Electric Power System Operation: A Petri Net Approach for Modeling and Control

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Abstract: In this paper the operation of electric power system is treated as a discrete event system and a Petri Net is used as formal tool to achieve it. Each component of power system is modelled as a single Petri Net and the complete model is reached by composition of these single Petri Nets. Properties such as: parallelism, conflict, concurrency and others are used to study the operation of the modelled system. In this study are detected undesired behaviors on the dynamics of system. The theory of supervisory control is used to avoid these undesired behaviors forcing the system to have a controlled behavior. To obtain a controlled system a set of constraint are modelled by linear inequalities and the systems is forced to obey it. A case study application is presented to illustrate the proposed model here.

1 INTRODUCTION

The power system is modeled using math tools such as, differential equations and the main goal is to obtain the optimal operation state. In the basic level, the power system generates, transforms and distributes the electric power to the loads.

Next, there is the layer of control equipment. This equipment helps maintain the power system at its normal voltage and frequency, generates sufficient power to meet the load and maintains optimum economy and security in the interconnected network. The control equipment is organized in a hierarchy of its own, consisting of local and central control functions.

The equipments control of electric power systems is governed by a set of procedures that are required to ensure that any intervention on the system and will be performed under safe condition, considering the inherent dangers coming from the presence of live conductors, parts and components. In the last years the formalization of the general rules and methods adopted in the preparation, control and actuation of electrical procedures has been proposed. The Electric Power System modeling using Petri nets is one of this formal methods used by researchers.

In this sense, the system evolution and its state changes is obtained in graphical and analytical form allowing the better understand of logic in the procedures and to allow its immediate consistency. In addition, the check of the correctness of the procedures’ operating sequences could be followed. In the work of (A. Ashouri and Noroozian, 2010); the Petri nets is used, in addition of SCADA system, to diagnose fault protection elements. In (Vescio et al., 2015) has been used Marked Petri nets to represent EPS and through its dynamics and mathematical structure to find undesirable conditions. In this work is presented the equipment control of electric power systems using supervisory control theory and Place Invariants of Petri nets (Lima and Dória, 2002).

In this sense, this paper is organized as follow: In the section 2 is presented the theoretical basis of supervisory control using place invariants of Petri Nets(PN). Electric Power Systems(EPS) is described in the section 3. The case study is treated in the section 4. Conclusions and future expectations end this work.

2 SUPERVISORY CONTROL

Many researchers used Petri nets as a tool for modeling, analyzing and synthesize control laws for DES. There are many works applied in the literature (Murata, 1989),(Boissel and Kantor, 1995),(Zhou and

Assuming the model of a given modeled plant in Rdp is \((N, M_0)\) where \(N = (P, T, D, W)\) and \(D = Post - Pre\) is a set of all reachable markings starting in the initial mark \(M_0\) in the corresponding PN. Assuming also that the control objective is to restrict the state plant evolution from state \(S\) to subset \(S' \subseteq S\). This constraint imposed by the control is described by a set of linear inequalities:

\[
L \cdot M(k) \leq b
\]  

(1)

Where \(L \in \mathbb{Z}^{m\times n}\), \(b \in \mathbb{Z}^n\), \(M \in \mathbb{Z}^m\).

The computed controller will act not only to allow the firing for a transition leading to the occurrence of an unwanted state. It sets an extreme condition of the controller action. To determine the unknown parameters of the controller: \(M_c(0) \in D_c\) is calculated as follows:

From the control specifications given in equation 1 is noted that

\[
L \cdot M(k) + M_c(k) = b,
\]  

(2)

where \(k = 0, 1, \ldots m\) and \(M_c(k)\) is a positive vector of integers assumed as a break to make the inequality in equality. For \(k = 0\) is determined that the driver mark is given by:

\[
M_c(0) = b - L \cdot M(0)
\]  

(3)

2 - Multiplying both sides of equation 5 by the matrix \([L, I]\) and applying invariance property from the equation 2, it is determined that the controller incidence matrix is given by:

\[
D_c = -L \cdot D
\]  

(4)

The equations 3 and 4 are used for solve control problems. The equation 3 is used to compute initial marking to the controller while equation 4 show how Places-controller are linked with transitions in the plant (EPS model) (Lima and Dórea, 2002), (Moody, 1998).

The controllers are places linked by arcs in the model \(pc_1, \ldots, pc_q\) and their marking \(M_c(k)\in N^n\) respectively. The initial marking and the way the controller is connected to transitions can be obtained by taking an extended Petri net with \((M, M_c)^T\). If a transition \(t_j\) fire, then the PN state change according the equation 5.

\[
\begin{pmatrix}
  M(k + 1) \\
  M_c(k + 1)
\end{pmatrix} = \begin{pmatrix}
  M(k) \\
  M_c(k)
\end{pmatrix} + \begin{pmatrix}
  D \\
  D_c
\end{pmatrix} \cdot \sigma_j
\]  

(5)

3 ELECTRIC POWER SYSTEM

The Electric Power Systems can be defined as a set of physical equipment and connected electrical circuit elements, which act in a coordinated manner in order to generate, transmit and distribute electrical energy to consumers. The generation makes up the function of converting some form of energy into electrical, transmission carries electricity from production centers to consumption centers or to other electrical systems, connecting them. Distribution distributes the power received from the transmission system to large, medium and small consumers (Vescio et al., 2015).

3.1 Modeling of Electric Power Systems (EPS)

The electrical system should be carefully represented by appropriate modeling tool. The tool has relationship to type of study to be performed. For protection studies, for example, values of short-circuit currents should be calculated. Therefore, each system component must be modeled and represented from the perspective of its behavior to short-circuit currents. This modeling is relatively easy due to the simplifications made in the equivalent circuits of the components. The suitability of the model for studies of short-circuit is made with the use of symmetrical components, which leads to the obtaining of three system models: positive sequence, negative sequence and zero sequence (Grainger and Stevenson, 1994).

3.2 Single Line Diagram of EPS

Because the system operates normally balanced, it replaces its three-phase representation by a symbolic representation, known as single-line diagram. The importance of the single-line diagram is clearly present the topology and concisely the significant power system data. The single-line diagram can contain different information depending on the type of study, such as for power flow, short circuit, stability and protection (Anderson, 1998). An example of single-line diagram is shown in Figure 1.

3.3 Elements of Electric Power System

The electric power systems can be composed of some basic components that together have the function generate, transmit, distribute or connect other electrical power systems (Grainger and Stevenson, 1994). Some of these elements are:

1. **Generator** - Element active power generator
2. **Power Transformer** - They increase or decrease the currents and voltages of the EPS
3. **Transformer as Measure Instrument** - parameters in order to monitor, control and protection.
4. **Bus** - elements used as points of interconnection between the EPS components.

5. **Breakers** - Switching used to turn on or off a EPS normal or abnormal condition.

6. **Switch** - Device designed to isolate (sectioning) parts (subsystems, equipment etc.) of electrical circuits. They are installed aiming at breaking the network to minimize the effects of planned outages or not, establish visible sectioning in equipment such as automatic circuit reclosers, switches oil, establish bypass in equipment such as voltage regulators, etc. (Grainger and Stevenson, 1994).

### 4 MODELING OF EPS OPERATION USING PLACE-TRANSITION PETRI NETS

This chapter is a case study of modeling a EPS using Place-Transition Petri net. Therefore, it has to be a single line diagram EPS consisting of 2 buses ($B_1$ and $B_2$) 6 circuit breakers ($D_1, D_2, \cdots, D_6$), 5 switches ($S_1, \cdots, S_5$) 1 transformer and 3 energy consumers. Figure 1 shows the single line diagram for this study.

#### 4.1 Modeling of the EPS Free Behavior

The development of the free behavior model in PN associated to elements of the EPS Figure 1, taking into account:

- breakers and switches have two possible states on or off
- The transformer $T$ and buses $B_1$ and $B_2$ will be modeled by three states, which are power off state, energized via feeder $PS_1$ and energized via feeder $PS_2$
- In this model will not be provided abnormal operation of the elements.

The Figures 2 and 3 show these representations in Petri net. The presence of token in a place will indicate the current state of the element.

From the individual model, determines the complete model of the system. The composition of $PS_1$ feeder is shown in Figure 4. The places of $P_1$ to $P_6$ represent the sets of circuit breaker and switches that will energize the input bus $B_1$, $P_7$ represents that Bus $B_1$ is energized via $PS_1$ and $P_8$ the opposite state. the complete model is made by 6 places representing $PS_2$ to $P_3$. The same way bus $B_2$, the transformer and the output bus $B_2$ have in their models information about power supply origin see Figure 3. They have three places those represent energized via $PS_1$ or $PS_2$ and another to represent the off-state. The transformer
is represented by the states $P_{19}$, $P_{20}$ and $P_{21}$ and the output bus $B_2$ by the states $P_{24}$, $P_{25}$ and $P_{26}$ respectively. Power up these elements are linked to $S_5$ and $D_3$ when $P_{18}$ and $P_{23}$ are marked. All consumers are powered up by individual circuit breakers represented by places $P_3$ to $P_5$ that enable energy consumption by two distinct paths $PS_1$ and $PS_2$. To represent consumers, the model has places of infinite capacity with dual transition. The enabled transitions provide information about the origin of the supplied energy. The even transitions are enabled and fired when the power supply is $PS_1$ and odd transitions $PS_2$.

### 4.2 Supervisory Control Theory Applied to EPS

The free behavior of the proposed example presents several EPS operating problems. There are characteristics of power components that make up a EPS to be taken into account when turn on or off the power. As shown in (Zhao and Mi, 2006) breakers are the EPS equipment designed to make power on and off. Switches should isolate or integrate a region of a EPS when in power off state.

To avoid the modeled example may have improper representation (e.g., open switch when circuit breaker is on state) should expand this PN model putting supervisors places distributed in all sectors of the prior PN. These places are dedicated to develop constraints in breakers and switches models as shown in (Zhao and Mi, 2006). The following constraints are identified in the studied model:

1. The procedures for energization via $B_1$ must start with the connection of the keys $S_3$ and $S_4$. For this, you should impose the following constraint, $M_{p_6} + M_{p_8} \leq 2$;

2. After the $P_{36}$ controller close the switches $S_1$ and $S_2$, the circuit breaker controller of $D_1$ receives the privilege for its opening or closing, $2 \times M_{p_8} + M_{p_6} \leq 2$;

3. Similarly, energization via $B_2$ must start with the connection of the keys $S_5$ and $S_6$. For this, you should impose the following constraint, $M_{p_6} + M_{p_8} \leq 2$;

4. After the controller $P_{38}$ close the switches $S_3$ and $S_5$, the circuit breaker controller of $D_2$ should gain the status to take any action on it. This restriction is described as follows: $2 \times M_{p_1} + M_{p_8} \leq 2$;

5. The switch $S_3$ may change status (on / off) when the input bus is power off. This is possible to create a Constraint from the state that is power the bus off $P_{16}$ and the state is $S_5$ connected $P_{38}$ then we have: $M_{p_{16}} + M_{p_{18}} \leq 1$;

6. The controller for handling breaker $D_3$ of the output bus energization will have autonomy to operate it only when the status of switch $S_5$ is on. This condition generates inequality linked with the marking of the states that represent them, $P_{17}$ and $P_{22}$. It is then: $M_{p_{17}} + M_{p_{22}} \leq 1$;

7. The last supervisor shall be responsible for routing the way which will be energized when the bus power down. The supervisor function is monitor the $B_1$ power down through the place $P_{16}$. It is write the following restriction: $M_{p_{16}} + M_{p_{18}} \leq 1$

To illustrate the implementation of the controllers consider the inequalities generated from the feeder free behavior model $PS_1$. As shown in the literature, see (Grainger and Stevenson, 1994), the moveables of switches and circuit breakers must comply their constructive features as real representative system. Thus, the model dynamic should express such conditions. So supervisor control theory will restrict the model dynamics. The new PN transform the free behavior in the actual operating conditions of EPS elements. From item 1 and 2 above, follow the equations:

a) Constraint $M_{p_6} + M_{p_8} \leq 2$

$$
\begin{pmatrix}
0 & 1 & 0 & 0 & 1
\end{pmatrix} \times \begin{pmatrix}
M_{p_1} \\
M_{p_2} \\
M_{p_3} \\
M_{p_6} \\
M_{p_8}
\end{pmatrix} \leq [2]
$$

b) Constraint $2 \times M_{p_6} + M_{p_{36}} \leq 2$

$$
\begin{pmatrix}
0 & 0 & 2 & 0 & 0 & 1
\end{pmatrix} \times \begin{pmatrix}
M_{p_1} \\
M_{p_2} \\
M_{p_3} \\
M_{p_6} \\
M_{p_8} \\
M_{p_{36}}
\end{pmatrix} \leq [2]
$$

Extracting the PNs for both cases the incidence matrices and the initial marking ($D_1$ and $M_{10}$) and ($D_2$ and
\[ M_{20} \text{ we have:} \]
\[
\begin{pmatrix}
-1 & 1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 \\
\end{pmatrix}
\]

\[ D_1 = \]
\[
\begin{pmatrix}
-1 & 1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 \\
\end{pmatrix}
\]

\[ e \; M_{01} = (1 \ 0 \ 1 \ 0 \ 1 \ 0)^T \]

\[ D_2 = \]
\[
\begin{pmatrix}
-1 & 1 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & -1 \\
\end{pmatrix}
\]

\[ e \; M_{02} = (1 \ 0 \ 1 \ 0 \ 1\ 2)^T \]

by these information and using equations 3 and 4 are found the controller 1 and 2 characteristics. Such information are represented in the expanded Petri net by places \( P_36 \) and \( P_37 \):

a) controller1 Features

\[ D_{c1} = (1 \ 1 \ 0 \ 0 \ -1 \ 1) \]

\[ M_{c10} = [2] \]

b) Controller2 Features

\[ D_{c2} = (1 \ -1 \ 2 \ -2 \ 1 \ -1) \]

\[ M_{c20} = [0] \]

Figure 5 shows the expanded model of Petri net feeder \((PS_1)\) bus of the EPS case study. The Figure 5 shows that the PN is started with switches and Breaker in off state \((P_1, P_3 \text{ and } P_5 \text{ with marking})\). In this condition, only Controller1 is enabled \((P_{36}, 2 \text{ marks})\) allowing the switches models \(S_1 \text{ and } S_2\) to go to the on state. \(t_1 \text{ and } \)

\[ t_5 \text{ firing takes } P_2 \text{ and } P_6 \text{ to receive tokens representing the switches were turned on. This sequence of firing takes disabling Controller1 and enabling Controller2 that received 2 marks. This condition enables the transition } t_5 \text{ firing takes the model of } D_1 \text{ to on state, meaning that } B_1 \text{ was energized. The same condition can be obtained by the equation } 5 \]

\[
\begin{pmatrix}
M_1 \\
M_2 \\
M_3 \\
M_4 \\
M_5 \\
M_6 \\
M_{c1} \\
M_{c2}
\end{pmatrix} = \begin{pmatrix}
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
2 & 0 \\
0
\end{pmatrix} + D \begin{pmatrix}
D_{c1} \\
D_{c2}
\end{pmatrix} \sigma
\]

Where

\[ (D) = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 \\
-1 & 1 & 0 & 0 & -1 & 1 \\
-1 & -1 & 2 & 1 & -1 & 0
\end{pmatrix} \]

and \( \sigma \) is the transition sequence that will fire, \( t_1 t_3 t_5 \)

computing get the following result:

\[
\sigma = (1 \ 0 \ 1 \ 0 \ 1 \ 0)^T
\]

The result shows the PN conditions obtained in previous analysis. Places \( P_2 \), \( P_4 \) and \( P_6 \) receive marks representing the circuit breaker and switches in on states. This condition makes the transition \( t_{13} \) enabled, if fire, place \( P_{16} \) loss marking \((Bus \; Off)\) to place \( P_{14} \) \((Bus \; On)\) indicating bus \( B_1 \) power on via \( PS_1 \). Note that transition \( t_{13} \) is connected to places by self-loops, this connection maintain marking after \( t_{13} \) firing. This is critical to guarantee the states of switches and breakers with bus power on. The Figure 6 is a simplified way to represent EPS PN model studied with controllers those restrict the free behavior of their switches and circuit breakers. The figure shows two-way arrows to show the flow of PN-controller information. Thus, if transition \( t_j \) fires, the controller receives or losses marking given by the weight of its arcs. The set of controllers \( C_1, C_2, C_3, C_4, C_3 \text{ and } C_6 \) receive information through their enabled input transitions or disable other output transition. these actions impose free behavior model(without control) comply the constraints that are imposed. Such dynamic avoid firing transitions those take the PN to forbidden

Figure 5: Petri Model of \( PS_1 \) Feeder.
The electric power system operation is completely modeled by a Petri nets. The model can be achieved through the creation of single PN for each part of system. The free behavior of each elements that make up the system then are interconnected to form the representation of power system. This representation is free of any constraint of the elements make up the EPS. This article was called free model or free behavior of the electric power system.

The existing memory effect on the opening and closing of a circuit breaker in the PN model was solved using a feature of Petri nets called self-loop. This is done by interconnecting one place and transition by two arcs in opposite directions to each other. Thus, the turn on circuit breaker do not lost a token by energizing a bus transition is enabled by the place up switch can shoot without any loss of the marking place in dynamic network.

The model of free behavior of the electric power system does not describe the actual behavior of EPS elements. Decoupling between individualized models of its elements provide the appearance of undesirable states or states that would cause risk to integrity of the EPS. To eliminate such states, the model was expanded using local supervisors. The Local supervisors are places inserted in the free model that have the ability to restrict undesirable states through inhibition of controllable transitions firing. The implementation of these controllers are made from inequalities that describe such constraint. These inequations determining the initial marking, the weights of input and output arcs for each supervisor. The evolution of the dynamics of the new network is presented as a sequence of markings of places that fully comply with the procedures and or care that you have to turn on or off an electric power system without weakening the actual functionality of the EPS.

The existing memory effect on the opening and closing of a switcher and breaker in the PN model was solved using a feature of Petri nets called self-loop. This is done by interconnecting one place and transition by two arcs in opposite directions to each other. Thus, the turn on switcher do not lost a token by energizing a bus transition is enabled by the place up switch can shoot without any loss of the marking place in dynamic network.

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the procedures and/or care that you have to turn on or
off an electric power system without weakening the
actual functionality of the EPS.

Going beyond that as future work should improve
the model by entering the protection elements and ap-
ply this model in the study of diagnosability.

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