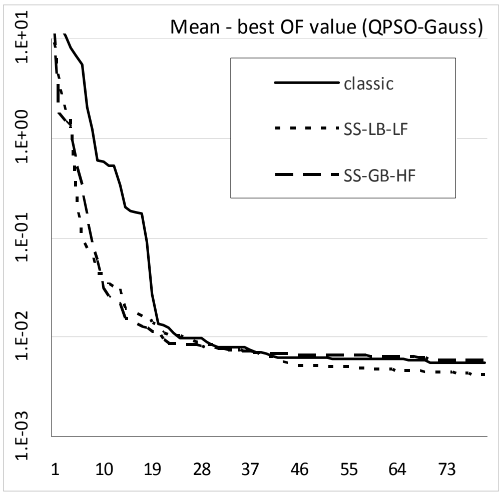

Figure 10: QPSO-Gauss mean for TEAM22.

For the Loney's solenoid benchmark, the classic algorithms performances are always improved when the algorithms are enhanced with neighborhood strategies. The most stable combinations (smallest mean-best, and standard deviation) are QPSO-RM with SS-LB-LF, and QPSO-Gauss with INF-HF. The overall best solution, which is among the best found in the literature (by our knowledge), was obtained with QPSO-RM with SS-LB-LF. In terms of frequency, while for the SS-LB strategy the low frequency is better, for SS-GB and INF strategies the high frequency provides better results.

For the TEAM22 benchmark the algorithms with neighborhood strategies provide most of the time better mean and standard deviation values, but the improvement for the best solution is not as significant as in the case of Loney's solenoid. Surprisingly, the best solution is obtained with the classic version of the basic QPSO, which offers a solution close to the well known best from the literature (1.8 E-3) (TEAM22, 2015). The most stable combinations are QPSO-Gauss with SS-GB-HF (LF), QPSO-RM with SS-LB-LF(HF), and QPSO with SS-GB-LF. Regarding the frequency change of particles connections, the small frequencies are suitable for obtaining better mean values while high frequencies lead to better standard deviations.

The improvements obtained with the algorithms enhanced with neighborhood strategies can also be seen from mean-best evolution during the optimization process. Besides the fact that statistical mean values are smaller, the neighborhood enhanced algorithms are more stable having a smoother evolution while the classic algorithms evolve (with some exceptions) in slopes.

6 CONCLUSIONS

The present paper studied the efficiency of neighborhood strategies when applied to QPSO based algorithms to solve benchmarks electromagnetic problems.

Three different neighborhood have been proposed and analyzed, one with unidirectional and two with bidirectional particle connections. In the first strategy, inspired from Clerc's SPSO, each link has an informant and an informed particle, thus each particle has its own neighborhood containing the informants. The other two strategies divide the swarm into disjoint subswarms and use to calculate the attractors the local best of the subswarm or the global best. For all the strategies the connections are dynamically changed, reset and randomly regenerated, if the solution is not improved for several iterations.

These strategies have been applied to the best QPSO algorithms available, such as QPSO-RM, QPSO-Gauss or basic QPSO, and were tested on two problems from electromagnetism, namely TEAM22 and Loney's solenoid.

In case of Loney's solenoid benchmark the QPSO algorithms enhanced with neighborhood strategies significantly improve the results for each of the combinations. The enhanced QPSO algorithms provide much small mean and standard deviation values. In the same time the overall best solution obtained with a QPSO-RM with SS-LB is one of the best solutions available in the literature.

In case of TEAM 22 problem the enhanced QPSO algorithms performed better in terms of stability providing smaller mean and standard deviation values. However, the best solution is given surprisingly by the basic QPSO.

For both testing problems the frequency of structure (connections) change has also been studied. A low frequency was more suitable for the SS-LB strategy. For the other two strategies a higher frequency leads most of the times to better results, but the optimal frequency also depends on the QPSO algorithm.

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APPENDIX

Table 1: Objective function values and standard deviation for Loney's solenoid.

Algorithm		Min-best OF value	Max - best OF value	Mean - best OF value	Standard deviation
QPSO	classic	9.73E-09	1.13E-06	7.72E-08	2.03E-07
	SS–LB–LF	4.09E-09	1.78E-07	**3.04E-08**	**3.47E-08**
	SS–LB–HF	9.89E-09	2.05E-07	3.87E-08	3.63E-08
	SS–GB–LF	1.02E-08	5.99E-07	6.47E-08	1.19E-07
	SS–GB–HF	**3.87E-09**	2.38E-06	1.21E-07	4.30E-07
	INF–LF	9.29E-09	3.33E-07	4.07E-08	5.99E-08
	INF–HF	8.64E-09	**1.71E-07**	3.33E-08	3.47E-08
QPSO RM	classic	6.02E-09	4.04E-07	3.99E-08	7.33E-08
	SS–LB–LF	**1.05E-10**	**1.80E-08**	**1.07E-08**	**5.33E-09**
	SS–LB–HF	2.89E-09	8.23E-08	2.42E-08	1.52E-08
	SS–GB–LF	1.17E-10	4.13E-07	3.84E-08	9.68E-08
	SS–GB–HF	3.93E-09	2.55E-07	2.64E-08	4.85E-08
	INF–LF	1.32E-08	3.78E-07	6.77E-08	8.62E-08
	INF–HF	7.41E-09	2.21E-07	4.37E-08	5.15E-08
QPSO Gauss	classic	8.03E-09	2.17E-07	3.01E-08	4.08E-08
	SS–LB–LF	4.33E-09	5.28E-08	1.93E-08	1.04E-08
	SS–LB–HF	**1.88E-09**	7.95E-08	2.18E-08	1.40E-08
	SS–GB–LF	7.09E-09	4.29E-07	4.03E-08	7.91E-08
	SS–GB–HF	1.23E-08	1.39E-07	3.43E-08	3.17E-08
	INF–LF	1.02E-08	8.76E-08	2.44E-08	1.73E-08
	INF–HF	1.25E-08	**3.47E-08**	**1.92E-08**	**5.34E-09**

Table 2: Objective function values and standard deviation for TEAM 22.

Algorithm		Min - best OF value	Max - best OF value	Mean - best OF value	Standard deviation
QPSO	classic	**2.23E-03**	2.76E-02	8.68E-03	6.46E-03
	SS–LB–LF	4.69E-03	3.17E-02	8.78E-03	5.84E-03
	SS–LB–HF	4.48E-03	1.65E-02	7.71E-03	3.15E-03
	SS–GB–LF	3.85E-03	**1.06E-02**	**6.76E-03**	**1.93E-03**
	SS–GB–HF	3.65E-03	2.71E-02	7.46E-03	5.08E-03
	INF–LF	5.21e-03	3.38e-01	3.46e-02	7.82e-02
	INF–HF	4.12e-03	3.56e-02	1.17e-02	8.71e-03
QPSO RM	classic	3.09E-03	2.17E-02	7.31E-03	4.85E-03
	SS–LB–LF	3.50E-03	1.40E-02	**5.33E-03**	2.55E-03
	SS–LB–HF	4.53E-03	**9.31E-03**	6.11E-03	**1.26E-03**
	SS–GB–LF	**2.78E-03**	1.77E-02	6.37E-03	4.01E-03
	SS–GB–HF	3.64E-03	1.01E-02	6.16E-03	1.80E-03
	INF–LF	3.21e-03	9.48e-03	5.93e-03	1.81e-03
	INF–HF	3.65e-03	1.02e-01	1.24e-02	2.15e-02
QPSO Gauss	classic	2.66E-03	9.54E-03	5.53E-03	1.54E-03
	SS–LB–LF	**2.58E-03**	8.95E-03	**4.27E-03**	1.56E-03
	SS–LB–HF	4.10E-03	9.25E-03	6.35E-03	1.54E-03
	SS–GB–LF	4.54E-03	1.17E-02	6.76E-03	2.18E-03
	SS–GB–HF	4.61E-03	**7.65E-03**	5.95E-03	**8.79E-04**
	INF–LF	4.40e-03	9.86e-03	6.71e-03	1.73e-03
	INF–HF	4.08e-03	1.69e-02	7.16e-03	3.08e-03