Numerical Research on Water Hammer in Propellant Filling Pipeline based on Spectral Method

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Abstract: In order to research the water hammer problem in the filling pipeline during the rocket propellant filling process for the spaceflight launch site, improved schemes are proposed. Chebyshev spectral method is adopted to solve the water hammer problem in the paper. The law of pressure change is analyzed when water hammer occurs, and the results calculated by the spectral method are compared with the results calculated by the characteristic line method and the experimental results. The results show that the proposed schemes can effectively weaken the water hammer in the pipeline during the filling process, improve the reliability and security of the filling system, and verify the feasibility that adopting the Chebyshev spectral method to solve the water hammer problem in the filling pipeline.

1 INTRODUCTION

The rockets propellant filling system is an important part of the spaceflight launch site, and the filling pipeline is a key assembly in the filling system. Accurately grasp the work state of the filling pipeline in the rocket propellant filling process is very important for the filling accuracy and the reliability and security of the system (Xiang, 2015).

The filling pipeline in the launch site can provide route way for the propellant transporting from the storehouse horizontal tank to the rocket tank. Its stability, reliability and security is very important for the success of the spaceflight launch. The water hammer is a water power phenomena in the pipeline that the water flow rate changed suddenly, leading to the pressure rise and fall sharply, caused by some external reasons, such as the valve suddenly open or close. In the process of rocket propellant filling in the spaceflight launch site filling system, it often takes place the phenomena that the pressure of the filling pipeline is far higher than the designed pressure, and it is far more than the normal working pressure range. It is a potential danger for the system.

The water hammer can damage equipment, increase the fault probability of the system. It also can cause violent vibration of the filling pipeline, result in measurement error for the vortex-shedding flowmeter. It will reduce the actual propellant filling precision. Therefore, it requires numerical research on water hammer problem in the filling system, and proposes effectively improved schemes. It is very important for improving the filling precision and guaranteeing the success of rocket launch.

The rest parts of the paper are organized as follows: Section 2 introduces the related work. Section 3 adopts the spectral method to solve the water hammer problem. The experimental results are compared and analyzed in section 4. Section 5 makes the conclusions.

2 RELATED WORK

The characteristic line method is widely applied for solving the water hammer problem. (Liu, 2005) Firstly it changes the partial differential equation into ordinary differential equation along the characteristic line, and then changes it into first order finite difference equation. The method can solve the water hammer problem of complicated piping system with boundary conditions, and the calculation accuracy of the method is high.

In literature (Yan, 2012), Yan Zheng researches the water hammer problem of the spacecraft propulsion system in the processes of priming and shutdown. On the basic of the established simulation...
model of the spacecraft propulsion system, the simulation research was conducted and the suppression effect of water hammer for the orifice and bent duct was analyzed. In literature (Lin, 2008), Lin Jing-song studies the fluid transients of the propellant pipes after the liquid rocket engine shut down, and carries out numerical simulation of water hammer in shutting liquid rocket engine based on the method of characteristic line. The correctness of the simulation results was approved by comparison with the experiment data. In literature (Nie, 2003), Nie Wan-sheng researches the pressure and the flow transients characteristic when the liquid rocket engine system shut down based on the method of finite difference characteristic line. In literature (Liu, 2010), Liu Zhao-zhi analyzes the water-hammer problem based on the characteristic line method for the actual pipeline structure in the liquid hydrogen filling system, and the useful measures are proposed to reduce peak pressure of the water-hammer.

The following are the steps that using the characteristic line to solve the water hammer problems. The first step: the partial differential equation that can’t directly to solve should be changed into a specific form of ordinary differential equation, namely characteristic line equation. The second step: carrying through integral calculus for the ordinary differential equations, getting the approximate algebraic integral formula, namely finite difference equation. The third step: according to the finite difference equation and bound condition equation of piping system to calculate. However, when adopting the spectral method to solve water hammer in the filling pipeline, the boundary conditions are complicated, the coordination of time step is difficult, and the nonlinear iterative convergence is slow.

The spectral method is discrete method for a kind of partial differential equation. It is a calculation method that takes orthogonal function or inherent function as the approximate function. The spectral approximation contains two approximate ways, that is function approximation and equation approximation (Wang, 2001). On the way of function approximation, the spectral method contains three methods: the Fourier method, the Chebyshev method and the Legendre method. The former is suitable for the periodic problem, and the latter is suitable for aperiodic problem. On the way of equation approximation, the spectral method contains Collocation method, Galerkin method and Pseudo-spectral method. The Collocation method is suitable for the nonlinear problem in the physical space. The Galerkin method is suitable for the linear problem in the spectral space. The Pseudo-spectral method is suitable for nonlinear term processing in the combination of physical space and spectral space.

The main characteristic of the spectral method is fast convergence speed, no phase error, higher precision and global. It makes the spectral method be widely adopted in high precision calculation. In literature (Chen, 2012), Chen Hong-yu proposes a new algorithm that adopting the Fourier spectral method to solve the nonlinear hyperbolic partial differential equations for governing the fluid transient. By adopting the method, it solves the water hammer and pressure oscillation formed in the pipeline when the valve is shut down. It proves the credibility of the method. In literature (Chen, 2013), Chen Hong-yu proposes the Chebyshev spectral method to solve the nonlinear hyperbolic partial differential equations for governing the fluid transient in the propellant pipelines. It solves the water hammer problem in the pipeline when the valve is shut down by the method, and proves the feasibility of the method.

In order to further analyzing the generating mechanism of water hammer problem in the filling pipeline and the water hammer change law influenced by the control process of filling system, and researching the scheme weakening the water hammer problem in the filling pipeline, the Chebyshev spectral method is adopted to solve the water hammer problem in the filling system in the paper.

3 SOLVE THE WATER HAMMER PROBLEM BASED ON THE CHEBYSHEV SPECTRAL METHOD

3.1 Basic Differential Equation of Water Hammer

The theoretical basic of the water hammer basic equation is the mechanics law and continuous principle of water flow movement. It includes the motion equation and the continuity equation which expressed in differential equation. It reflects the flow velocity of instability flow and the changing rule of water head in the process of hydraulic transient (Xiang, 2015), (Lin, 2007).

The continuous differential equation of water hammer is:

\[
\frac{d}{dx} \left( \rho \frac{dU}{dx} \right) = -g \rho h \frac{dU}{dx} + F
\]
The motion differential equation of water hammer is:

\[
\frac{\partial h}{\partial t} + v \frac{\partial h}{\partial s} + \frac{c^2}{g} \frac{\partial v}{\partial s} = 0
\]

(1)

In the equation, \(v\) is the flow velocity of water hammer, \(h\) is the piezometric head of water hammer, \(c\) is the wave velocity of water hammer, \(s\) is the distance, \(\theta\) is the angle between pipeline and horizontal plane, \(g\) is the acceleration of gravity.

\(D\) is the pipe diameter, \(f\) is the pipe friction coefficient, \(\theta\) is the pipe wall thickness.

3.2 Solve the Problem by the Chebyshev Spectral Method

The basic differential equation of water hammer is a first order quasilinear hyperbolic partial differential equations thanks to considering the loss of the frictional head. The equations contains two dependent variables and two independent variables. It is difficult to accurately solve the equations.

\[
\frac{\partial h}{\partial t} + \frac{\partial}{\partial s} \left( \frac{c^2 h}{g} \right) + \frac{f}{2D} h = 0
\]

(2)

The basic differential equation of Water Hammer is solved based on the Chebyshev spectral method in this paper. Chebyshev spectral method takes the Chebyshev polynomial as the basis function. The function defined in the computational domain can be solved based on the Chebyshev spectral method in this paper. Chebyshev Spectral Method Solve the Problem by the weighted residual method.

When adopting the Chebyshev spectral method, firstly we take \(N+1\) Chebyshev-Gauss-Lobatto (CLG for short) (Yang, 2015) points in the interval of [-1,1]. Namely, \(\tau_n = \cos \frac{\pi}{N} n, n=0,1,2,\cdots,N\).

Then the Chebyshev polynomial is as follow:

\[
T_m(\tau_n) = \cos[m \arccos(\tau_n)], m=0,1,2,\cdots,N
\]

(3)

The approximate values of state variable and control variable for the basic differential equation of water hammer are:

\[
h(\tau) \approx h_N(\tau) = \sum_{k=0}^{N} h(\tau_k) \varphi_k(\tau)
\]

(4)

\[
v(\tau) \approx v_N(\tau) = \sum_{k=0}^{N} v(\tau_k) \varphi_k(\tau)
\]

(5)

For \(k=0,1,2,\cdots,N\), the N-order Lagrange polynomial is as follow:

\[
\varphi_k(\tau) = \frac{(-1)^{k+1}}{N! c_k}, \frac{(1-\tau^2)}{T_N'(\tau)}
\]

(6)

In the above equation, \(T_N'(\tau)\) is the derivative of \(T_N(\tau)\) which is the N-order Chebyshev polynomial, and parameter \(c_k\) satisfy the following condition:

\[
c_k = \begin{cases} 2 & k = 0, N, \text{ and } D_{jk} = \varphi_k'(\tau_j) \cdot D_{jk} \text{ is the Chebyshev differential matrix, the expression is as follow:} \\
0, 1 \leq k \leq N - 1
\end{cases}
\]

(7)

The water hammer problem has discontinuous solutions, and larger oscillation can be produced near the discontinuity point. In order to solve the problem, the viscous term is introduced in the equation (Chen, 2013), (Ma, 2006). For the continuous differential equation, the viscous term is \(\varepsilon(-1)^{i+1}Q^2 h_N\). For the motion differential equation, the viscous term is \(\varepsilon(-1)^{i+1}Q^2 v_N\). In the viscous term, \(\varepsilon\) is the viscous amplitude, \(Q\) is the viscous operator, and \(\varepsilon = CN^{1-2i}\).

The calculation formula of water hammer wave velocity is (Xu, 2012): \(c = \sqrt{\frac{K/\rho}{1 + (K/E) (D/\delta)}}\). In the formula, \(K\) is the fluid bulk modulus, \(\rho\) is the fluid density, \(E\) is the piping materials elastic modulus, \(D\) is the pipe diameter, \(\delta\) is the pipe wall thickness. According to the calculation formula of water hammer wave velocity, the water hammer wave velocity of oxidant pipeline in the propellant filling system can be get through calculation, \(c = 850\text{m/s}\).

4 EXPERIMENTAL RESULT

The pipeline model for the rocket propellant filling system is established as Fig.1. In the Fig, the rocket...
tank is vertically located on the launch pad and on one horizontal plane. Except that, the equipments such as pump, flowmeter, valve and pressure gauges, are all located in the pump room of the filling storehouse and on the same horizontal plane. The height from the 125# valve to the filling port of the rocket oxidizer tank is 30 meter or so, and they are connected together through the filling pipeline.

![Diagram of pipeline model for the filling system](image)

**Figure 1: Pipeline model for the filling system.**

The fluid in the pipeline is N₂O₄, and the physical parameter of N₂O₄ at the temperature of 20°C is: the density \( \rho = 1.446 \text{g/cm}^3 \), the viscosity \( \mu = 0.4189 \times 10^{-3} \text{Pa·s} \), the saturation pressure \( P_s = 0.096 \text{MPa} \), the fluid bulk modulus \( K = 1.27 \text{GPa} \).

Before water hammer in the filling pipeline occurs, the initial state is: the opening of electric control valve is 30%, the frequency of pump inverter is 50Hz, the state of the 121# 124# and 134# valve is open, and the state of the 122# and 125# valve is close.

In order to verify the validity of adopting the Chebyshev spectral method to solve water hammer problem in the filling pipeline, the following four experiments that reduce water hammer in the pipeline are calculated. The results calculated by the Chebyshev spectral method are compared with the results calculated by the characteristic line method and the experimental results.

**Experiment 1:** According to the existing filling process, that is: open up 125# valve, delay of 1 second, at the same time close 124# and 134# valve, the pump speed is 50Hz, the opening of electric control valve DT4 is 30%. The calculation results are shown in Fig. 2.

Fig. 2 shows the comparison and pressure change law when water hammer happens under the existing process.

![Graph showing comparison and pressure change law under the existing process](image)

**Figure 2: Comparison and pressure change law under the existing process.**

The ordinate shows the pressure, and the abscissa shows the test time. The read curve shows the results calculated by the Chebyshev spectral method, the black curve shows the results calculated by the characteristic line method, and the blue curve shows the real experimental data. From the Fig we can know, before water hammer happens, the pressure of the filling pipeline is 0.25MPa. When water hammer happens, the pressure increases rapidly, and the peak pressure calculated by the Chebyshev spectral method is as high as 3.22MPa. The pressure in the pipeline has changed dramatically, and there is 13 times difference of the pressure when water hammer happens.

**Experiment 2:** On the basis of the above experiment, we change the closed sequential of the related valve when water hammer happens. The closed sequential of the related valve are changed as follow: close the 124# valve, 1 second later close the...
134# valve. The calculation results are shown in Fig.3.

Fig. 3 shows the comparison and pressure change law when water hammer happens after changing the sequential. The ordinate shows the pressure, and the abscissa shows the test time. Through comparing Fig.2 and Fig.3, we can know that the scheme which changing the closed sequential of the related valve can effectively weaken the water hammer problem in the filling pipeline. Compared with the results in experiment 1, the peak pressure calculated by the Chebyshev spectral method is reduced from 3.22MPa to 2.81MPa, reduced by 12.7%.

Experiment 3: On the basis of the above experiment, we change the speed of the filling pump when water hammer happens. The speed of the filling pump is changed from 50Hz to 40Hz. The calculation results are shown in Fig.4.

Fig. 4 shows the comparison and pressure change law when water hammer happens after changing the speed of the filling pump. The ordinate shows the pressure, and the abscissa shows the test time. Through comparing Fig.3 and Fig.4, we can know that the scheme which changing the speed of the filling pump can effectively weaken the water hammer problem in the filling pipeline. Compared with the results in experiment 2, the peak pressure calculated by the Chebyshev spectral method is reduced from 2.81MPa to 2.5MPa, reduced by 11%.

Experiment 4: On the basis of the above experiment, we change the opening of the electric control valve DT4 when water hammer happens. The opening of the electric control valve DT4 is changed from 30% to 60%. The calculation results are shown in Fig. 5.

Fig. 5 shows the comparison and pressure change law when water hammer happens after changing the opening of electric control valve DT4. The ordinate shows the pressure, and the abscissa shows the test time. Through comparing Fig.4 and Fig.5, we can know that the scheme which changing the opening of electric control valve can effectively weaken the water hammer problem in the filling pipeline. Compared with the results in experiment 3, the peak pressure calculated by the Chebyshev spectral method is reduced from 2.5MPa to 2.28MPa, reduced by 8.8%.

Experiment 4: On the basis of the above experiment, we change the opening of the electric control valve DT4 when water hammer happens. The opening of the electric control valve DT4 is changed from 30% to 60%. The calculation results are shown in Fig. 5.

Experiment 4: On the basis of the above experiment, we change the opening of the electric control valve DT4 when water hammer happens. The opening of the electric control valve DT4 is changed from 30% to 60%. The calculation results are shown in Fig. 5.

The comparison of water hammer peak pressure under different experimental conditions is show in Fig.6. From the Fig, we can know that the water hammer peak pressure in experiment 1 is the highest, it decreased gradually in experiment 2, 3 and 4, and the water hammer peak pressure in experiment 4 is the lowest. There is little difference among the three peak pressures.
Overall, when adopting the spectral method to solve water hammer problem, boundary conditions is simple and computational efficiency is high. The results calculated by the spectral method under different experiments are well consistent with the results calculated by the characteristic line method and the experimental results. It shows that the spectral method can well solve the water hammer problem in propellant filling pipeline as well as the characteristic line method.

5 CONCLUSION

This paper researches the water hammer problem in the rocket propellant filling pipeline under the filling process of the spaceflight launch site, and analyzes the effects of filling process on water hammer. The law of pressure change is analyzed when water hammer happens. Improved schemes are proposed to weaken the water hammer in the filling pipeline. We adopt the Chebyshev spectral method to solve the water hammer problem, and present the calculation results. We can come to the following conclusions: (1) The Chebyshev spectral method can well solve the water hammer problem in propellant filling pipeline. (2) The proposed schemes can effectively weaken water hammer in the pipeline during the filling process, and improve the reliability and security of the filling system. Through numerical analysis for the different experiments, it can provide theoretical basis and data support for weakening water hammer problem in the filling system and optimizing filling process.

REFERENCES


