The Sourcing Problem
Energy Optimization of a Multisource Elevator

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Abstract: As the interest in regulating energy usage and in the demand-response market is growing, new energy management algorithms emerge. In this paper, we propose a formalization of “the sourcing problem” and its application to a multisource elevator. We propose a linear formulation that, coupled with a low level rule-based controller, can solve this problem. We show in the experiments that a compromise between reducing consumption peaks and minimizing the energy bill has to be reached.

1 INTRODUCTION
Reducing energy consumption is a major issue nowadays; not only in order to restrain the ecological impact on the planet, but also to both respect building norms and minimize industrial and residential activities’ energy bill. In order to achieve this goal, one can act on energy consumption by optimizing the amount of energy consumed, or by shifting consumption during the day. But one can also control one’s energy consumption impact, by adding renewable energy sources and storage units. Thus, an “energy hub” is created.

If one chooses this second possibility, a new optimization problem arises. The energy hub has to decide which energy source to be used at which moment. We call this problem “the sourcing problem” and formalize it in Section 2.

In this paper, we solve the sourcing problem for a multisource elevator, as the newest generations of elevators are equipped with energy storage to ensure a minimum autonomy in case of power failure. This is crucial for safety (e.g. to evacuate people with reduced mobility) and energy storage may also offer flexibility in power management. In Section 3, the reader is given an glimpse of what can be done for dealing with energy sourcing issues of those multisource elevators.

Section 4 gives details on our solution to solve the sourcing problem. First by describing interactions between our two coupled controllers. Then by explaining the linear formulation used to compute a sourcing strategy. And finally by giving some details on how our low-level rule-based controller takes strategy into account.

Afterwards, experiments are conducted in Section 5 to highlight advantages and drawbacks of that method, as well as parameters influencing its achievements.

Finally, Section 6 concludes the paper and introduces future work.

2 THE SOURCING PROBLEM
We call “prosumers”, entities that consume and/or produce energy. An energy hub allows each prosumer to consume power produced by all other prosumers at the same time.

Definition 1. Let \( P \) be a set of \( n_p \) prosumers, all connected to the same energy hub \( h \).
We differentiate three kinds of prosumers: the set \( S \) of storage units, the set \( P \) of controllable producers and the set \( E \) of the others. Thus, these sets form a partition of the whole set of prosumers.

**Definition 2.** Let \( S = \{ \pi^S_1, \ldots, \pi^S_n \} \) be the set of storage units, \( P = \{ \pi^P_1, \ldots, \pi^P_m \} \) be the set of controllable producers or consumers, and \( E = \{ \pi^E_1, \ldots, \pi^E_k \} \) be the set of the other prosumers. Then, \( \mathcal{P} = S \cup P \cup E \).

Each prosumer can consume and/or produce power, depending on its physical capabilities.

**Definition 3.** For a given prosumer \( \pi \in \mathcal{P} \), let \( p_i^{\min} \leq 0 \) (resp. \( p_i^{\max} \geq 0 \)) be the minimum (resp. maximum) instantaneous power of \( \pi \). Then, at a given time \( t \), let \( p_i^{\min} \leq p_i[t] \leq p_i^{\max} \) be the instantaneous power produced by \( \pi \) if \( p_i[t] \) is positive, or consumed by \( \pi \) if \( p_i[t] \) is negative.1

A system composed of an energy hub and its prosumers can be represented by a star oriented graph.

**Definition 4.** Let \( \mathcal{G} = (\mathcal{P} \cup h, \mathcal{A}) \) be a star oriented graph rooted in \( h \) where \( \mathcal{P} \cup h \) are the nodes of the graph and \( \mathcal{A} \) are the weighted arcs. There is an arc \((\pi, h)\) if \( p_i^{\max} > 0 \) and the weight of the arc is \( p_i^{\max} \). In the same way, there is an arc \((h, \pi)\) if \( p_i^{\min} < 0 \) and the weight of the arc is \( p_i^{\min} \).

We suppose that time can be sampled in a regular, uniform way.

**Definition 5.** Let \( \tau \in \mathbb{R} \) be the sampling period (expressed in hours), and \( H \in \mathbb{N} \) be the number of periods considered. Then time-steps are expressed in the following way: \( t_i = t_{i-1} + \tau = i \times \tau \). \( \forall i \in \{0, \ldots, H \} \).

Finally, each controllable producer has an energy cost function that gives the price associated to an energy consumption or production of this prosumer.

**Definition 6.** \( \forall \pi \in \mathcal{P} \), let \( c_{\pi_i} : [p_i^{\min}, p_i^{\max}] \rightarrow \mathbb{R} \) be the energy cost function associated to \( \pi_i \).

Then, we can define sourcing problems:

**Instance:** a set of prosumers \( \mathcal{P} = \mathcal{P} \cup \mathcal{S} \cup \mathcal{E} \),
a graph \( \mathcal{G} = (\mathcal{P} \cup h, \mathcal{A}) \),
a period \( \tau \in \mathbb{R} \),
a time horizon \( H \in \mathbb{N} \),
a set of cost functions:
\[
\{ c_{\pi_i}, \forall \pi_i \in \mathcal{P} \}
\]

**Solution:** \( S \), a \( n_H \times H \) matrix of \( p_i[t] \)

---

1Power is expressed in Watts and energy in Watt hours.

**Question 1:** Given \( p_{hub}^{\max} \in \mathbb{R} \) the allowed residual power of the energy hub, does a matrix \( S \) exist such that:
\[
\forall l \in \{0, \ldots, H - 1\},
- p_{hub}^{\max} \leq \sum_{i=1}^{n_H} p_i[t] \leq p_{hub}^{\max}
\]

Can the energy hub be autonomous?

**Question 2:** Given \( p_{\pi_i}^{\max} \in \mathbb{R} \) the allowed power peak of controllable producers, does a matrix \( S \) exist such that:
\[
\forall l \in \{0, \ldots, H - 1\},
\max_{\pi_i \in \mathcal{P}} p_{i}[t] \leq p_{\pi_i}^{\max}
\]

Can power peaks purchased from controllable producers stay below a given value?

**Question 3:** Given \( cost_{\text{hub}}^{\max} \in \mathbb{R} \) the allowed energy bill, does a matrix \( S \) exist such that:
\[
\sum_{l=0}^{H-1} \sum_{\pi_{i} \in \mathcal{P}} cost_{\pi_i} \times \pi_{i}[t] \times \tau \leq cost_{\text{hub}}^{\max}
\]

Can the energy bill stay below a given value?

These three objectives are sometimes antagonistic.

In this paper, we consider the following application of the sourcing problem. The set \( \mathcal{P} \) of prosumers is composed of: an elevator \( \pi_1 \), that can get energy from a battery \( \pi_2 \), a supercapacitor \( \pi_3 \), the grid \( \pi_4 \) and a dissipation resistor \( \pi_5 \).
a solar panel $\pi_5$. The supercapacitor is here to absorb power peaks above the maximum power capability of the battery. But the former is more expensive than the latter. Moreover, energy can be recovered from the elevator when the brakes are applied. Finally, energy can be dissipated in a resistor $\pi_6$ if there is too much. The partition of $\mathcal{P}$ is the following: $E = \{\pi_1, \pi_3\}$, $\mathcal{P} = \{\pi_2, \pi_6\}$, $S = \{\pi_2, \pi_3\}$. An illustration of the elevator sourcing problem is given in Figure 1.

As it is a real-life application, the three objectives corresponding to questions above have to be achieved simultaneously. The goal is to minimize power peaks purchased from the grid and the energy bill while ensuring that the hub is autonomous.

3 STATE OF THE ART

Energy optimization systems have been widely studied in the literature, especially in the past few years. In the following state of the art, we present research works dedicated to energy sourcing of multisource elevators.

A very comprehensive work on energy systems for elevators is proposed by (Paire et al., 2010). They have designed a physical multi-source system to power an elevator. In the paper, rules are used to charge or discharge batteries depending on whether the electrical current is below or above a given reference. This control method may be reduced to a simple “if, then, else” structure achieved by physical components. Moreover, energy can be recovered from the elevator. These methods allow controlling very reactively the system, but cannot take into account optimally external considerations such as the electricity tariff or battery state of health. Therefore, these control methods may not be efficient regarding economical objectives.

On the other hand, (Bilbao and Barrade, 2012) have proposed a General Energy and Statistical Description (GESD) of the possible missions of an elevator and an energy manager based on dynamic programming. Their energy manager is inspired by stock management theory and minimizes the sum of energy (i) absorbed from the grid, (ii) dissipated in the braking resistor and (iii) not provided to the elevator. The optimization is done off-line. This method deduces economically optimal solutions from consumption probabilities. But elevator usage is unpredictable by nature and there is no given alternative when the strategy is not applicable.

Finally, in (Sachs, 2005), authors summarize different ways to optimize choices of elevator physical components (motor, drive, etc). An appropriate sizing of these components is a way to optimize energy consumption but it should be coupled with a good control algorithm of multiple sources of energy.

From these observations, we have decided to propose a two-layer optimization that can achieve reactive control of low-level equipment as well as compute economically optimal sourcing strategy. As part of the European Arrowhead project, we published a first description of a linear program to solve the multisource elevator problem in (Desdouits et al., 2015). We also described, in (Boutin et al., 2014), the interactions between our control method and partner components. In the current paper, we formalize the sourcing problem, improve our linear formulation and give details on Local Controllers. More accurate experimental results than before are additionally given.

4 PROPOSED SOLUTION

In this section, we give a centralized rule-based algorithm for the energy hub controller, but we also could overlay another controller already implemented. We call these controllers “Local Controllers”, and we abbreviate LC. They have to be embedded and highly reactive, thus they cannot compute the best sourcing strategy on a long time-frame. Therefore, we decided to compute a sourcing strategy with a “Strategic Optimizer” (abbreviated SO), and to send next strategic instructions to LC regularly.

Definition 7. Let us call the plan computed by SO a strategy, and the set-point computed by LC a tactic.

Hereafter, we start by presenting data that feed components and interactions between them.

4.1 Data and Interactions

In this sub-section, we first describe how we draw samples of elevator usage. Then we explain how we compute forecasts used by SO. Finally, we show components interactions and their dynamic.

4.1.1 Elevator Usage Description

The building considered is a business tower with nine floors and the elevator has the following characteristics: standby consumption: 50 W, cabin mass: 750 kg, counterweight mass: 850 kg, nominal velocity: 1.0 m/s.
We simulate user calls to the elevator with a statistical model. This model distinguishes multiple types of travels: morning and afternoon arrivals and departures, lunch breaks, inter-floor travels, arrivals and departures of external visitors. Statistical laws are identified based on historical data. For each travel type and relevant pair of floors, these laws provide information on the number of people moving during the day (depending on day-of-week, week-of-year, etc.), the distribution of their weights, the distribution of the times of the people movements during the day, and the probability that two similar movements are grouped (e.g., several people going to lunch together). A random generator is used on this basis to generate scenarios.

Moreover, we have implemented a tactic to answer user calls to the elevator. That tactic considers calls in chronological order to choose the destination of the elevator. But, it stops the elevator along the way if another call destination is on this way. That seems to be the tactic implemented in the biggest part of the elevators.

On the other hand, we have an energy model of the elevator that allows us to compute energy consumption regarding the chosen travel and the weight of passengers. The data used in this model include the weight of the cabin and counterweight, the length and weight of the cable, the elevator’s base power and nominal speed, the altitude of the departure and arrival floors, and efficiency for both energy-consuming and energy-producing travels.

### 4.1.2 Forecasts

SO is fed by a forecast \( f \), that is a \( |\mathcal{P} \cup \mathcal{E}| \times H \) matrix, with \( H \) the number of periods considered by SO. For prosumers in \( \mathcal{P} \), the tariff is forecasted, while for prosumers in \( \mathcal{E} \), the produced and consumed quantity is forecasted. In this paper, forecasts are considered exact (issues concerning robustness to forecasts uncertainties will be considered in a future paper).

In our use case, a forecast is composed of: (i) the predicted amount of energy consumed / produced by the elevator, (ii) the predicted solar production, (iii) the predicted grid energy tariff. Dissipation is assumed to have no cost.

The ideal solar production prediction uses predicted irradiance data of a typical sunny day. We consider that 2 square meters of solar panels are installed on the roof top of the building, and are dedicated to the elevator. These solar panels are supposed well oriented towards the sun and having a yield of 15%.

The electricity tariff considered in the experiments is a typical peak / off-peak French tariff: 0.00015 / 0.00010 €/Wh.

Regarding the elevator, we use data described in sub-section 4.1.1 to simulate several daily scenarios and compute the average energy production or consumption for each SO period. An alternative could be to use standard machine learning techniques to directly forecast energy production or consumption for each SO period.

### 4.1.3 Interactions and Dynamic

LC has to be highly reactive. For the multisource elevator application, we estimate a relevant time-step is one second, iterated every seconds. On the other hand, SO has to consider sufficiently long time-steps to get relevant forecasts, and a sufficiently long horizon to take into account energy price variations. Thus, a fifteen minutes period with a 24h horizon is relevant for the multisource elevator, and the problem is resolved every hours. The dynamic of interactions is illustrated on Figure 2.

In a real product, LC would probably be embedded into the energy hub. While SO could be proposed as a web service. We can see on the figure that LC and SO are separated components that communicate only through a strategic instruction. An instruction is composed of a target time, and an array of \( n_p \) cells: one per prosumer connected to the energy hub. For storage units, an instruction is expressed as a target state of charge. For controllable producers, an instruction is expressed as mean power. For the other prosumers (that are supposed non controllable), instructions are empty. An instruction can be sent over a network or shared by components running into the same computer. That allows flexible business models.

Moreover, SO needs to know the current state of charge of every storage units and the current availability of controllable producers.

Finally, LC applies the computed tactic that is a vector of \( n_p \) power values, on the multisource system. LC is fed with solar radiation, electricity tariff and users’ calls to the elevator.

![Software components interactions](image)
4.2 Strategic Optimizer

The problem solved by SO consists of finding the best sources of energy to be used, factoring in the storage capacities, during a long time frame. The goal is to minimize costs related to energy purchasing and battery usage within the time frame, while ensuring the hub is autonomous.

4.2.1 A Generic Formulation Depending on Prosumers Kind

Let us suppose that the optimization period is a constant $\tau$, and that the number of periods in the optimization horizon is denoted $H$.

**Definition 8.** Let $e_i[t] = p_i[t] \times \tau$ be the energy amount produced by prosumer $\pi_i$ over a period $\tau$.

Thereafter decision and state variables are defined, depending on prosumers kind:

- **Storage units:** $\forall \pi_i \in S$, $0 \leq e_i^{ch}[t] \leq e_i^{min} \times \tau$ (resp. $0 \leq e_i^{dis}[t] \leq e_i^{max} \times \tau$) the amount of energy charged into (resp. discharged from) the prosumer $\pi_i$ (that is a storage unit) between time $t_i$ and time $t_i+1 = t_i + \tau$. Thus $e_i[t] = e_i^{ch}[t] - e_i^{dis}[t]$.

- **Controllable producers:** $\forall \pi_i \in P$, $0 \leq e_i^{purch}[t] \leq e_i^{max} \times \tau$ (resp. $0 \leq e_i^{sold}[t] \leq e_i^{min} \times \tau$) the amount of energy purchased from (resp. sold to) the prosumer $\pi_i$ (that is a controllable producer) between time $t_i$ and time $t_i+1 = t_i + \tau$. Thus $e_i[t] = e_i^{purch}[t] - e_i^{sold}[t]$.

- **Non-controllable producers:** $\forall \pi_i \in E$, $0 \leq e_i^{forecast}[t] \leq e_i^{max} \times \tau$ the forecasted production (if positive) or consumption (if negative) of the prosumer $\pi_i$ between time $t_i$ and time $t_i+1 = t_i + \tau$. These $e_i$ are not decision variables but constant data given by an external forecast.

**Remark:** Although storage units charge and discharge could be modeled as a single variable $e_i$, two variables ($e_i^{ch}$ and $e_i^{dis}$) are used in our model, because there are two different yields that impact the charge and the discharge. However, we do not want to charge and discharge the same storage unit at the same time, thus both variables are minimized in the objective function.

**Lemma 4.1.** In all optimal solutions, $\forall l \in \{0, \ldots, H - 1\}, \forall \pi_i \in S$, $e_i^{ch}[t] = 0 \lor e_i^{dis}[t] = 0$.

**Proof.** Assume $\hat{e}$ an optimal solution, such that there exists $0 \leq l \leq H - 1$ where both $e_i^{ch}[t]$ and $e_i^{dis}[t]$ are strictly positive. Then, let us consider another solution $\hat{e}'$ where all decision variables have the same value except that:

$$e_i^{ch}[t] = e_i^{ch}[t] - \min (e_i^{ch}[t], e_i^{dis}[t])$$
$$e_i^{dis}[t] = e_i^{dis}[t] - \min (e_i^{ch}[t], e_i^{dis}[t])$$

As $e_i^{ch}[t]$ and $e_i^{dis}[t]$ are both penalized in the objective function, then solution $\hat{e}'$ admits a lower cost than solution $\hat{e}$ that was optimal by assumption.

The following constraints must be taken into account:

- A minimum energy amount must be kept into storage units.

$$\forall \pi_i \in S, \forall l \in \{0, \ldots, H\}, x_i[t] + p_i^{minSOC}[t] \geq e_i^{minSOC}, \quad (4)$$

For this constraint, a new slack variable is defined: $0 \leq p_i^{minSOC}[t] \leq e_i^{minSOC}$ is the percentage of storage unit state of charge under a given minimum value $e_i^{minSOC}$ at time $t_i$. Then, $p_i^{minSOC}$ is the ratio of the storage unit state of charge that the energy hub needs to ensure security in case of grid failure. The $p_i^{minSOC}$ variables must be null except in the case of grid failure, so they are penalized in the objective function (cf Equation (9)).

- The energy-related equation of the energy hub must be satisfied: the sum of consumed and produced energy between time $t_i$ and time $t_i+1 = t_i + \tau$ must be equal.

$$\forall l \in \{0, \ldots, H - 1\}, \sum_i (e_i[t]) = 0 \quad (5)$$

- The state of charge of storage units must be updated at each time-step with the energy charged and discharged.

$$\forall \pi_i \in S, \forall l \in \{0, \ldots, H - 1\}, x_i[t+1] = x_i[t] + e_i^{ch}[t] - e_i^{dis}[t]$$

$$- \frac{1}{e_i^{cap}} e_i^{cap} x_i^{cap} \times e_i^{ch}[t]$$

where $e_i^{cap}$ is the energy capacity of the storage unit and $e_i^{cap}$ (resp. $e_i^{ch}$) is the charging (resp. discharging) yield of the storage unit. Yields are normalized between 0 and 1.
Let a new slack variable ∀l ∈ {0, ..., H − 1}, 0 ≤ ρ_i^{stab}[t_l] ≤ p_{i}^{max} − p_{i}^{min} be the difference between the amount of energy purchased from a controllable producer π_i at time t_l and the amount of energy purchased from the same controllable producer at time t_{l−1}. This value has to be penalized in the objective function. The associated constraints are:

\[ ∀π_i ∈ P, ∀l ∈ \{0, \ldots, H − 1\}, \]
\[ e_i[t_{l−1}] − e_i[t_l] − ρ_i^{stab}[t_l] ≤ 0 \]  
\[ e_i[t_l] − e_i[t_{l−1}] − ρ_i^{stab}[t_l] ≤ 0 \]  

Please note that, for the first period, e_i[t_{l−1}] is set to the instruction computed for prosumer π_i during SO last run. If the current execution is the first one, e_i[t_{l−1}] is set to zero.

• Depending on the exact use case, cyclical constraints (or additional cost factors), such as requiring the battery to be full at the beginning of the morning can be added.

Given those constraints, our economical objective function is given by Equation (9).

\[
\text{Minimize } \sum_{l=0}^{H-1} \left[ \sum_{π_i ∈ P} \left( -c_i^{old}[t_l] \times e_i^{old}[t_l] + c_i^{purch}[t_l] \times e_i^{purch}[t_l] \right) + \min \left( \frac{\min(ε_i^{purch}[t_l])}{π_i ∈ S}, \frac{ε_i^{aging}[t_l]}{2} \right) \times p_i^{stab}[t_l] \right] + \sum_{π_i ∈ S} \left( \frac{c_i^{aging}}{2} \times ε_i^{ch}[t_l] + \frac{c_i^{aging}}{2} \times ε_i^{dis}[t_l] \right) + 2 \times \max_{π_i ∈ S} (ε_i^{purch}[t_l]) \times p_i^{minSOC}[t_l] \right] \]  

\[ (9) \]

where ε_i^{purch}[t_l] is the electricity buying price at time t_l and ε_i^{sold}[t_l] is the electricity selling price at time t_l thus \(-c_i^{old}[t_l] \times ε_i^{old}[t_l] + c_i^{purch}[t_l] \times ε_i^{purch}[t_l] \) is the electricity bill for the l-th period and prosumer π_i. These constants are given by the cost function cs_π, which is considered linear in the current formulation.

On the other hand, ε_i^{aging} is a coefficient that allows to have a linear approximation of the impact of the storage unit usage on its aging: \( ε_i^{aging} = \frac{ε_i^{inv}}{ε_i^{cye}} \). the constant ε_i^{inv} represents the investment cost of the storage unit and ε_i^{cye} is the mean number of cycles that the storage unit can bear.

Remark: As the battery aging cost is just a way to discourage the controller from using the battery, a first order approximation was chosen. In reality, "small" charges and discharges further impact storage units but we ignore this effect here. This cost could be tuned depending on the results of long-term simulations (typically several years) of the controller and its impact on the battery lifetime. On the other hand, auto-discharge of storage units is neglected for the moment.

Moreover, a minimum energy amount must be kept into storage units in order to ensure autonomy in case of grid failure. We cannot ensure that with a hard constraint because we need to allow consuming this reserve during a grid failure. Thus, we use the soft constraint (4) with a slack variable ρ_i^{minSOC} that is minimized in the objective function. We could have considered that, in case of grid failure, a different operating mode that allows violating this constraint would be set. But a soft constraint, with a soundly chosen penalization, does the same job in a simpler way.

Finally, in order to reduce the chattering of the energy purchased from controllable producers, we minimize a slack variable called ρ_i^{stab}.

Note that all variables are weighted with a fraction of the energy tariff in order to ensure the right setting of the objective (reserve for grid failure, electricity cost, ...).

The overall linear formulation is then:

\[
\text{Minimize (9)}
\]

Subject to (4) – (8)

4.2.2 The Multisource Elevator Formulation

In the elevator context, we choose to set ε_2^{minSOC} to 0.2 and ε_3^{minSOC} to 1.0 because the super-capacitor has to be fully charged in case of grid failure and a 20% charged battery can supply the super-capacitor for several travels. Moreover, the battery is a lead-acid battery with an energy capacity of 3000 Wh, an investment cost of 300 € and a maximum power of 2880 W. The super-capacitor has an energy capacity of 120 Wh, an investment cost of 2400 € and a maximum power of 57600 W. Thus, storage units aging costs are the following ones:

\[ ε_2^{aging} = \frac{c_2^{inv}}{c_2^{cye} \times ε_2^{cye}} = \frac{300}{20000 \times 3000} = 0.0000005 \]

\[ ε_3^{aging} = \frac{c_3^{inv}}{c_3^{cye} \times ε_3^{cye}} = \frac{2400}{20000 \times 120} = 0.001 \]

Table 1 shows the linear program applied to the multisource elevator case, in vector form. Variables
Table 1: A linear formulation for the multi-source elevator sourcing problem.

Minimize \[
-\mathbf{c}_{\text{purch}}^4 \odot \mathbf{e}_{\text{purch}}^4 + 5 \times 10^{-6} \times \mathbf{x}_1 + 0.001 \times \mathbf{x}_3 \\
+ 2 \times \mathbf{c}_{\text{purch}}^4 \odot \mathbf{e}_{\text{stable}}^4 + 2 \times \mathbf{c}_{\text{purch}}^4 \odot \rho_{\text{minSOC}}^4 \\
+ \min \left( \frac{\mathbf{c}_{\text{purch}}^4}{25000} \times 0.0005 \right) \odot \rho_{\text{stab}}^4 \\
+ \sum_{i=1}^{6} \mathbf{e}_i = 0 \\
\mathbf{e}_{\text{old}}^4 - \mathbf{e}_{\text{old}}^4 - \rho_{\text{stab}}^4 \leq 0 \\
\mathbf{e}_{\text{old}}^4 - \mathbf{e}_{\text{old}}^4 - \rho_{\text{stab}}^4 \leq 0 \\
\mathbf{x}_2 - \mathbf{x}_3[t_1] + \mathbf{A} \mathbf{e}^{\text{ch}} + \mathbf{B} \mathbf{e}^{\text{dis}} \leq 0 \\
\mathbf{x}_3 - \mathbf{x}_3[t_1] + \mathbf{A} \mathbf{e}^{\text{ch}} + \mathbf{B} \mathbf{e}^{\text{dis}} \leq 0 \\
\mathbf{x}_3 - \mathbf{x}_3[t_1] + \mathbf{A} \mathbf{e}^{\text{ch}} + \mathbf{B} \mathbf{e}^{\text{dis}} \leq -0.2 \\
\mathbf{x}_3 - \mathbf{x}_3[t_1] + \mathbf{A} \mathbf{e}^{\text{ch}} + \mathbf{B} \mathbf{e}^{\text{dis}} \leq -1.0 \\
0 \leq \mathbf{e}^{\text{ch}} \leq -\rho_{\text{min}}^4 \times \tau, \forall \pi_i \in \mathcal{S} \\
0 \leq \mathbf{e}^{\text{stab}} \leq -\rho_{\text{min}}^4 \times \tau, \forall \pi_i \in \mathcal{S} \\
0 \leq \mathbf{e}^{\text{dis}} \leq -\rho_{\text{min}}^4 \times \tau, \forall \pi_i \in \mathcal{S} \\
0 \leq \mathbf{c}_{\text{purch}}^4 \leq 50000 \times \tau, \forall \pi_i \in \mathcal{S} \\
0 \leq \mathbf{e}_i \leq 1, \forall \pi_i \in \mathcal{S} \\
0 \leq \mathbf{e}_i \leq 1, \forall \pi_i \in \mathcal{S} \\
0 \leq \mathbf{e}_i \leq 20 \times 10^{-6}, \forall \pi_i \in \mathcal{S}
\]

in bold are column vectors of dimension \( H \), and matrices. Let \( A, B, A' \) and \( B' \) be \( H \times H \)-matrices:

\[
A = \text{diag} \left( \begin{array}{cccc}
-\mathbf{c}_1^2 & 0.9 & 0.9 & 0.9 \\
0 & 0.9 & 0.9 & 0.9 \\
0 & 0 & 0.9 & 0.9 \\
0 & 0 & 0 & 0.9
\end{array} \right)
\]

\[
B = \text{diag} \left( \begin{array}{cccc}
\frac{1}{\mathbf{c}_1^2} & \frac{1}{\mathbf{c}_2^2} & \frac{1}{\mathbf{c}_3^2} & \frac{1}{\mathbf{c}_4^2} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array} \right)
\]

\[
A' = \text{diag} \left( \begin{array}{cccc}
-\mathbf{c}_1^2 & 0.9 & 0.9 & 0.9 \\
0 & 0.9 & 0.9 & 0.9 \\
0 & 0 & 0.9 & 0.9 \\
0 & 0 & 0 & 0.9
\end{array} \right)
\]

\[
B' = \text{diag} \left( \begin{array}{cccc}
\frac{1}{\mathbf{c}_1^2} & \frac{1}{\mathbf{c}_2^2} & \frac{1}{\mathbf{c}_3^2} & \frac{1}{\mathbf{c}_4^2} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array} \right)
\]

Moreover, the symbol \( \odot \) states for the Hadamard product (element-wise product).

As we choose 15 minutes periods (\( \tau = 0.25 \)) and a 24 hours horizon, \( H = 24 \times 0.25 = 96 \). That gives us a linear program with 96 periods. In practice, we have 12 vectors of \( H \) decision variables each and 10 constraints per time step. Thus every hour we solve a linear problem with 1152 variables and 960 constraints. Building and solving the problem with GLPK (GNU, 2014) takes about one second.

### 4.3 Local Controller

Our LC is a centralized rule-based controller that computes, in real-time and for a single time-step, a sourcing tactic. The tactic depends on: 1) the relative priority associated to every prosumers, 2) current flexibilities of every prosumers, 3) the current strategic instruction if any, or a default instruction otherwise.

As SO is based on energy forecasts, some strategic instructions may be infeasible at some points, and LC has to find the best trade-off between instruction and current situation.

#### 4.3.1 Principles of the Rule-based Algorithm

Each prosumer connected to the energy hub is associated with a priority number. A priority list is defined as a permutation of the prosumers set \( \mathcal{P} = (\pi_1, \ldots, \pi_{\epsilon_p}) \), ordered by decreasing priority numbers. A priority number represents the importance of satisfying a prosumer relatively to the others.

Moreover each prosumer has a list of flexibilities, that can be discrete power values:

\[
(p_1, p_1^2, \ldots, p_1^m)
\]

or power intervals:

\[
([p_1^{\text{min}}, p_1^{\text{max}}], \ldots, [p_1^{\text{min}}, p_1^{\text{max}}])
\]

Flexibilities are ordered by decreasing preference order of the prosumer. The preference order of the prosumer is linked to its quality of service.

**Example:** If an elevator \( \pi_1 \) is stopped and empty, it can move to a higher floor (flexibility \( p_1^1 \)), or move to a lower floor (flexibility \( p_1^2 \)), or stay still and consume standby power (flexibility \( p_1^3 \)). The preferred flexibility \( p_1^1 \) of the elevator is to go to the floor that corresponds to the first user call.

On the other hand, some prosumers have no discrete flexibilities but a set of possible intervals. For example, a battery can consume or produce a power value bounded by its minimum and maximum power bound.

The third and last, parameter that influences LC is the strategic instruction. If there is no strategic instruction available, LC decides itself of a default instruction. That allows LC to work alone if its link with SO is broken. Setting the default instruction influences performances of the tactic.

Given these three inputs, LC builds a decision tree for the current time step. The decision tree has a level per prosumer, ordered by decreasing priority order. In a given level, every node has as many children as the number of flexibilities of the prosumer corresponding to the next level. A tactic is obtained by a depth-first search in the tree, and composed of a power value per prosumer.

During the depth first search, when the node holds only the power value (example, a battery can consume or produce a power value bounded by its minimum and maximum power bound).
Table 2: Three LCs and their parameters.

(a) Priority orders.

<table>
<thead>
<tr>
<th></th>
<th>MinPeaks</th>
<th>Opportunistic</th>
<th>Secure</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>S₁</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>S₂</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>S₃</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

(b) Default instructions.

<table>
<thead>
<tr>
<th></th>
<th>MinPeaks</th>
<th>Opportunistic</th>
<th>Secure</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>standby cons</td>
<td>0 W</td>
<td>0 W</td>
</tr>
<tr>
<td>S₁</td>
<td>(c^\text{minSOC}_i)</td>
<td>(c^\text{minSOC}_i)</td>
<td>(c^\text{minSOC}_i)</td>
</tr>
<tr>
<td>S₂</td>
<td>100%</td>
<td>(x_i)</td>
<td>100%</td>
</tr>
<tr>
<td>S₃</td>
<td>100%</td>
<td>(x_i)</td>
<td>(x_i)</td>
</tr>
</tbody>
</table>

reached, the algorithm checks if the sum of the chosen power values is equal to zero. If not, the difference is compensated, as much as possible, by each node through backtracking in the tree. When the tree root is reached, if the sum of the control vector is null, the solution is kept. If not, the depth first search continues.

That way, the first found solution is the best one regarding the prosumers priority order, the preferences of each prosumer and the strategic instruction.

4.3.2 Parameters Values and Objectives

This rule-based algorithm can be tuned depending on the objective, through parameters value described above. We cannot influence prosumers flexibilities but we can choose priority order and default instruction. There are three versions of LC:

MinPeaks. The first controller considered seeks to minimize power peaks purchased from controllable producers.

Opportunistic. The second controller considered seeks to minimize dissipated energy and thus the energy bill.

Secure. The third controller considered seeks to minimize storage units usage while guaranteeing that storage units will be ready in case of grid failure.

Note that the Opportunistic LC’s behavior is comparable to a classical rule-based controller.

Priority order and default instruction associated with each of these three controllers are shown in Table 2. For that purpose, storage units are classified in three categories:

- those under their minimum state of charge \(S₁ = \{π_i \in S | x_i < c^\text{minSOC}_i\}\),
- those usable to absorb power peaks \(S₂ = \{π_i \in S \setminus S₁ | p^\text{max} \geq \max_{x_i \in \mathbb{R}} (-p^\text{min}_j)\},\)
- the others \(S₃ = S \setminus (S₁ \cup S₂)\).

5 RESULTS

In order to evaluate the proposed solution, we need to compare SO plans with the results obtained by LC following the instructions. Then, we have to compare the different tactics between them, without strategic instructions.

For the simulation purpose, we developed in Matlab (The MathWorks Inc., 2015) a simulation engine with dynamic time steps depending on events occurring.

5.1 A Typical Strategy

Results obtained by SO, during a typical day, are illustrated on Figure 3. The first sub-figure is an energy layers plot where positive power represents power that is injected in the energy hub (produced by prosumers) and negative power represents power that is taken from the energy hub (consumed by prosumers). The second sub-figure presents the storage units state of charge and the grid tariff. We can see that, when the electricity is cheap (before 09:00), the energy is purchased from the grid (in red), and stored in the battery (in green). After 09:00, no more energy is purchased and the energy needed is discharged from the battery.

In this situation, SO tries to take advantage of the off-peak tariff to charge the battery and avoid to purchase energy from the grid during the peak tariff. We can see that the battery is charged just enough to achieve this goal (about 40% at 09:00).
5.2 Strategic Controller Error Induced by Aggregating Periods

In this sub-section, the point is to characterize the error made by SO when computing the objective value (i.e., energy bill and aging costs). This error is evaluated (on a typical day) by comparing results of the MinPeak LC following the strategy with results computed by SO before the beginning of the day.

The plan computed before the day by SO is the same as on Figure 3 and the corresponding MinPeaks tactic is illustrated on Figure 4. The first sub-figure represents the power consumption and production of the different prosumers. We can see that the consumption peaks of the elevator (in blue; that reaches 8 kW) are absorbed by storage units (in green). While in every cases, the maximum power peak purchased from the grid is under 400 W, that is far less than 8 kW. The second sub-figure shows the evolution of the storage units state of charge and the grid tariff. In brown, we can see the strategic instructions that are quite well followed by the battery.

Now, let us compare numeric results of LC and SO. As the accuracy of SO depends on 1) charging and discharging yields of storage units, and 2) forecasts accuracy. We decided to consider ideal yields (100% and realistic yields (90%). On the other hand, forecasts are exact as described in sub-section 4.1.2. Table 3 summarizes the results obtained by SO and by the MinPeaks LC following the instructions, in three cases.

When storage units yields are ideal, the energy bill and the amount of energy purchased obtained by the strategy are very close to ones obtained by the tactic. Both values are less than 3% higher with tactic than with strategy. Though all parameters are ideal, this small error is due to the energy aggregation in 15 mn periods into forecasts. Indeed, when summing positive and negative energy values over a period, only the difference is kept. Then, SO take into account only this small amount of energy to purchase (or discharge). In reality, if production occurs before consumption, produced energy is stored and discharged later to supply consumption. But, if consumption occurs before production, the energy has to be found elsewhere before the production could be stored. When yields are ideal and storage units not empty, that has no impact on the energy bill. But when storage units are empty the energy has to be purchased and that explains the small differences observed above. On the other hand, the aging cost computed by LC is far bigger than those estimated by the SO because the latter did not take into account power peaks. Indeed, to achieve instructions at the end of a strategic period, LC cannot always charge storage units with average power computed from the instruction. It has to absorb consumption and production peaks, and thus to charge and discharge storage units many times within the period. The amount of energy charged into and discharged from storage units is thus far bigger (about 14 times in this case) than computed by SO.

Now let us look at the impact of realistic yields on these results. We can see that, in the three metrics, results are worse than before. The reason is that non ideal yields drive non null energy losses. Thus, the above mentioned discrepancy due to aggregation is emphasized in presence of non ideal yield.

In order to make SO take into account almost all energy has to transit through storage units, we integrate the impact of storage units yield into consumption forecasts of prosumers in E. The strategy becomes pessimistic because not all energy goes through storage units. But the yield taken into account in forecasts becomes a tunable parameter for the pessimistic prediction. Moreover, re-computing a strategy every hour prevents LC to deviate too far away from the target, even if the strategy is not perfectly accurate. In this experiment, we choose a yield value of 0.9, that is the real storage units yield. We can see that the error of SO is really reduced compared to the previous experiment. That also improves LC results, especially on aging costs.

The last thing that has to be explained in the table is: why do tactic aging costs are higher when yields are ideal than when yields are realistic and when forecasts take yields into account? Aging costs are higher in the former because, the battery is emptied at the beginning of some periods and the supercapacitor has to supply the elevator before being refilled by produced energy. On the other hand, when the strategy is pessimistic, this situation occurs less frequently. Since supercapacitors are much more expensive than regular batteries, aging costs in the first experiment are higher than in the third one.
### Table 3: Strategical and tactical results with different yields.

<table>
<thead>
<tr>
<th>viewpoint</th>
<th>yield</th>
<th>forecast</th>
<th>energy bill</th>
<th>aging costs</th>
<th>energy purchased</th>
</tr>
</thead>
<tbody>
<tr>
<td>SO</td>
<td>100 %</td>
<td>exact</td>
<td>0.1164 €</td>
<td>0.0097 €</td>
<td>1164.3 Wh</td>
</tr>
<tr>
<td>LC</td>
<td>100 %</td>
<td>exact</td>
<td>0.1198 €</td>
<td>0.1318 €</td>
<td>1187.8 Wh</td>
</tr>
<tr>
<td>ratio LC SO</td>
<td>100 %</td>
<td>exact</td>
<td>1.03</td>
<td>13.59</td>
<td>1.02</td>
</tr>
<tr>
<td>SO</td>
<td>90 %</td>
<td>exact</td>
<td>0.1382 €</td>
<td>0.0104 €</td>
<td>1382.2 Wh</td>
</tr>
<tr>
<td>LC</td>
<td>90 %</td>
<td>exact</td>
<td>0.1745 €</td>
<td>0.3285 €</td>
<td>1640.5 Wh</td>
</tr>
<tr>
<td>ratio LC SO</td>
<td>90 %</td>
<td>exact</td>
<td>1.26</td>
<td>31.59</td>
<td>1.19</td>
</tr>
<tr>
<td>SO</td>
<td>90 %</td>
<td>exact + yield</td>
<td>0.1706 €</td>
<td>0.0117 €</td>
<td>1705.7 Wh</td>
</tr>
<tr>
<td>LC</td>
<td>90 %</td>
<td>exact + yield</td>
<td>0.1655 €</td>
<td>0.1015 €</td>
<td>1654.5 Wh</td>
</tr>
<tr>
<td>ratio LC SO</td>
<td>90 %</td>
<td>exact + yield</td>
<td>0.97</td>
<td>8.67</td>
<td>0.97</td>
</tr>
</tbody>
</table>

#### 5.3 Three Tactics, One Strategy

Let us compare results of the different LCs, averaged over fifty elevator usage samples drawn. For this purpose, we use an additional Key Performance Indicator (KPI) that is the net daily gain \( g_{\text{net}} \). Let: \( c_{\text{ebill}} \) be the energy bill of the whole day, \( c_0 \) be the energy bill that would have been obtained without storage units, \( c_{\text{aging}} \) be the aging cost associated to the storage units usage that have been done during the day. Moreover, the initial state of charge of storage units are: 20% for the battery and 100% for the supercapacitor. But depending on the controller, final states of charge can be different. Then we note \( c_{\text{soc}} \) the cost associated to refill (or empty) storage units to match their initial state of charge. We consider that corresponding energy is purchased (or sold) during off-peak hours. Then, \( g_{\text{net}} = (c_{\text{ebill}} - c_0 + c_{\text{aging}} + c_{\text{soc}}) \) is an additional KPI for the following experiments.

First of all, let us look at Figures 5(a), 5(b) and 5(c) that allow us to compare the three different tactics, without any strategy.

Figure 5(a) corresponds to the MinPeaks LC and we can see that there are a few power peaks from the grid at the beginning of the day. This is because not enough energy was produced before consumption, so energy had to be purchased to supply the elevator. Moreover at the end of the day, the battery is not at its minimum state of charge and the supercapacitor is full.

Figure 5(b) corresponds to the Opportunistic LC. With this tactic, all available energy is used as soon as possible, thus the supercapacitor supplies the elevator at the beginning of the day and is emptied. Then, there are many power peaks purchased from the grid in the morning and in the middle of the afternoon.

Figure 5(c) corresponds to the Secure LC. This tactic purchases from the grid all energy needed to preserve storage units from aging. The supercapacitor, that allows the elevator to travel during grid failures, is maintained full. The battery, that has to supply supercapacitor during grid failures, is maintained at its minimum state of charge.

Now, let us look at numerical results of these three tactics and of the MinPeaks Controller following strategy. The latter is illustrated on Figure 4. Please note that a LC on its own does not take into account electricity tariff at all. The \( c_0 \) value, in this example, corresponds to the \( c_{\text{ebill}} \) value of the Secure LC.
Table 4: Net daily gain of different tactics.

<table>
<thead>
<tr>
<th>SO</th>
<th>LC</th>
<th>$p_{\text{max}}$</th>
<th>$c_{\text{e}}$</th>
<th>$c_{\text{aging}}$</th>
<th>$c_{\text{soc}}$</th>
<th>$g_{\text{net}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓</td>
<td>MinPeaks</td>
<td>7839</td>
<td>0.19</td>
<td>0.51</td>
<td>-0.03</td>
<td>-0.42</td>
</tr>
<tr>
<td>×</td>
<td>MinPeaks</td>
<td>8775</td>
<td>0.24</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>×</td>
<td>Opport.</td>
<td>8627</td>
<td>0.12</td>
<td>0.07</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>×</td>
<td>Secure</td>
<td>317.7</td>
<td>0.13</td>
<td>0.12</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

LC: $c_0 = 0.24 \, \text{€}$.

On simulated days, we can see that the only efficient way to reduce the maximum power peak purchased from the grid is the MinPeaks Controller following the strategy. That way, the purchasing peak is less than 5% of the maximum possible power peak. The tactics on their own do not absorb power peaks, because a predictive strategy is necessary to achieve such a goal. On the other hand, the MinPeaks Controller following the strategy only succeeds to compensate aging costs by a gain on the energy bill. Thus, its net daily gain is null in this context. Please note that, if the gap between on- and off-peak hourly cost grows, the net daily gain of this tactic following the strategy also increases.

If the MinPeaks LC is on its own (for example during a long network failure), the net daily gain becomes negative but the number of (and the maximum) power peaks stays below the other controllers alone.

However, if the goal is only to minimize the net daily gain, without taking into account power peaks, the Opportunistic LC alone is the best (as far as the ratio between the high/low prices is moderate).

Finally, the Secure tactic does not use storage units at all and thus, has a higher energy bill, but a null aging cost.

6 CONCLUSIONS

In this paper we formalize what we call the sourcing problem and its application to a multisource elevator. We give a method to solve it, composed of a Local Controller coupled with a Strategic Optimizer. That allows us to tackle real-time issues while taking into account long-term objectives based on forecasts. A linear formulation allows us to compute a strategy and a centralized rule-based algorithm gives us a tactic. Three Local Controller parametrizations are illustrated, and one will choose the most adapted to its studied energy hub.

A legitimate criticism of this work could be that gains in euros are very low. First, let us recall that these results are for a unique elevator, while in practice several elevators could share the same battery. Second, reducing power consumption peaks could be very useful to be demand-response aware and to respect future energy limitation laws. In those cases, minimizing the energy bill is only an appreciable addition to the consumption peaks minimization. For the multisource elevator use case, having storage units allows the elevator to evacuate disabled people in case of fire. Using these storage units to minimize the energy bill could amortize the investment. Third, energy prices are going to increase in future years, as will storage units performances. That will also increase the benefit of this solution. Finally, the proposed solution can be applied to other multisource systems where it can be much more profitable. In fact, as the consumption increases, the profitability also increases, especially if reselling energy is possible.

On the other hand, we will have to compare the cost of maintaining our rather complex solution, regarding the customer value in each use case. If the maximum power peak purchased from the grid is not an issue and the energy tariff considered is a typical French peak/off-peak tariff, the Opportunistic LC (or a similar classical rule-based controller) should be used.

As future work, we will study our method robustness to forecast uncertainties. This is a critical issue and studying it could allow us to give performance guarantees to potential customers. A sensibility study of controller parameters will also be conducted. Finally, a real-life experiment, in a building equipped with a BMS, would worth being conducted.

ACKNOWLEDGEMENTS

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