Low Order Aberrations Compensation by Direct Adjustment of the Reflective Beam Shaper in Slab Laser

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Abstract: A direct method for compensation of low order aberrations with large PV value was presented in this paper. In which, the relationship between the optical layout parameters and the output aberration coefficients were derived by ray matrix method. Then, the adjustment parameters calculated by the equations were used to change the optical layout parameters to compensate the low order aberrations with defocus and astigmatism. The effectiveness of this method was verified by simulations based on the optical models built in a optical design software. It shows that the low order aberrations can be accurately compensated to a level below 0.5λ by the direct method.

1 INTRODUCTION

In the development of high power slab lasers, both output power and beam quality are crucial parameters to be considered. Although power scaling of slab laser can be realized by MOPA (Master Oscillator and Power Amplification) configuration, preserving high beam quality in high power slab laser is a real challenge (Redmond et al, 2007). In high power slab lasers, the Peak-Valley (PV) value of thermally induced wave-front distortion can be dozens of micrometers (Ganija et al., 2013), and it’s difficult to be corrected by a deformable mirror with limited correction range (typically in the range of 6 μm). Multiple deformable mirrors are proposed to correct the wave-front distortions in high power lasers (Xiang et al., 2012; Conan et al., 2007). However, this solution is both expensive and complex. Some experiments have been done to analysis the characteristics of the wave-front in the high power slab laser (Liujing et al., 2011) It shows that in the distortions, low order aberrations, mainly consist of defocus and astigmatism, are the main contributors. So the two-steps beam cleanup concept is proposed as a cost-effective approach to get high beam quality. That is, low order aberrations are compensated by one compensator firstly. And next, the high order aberrations are corrected by one deformable mirror. Static phase corrector (W Qiao, et al, 2014) can be used to compensate some low order aberrations, but it doesn’t work well when the operational conditions were changed, and it can also be thermally distorted under high power flux. A reflective beam shaper with two cylindrical mirrors and one spherical mirror was proposed to compensate the low order aberrations by active adjustment of the optical parameters with PID algorithm (Wenguang et al., 2014). Due to the respond speed of the motorized linear stage used, the convergence of PID controller may take about 20s.

In this paper, for the purpose of speeding the compensation process of low order aberrations in slab laser, a direct method was proposed, in which the relationships between the low order aberrations and adjusting parameters were derived from ray matrix equations. Simulations were done to verify the correctness of the method.

2 THEORITICAL DERIVATION

2.1 Layout of the Reflective Beam Shaper

A reflective beam shaper is often used to transform a narrow beam to a square one in slab laser system. The beam shaper can also be used to compensate the low...
order aberrations, as shown in Fig.1, where the beam shaper mainly consists of two cylindrical mirrors (y-oriented mirror \( M_y \), x-oriented mirror \( M_x \)), one spherical mirror (\( M_B \)). The distance between \( M_y \) and \( M_B \) is \( L_1 \), and the distance between \( M_B \) and \( M_x \) is \( L_2 \). Four plane mirrors (\( M_1 \), \( M_2 \), \( M_3 \), \( M_4 \)) are used to fold the optical path, for the purpose of keeping the output beam position unchanged while \( L_1 \) and \( L_2 \) are adjusted to compensate the defocus and 0 degree astigmatism, and \( M_x \) can be rotated a angle of \( \kappa \) about z-axis to compensate the 45 degree astigmatism. The PID algorithm was used to adjust \( L_1 \) and \( L_2 \), and \( \kappa \) to slowly compensate the low order aberrations.

![Figure 1: The optical layout of a reflective beam shaper and adjustment parameters for compensation of low order aberrations.](image)

### 2.2 Matrix Analysis of the Low Order Aberration Compensator

In this paper, matrix methods are used to analyze the relationship between low order aberrations and the adjustment of \( L_1 \) and \( L_2 \), and \( \kappa \), to compensate the aberrations directly and quickly without PID algorithm.

Ray tracing are taken from \( M_i \) to the output plane \( S_{out} \), as shown in Fig.1. Using a Cartesian-azimuth representation, an incident ray on \( M_i \) can be written as:

\[
V_i = \begin{bmatrix} x \\ y \\ \alpha \\ \beta \end{bmatrix}
\]  

(1)

For a reflective y-oriented cylindrical mirror of curvature \( R \), the matrix is:

\[
M_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{2}{R} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]  

(2)

The propagation matrix between \( M_i \) and \( M_B \) is:

\[
M_{1i} = \begin{bmatrix} 1 & L_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]  

(3)

The reflective matrix of spherical mirror \( M_B \) of curvature \( R_B \) is:

\[
M_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{2}{R_B} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]  

(4)

The propagation matrix between \( M_B \) and \( M_x \) is:

\[
M_{2i} = \begin{bmatrix} 1 & L_2 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]  

(5)

The coordinate transform matrix with rotation angle of \( \kappa \) about z-axis is:

\[
R_{z\kappa} = \begin{bmatrix} \cos \kappa & 0 & -\sin \kappa & 0 \\ 0 & \cos \kappa & 0 & -\sin \kappa \\ \sin \kappa & 0 & \cos \kappa & 0 \\ 0 & \sin \kappa & 0 & \cos \kappa \end{bmatrix}
\]  

(6)

For a reflective x-oriented cylindrical mirror of curvature \( R_x \) the matrix is:

\[
M_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{2}{R_x} & 1 \end{bmatrix}
\]  

(7)

And the coordinate transform matrix with rotation angle of \( -\kappa \) about z-axis is:

\[
R_{z\kappa} = \begin{bmatrix} \cos \kappa & 0 & \sin \kappa & 0 \\ 0 & \cos \kappa & 0 & \sin \kappa \\ -\sin \kappa & 0 & \cos \kappa & 0 \\ 0 & -\sin \kappa & 0 & \cos \kappa \end{bmatrix}
\]  

(8)

The propagation matrix between \( M_y \) and \( S_{out} \) is:
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Matrix of reflective beam shaper in this paper can be calculated by the matrix product of the component matrices:

\[
M = M_3L \cdot R_y \cdot M_5 \cdot R_y \cdot M_7 \cdot M_y \cdot M_3L \cdot M_y.
\]  

(10)

2.2.1 Functions for Compensating the Defocus and 90° Astigmatism

In Cartesian coordinates, the combination of defocus and 90° astigmatism can be written as:

\[
w(x, y) = \frac{x^2}{2R_y} + \frac{y^2}{2R_y}
\]

where \(R_x\) is the curvature of beam divergence in XOZ plane, and \(R_y\) is the curvature in YOZ plane (Geovanni et al., 2014). So the ray incident \(M_y\) with defocus and 90° astigmatism in matrix form is:

\[
V = \begin{bmatrix}
x \\
y \\
\alpha \\
\beta
\end{bmatrix}
\]

\[
= \begin{bmatrix}
x \\
y \\
\frac{\partial w}{\partial x} \\
\frac{\partial w}{\partial y}
\end{bmatrix} = \begin{bmatrix}
x \\
y \\
R_x \\
R_y
\end{bmatrix}
\]

Rewrite the distance as:

\[
L = (R_x + R_y) / 2 + \Delta L
\]

where \(\Delta L\) and \(\Delta L_2\) are the adjustment of distance desired to compensate the defocus and 90° astigmatism. Set rotation angle \(\kappa = 0\) for the simplification of derivation, The rays leaving \(M_y\) is calculated by

\[
V_{out} = \begin{bmatrix}
x' \\
y' \\
\alpha' \\
\beta'
\end{bmatrix} = M \cdot V =
\]

\[
\begin{bmatrix}
1 & L_y & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & L_y \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(9)

From Eq. (13) the adjustment of distance \(\Delta L_1\) and \(\Delta L_2\) are obtained when \(\alpha' = 0, \beta' = 0\):

\[
\Delta L = \frac{R_x^2}{2(2R_y - R_x)}
\]

(14)

\[
\Delta L = \frac{R_y^2}{2[2R_x + R_y + 2\Delta L_y]}
\]

(15)

It means that defocus and 90° astigmatism can be compensated by proper adjustment of \(\Delta L_1\) and \(\Delta L_2\). However, in the practical compensation process, Wave-front aberrations on \(S_{out}\) are often expressed as Zernike coefficients in most wave-front sensor, such as Hartman-Shack sensors. So it is convenience to express \(\Delta L_1\) and \(\Delta L_2\) as the functions of Zernike coefficients detected by H-S sensor on output plane \(S_{out}\).

Before adjustment, \(\Delta L_1 = 0, \Delta L_2 = 0\). From Eq. (13), the relationship between curvature of divergence beam on the output plane \(R_x'\) and the curvature on the input plane \(R_x\) is:

\[
R_x' = R_x + R_y + R_x\frac{R_y^2}{2R_y^2} - R_x^2 + 2R_x\Delta L
\]

(16)

We can rewrite Eq. (16) as:

\[
R_x' = \frac{2R_x^2 R_y^2 + R_y^2 R_x R_y - 2R_x^2 R_y^2 - 2R_x^2 R_y}{2R_y^2}
\]

(17)

In the same manner, the relationship of \(y\)-curvature of divergence beam on the output plane \(R_y'\) and the curvature on the input plane \(R_y\) is:

\[
R_y' = \frac{2R_x^2 R_y - R_y^2 R_x + R_x^2 R_y - 2R_x^2 R_y}{2R_x^2}
\]

(18)

The relationship between Zernike coefficients and beam divergence curvature on the output plane \(S_{out}\) is:

\[
\eta = \frac{r_y^2}{2\lambda(2\sqrt{3}Z_y - \sqrt{6}Z_x)} = \frac{\eta_k}{2}
\]

(19)

\[
\eta = \frac{r_y^2}{2\lambda(2\sqrt{3}Z_y + \sqrt{6}Z_x)} = \frac{\eta_k}{2}
\]

(20)

Where

\[
\eta = \frac{r_y^2}{2\lambda k_y} = \frac{1}{(2\sqrt{3}Z_y - \sqrt{6}Z_x)}
\]

\[
k_y = \frac{1}{(2\sqrt{3}Z_y + \sqrt{6}Z_x)}
\]

\(r_y\) is the normalized aperture on the output plane, and \(\lambda\) is the wavelength used in the beam shaper, \(Z_4\) is the...
Zernike coefficient of defocus term, and $Z_6$ is the coefficients of 90° astigmatism defined in the wave-front sensor. Insert Eq. (17)–(20) into Eq. (14) and Eq. (15), adjustment of distance $\Delta L_1$ and $\Delta L_2$ can be determined according to the Zernike coefficients from wave-front sensor on output plane:

$$\Delta L_1 = \frac{R_i^2}{2(\eta k_y - R_i - 2L_y)}$$ (21)

$$\Delta L_2 = \frac{R_i^2}{2[\eta k_y + (R_i - 2L_y + 2\Delta L_i^2 / R_i^2)]}$$ (22)

### 2.2.2 Functions for Compensating the 45° Astigmatism

In Cartesian coordinates, the 45° astigmatism can be written as:

$$w(x, y) = \frac{2}{R_i} xy$$ (23)

where $R_i$ is the curvature parameter of 45° astigmatism[9].

So the ray incident $M_i$ with 45° astigmatism in matrix form is:

$$V_{\alpha i} = \begin{bmatrix} x \\ \alpha \\ y \\ \beta \end{bmatrix} = \begin{bmatrix} 2 \frac{x}{R_i} \\ \frac{y}{R_i} \\ 2 \frac{y}{R_i} \\ \frac{x}{R_i} \end{bmatrix}$$ (24)

The rays leaving $S_{out}$ can be calculated by

$$V_{\alpha 45} = M \cdot V_{\alpha i}$$

In the derivation of $V_{\alpha 45}$, both $\Delta L_1$ and $\Delta L_2$ is set to zeros for the simplicity of derivation, and the terms of $\sin^2 \kappa$ are omitted for it’s a high order quantity in the functions. The rays leaving $S_{out}$ can be written as:

$$V_{\alpha 45} = \begin{bmatrix} \frac{x}{R_i} \\ \frac{y}{R_i} \\ \frac{y}{R_i} - \frac{x}{R_i} + \alpha' L_i \\ \frac{x}{R_i} - \frac{y}{R_i} + \beta' L_i \end{bmatrix}$$

$$\begin{bmatrix} R_i R_R R_i R_R R_i R_R R_i R_R \sin(2x) R_i R_R R_i R_R R_i R_R \cos(2x) + \cos(2x) R_i R_R R_i R_R R_i R_R \cos(2x) \sin(2x) + \cos(2x) R_i R_R R_i R_R R_i R_R \cos(2x) \sin(2x) + \cos(2x) R_i R_R R_i R_R R_i R_R \cos(2x) \sin(2x) + \cos(2x) \end{bmatrix}$$ (25)

The rotation of $M_i$ with an angle of $\kappa$ is to eliminate the 45° astigmatism, that is, the terms about $y$ in $\alpha'$ become zeros, and the terms about $x$ in $\beta'$ also become zeros by proper rotating of $M_i$ with an angle of $\kappa$. From Eq. (25), we can derive the relationship between rotation angle $\kappa$ and the input 45° astigmatism parameter $R_c$:

$$\tan \kappa = \frac{R_i}{R_c}$$ (26)

When $\kappa$ is a small angle

$$\sin \kappa = \tan \kappa = \frac{R_i}{R_c}$$ (27)

Insert Eq. (26) into Eq. (25), we can found that after the 45° astigmatism is compensated, there still are some small defocus:

$$\alpha' = 2 \frac{R_i R_i R_i R_i^2}{R_i R_i R_i R_i^2} x$$

$$\beta' = 2 \frac{R_i R_i R_i R_i^2}{R_i R_i R_i R_i^2} y$$ (28)

In most situations, the defocus introduced is small enough that could be omitted, and also it can be compensated by adjusting of $L_1$ and $L_2$ later if it is necessary.

The relationship between the coefficient of 45° astigmatism on the input plane and output plane can be derived when we let $\kappa=0$:

$$R_i = \frac{R_i}{R_c} R_c$$

(30)

The relationship between the Zernike coefficient $Z_5$ and $R_c$ is:

$$R_c = \frac{1}{\lambda \sqrt{6Z_5}} = \eta k_{x y}$$ (31)

Where

$$k_{x y} = \frac{1}{\sqrt{6Z_5}}$$

Insert Eq. (30) and Eq. (31) into Eq. (26), the equation between the rotation angle $\kappa$ and the Zernike coefficient of 45° astigmatism on the output plane can be derived as:

$$\tan \kappa = \frac{R_i}{\eta k_{x y}}$$ (32)

From Eq. (32), we can find that the rotation angle $\kappa$ have a very simple linear relationship with the Zernike coefficient $Z_5$ on the output plane.
3 Verification of the Method

In the derivation of the relationships between the compensating parameters \((ΔL_1, ΔL_2, ... \)) and the Zernike coefficients \((Z_0, Z_2, Z_4, \kappa)\) on the output plane, some higher order quantities have been omitted. To verify the correctness of the theoretical derivation, the optical model of the reflective beam shaper designed in Sec.2 was built in commercial optical design software, where \(R_1=516\text{mm}\), \(R_2=800\text{mm}\), \(R_3=206\text{mm}\), \(L_1=(R_1+R_2)/2\), \(L_2=(R_2+R_3)/2\). In the model, the input aberrations were generated by adding a phase plate with different combination of Zernike coefficients. And the Zernike coefficients of aberrations on the output plane and normalized radius \(r_0\) can be generated by the commercial software, to serve as the H-S sensor in Fig.1, which is needed in Eq. (21), (22) and (31). In the calculations of the adjusting parameters, the rotational angle \(\kappa\) was the first parameter to be calculated, then the adjustment of distance \(ΔL_1\) was calculated, at last, \(ΔL_2\) was calculated. After \(\kappa, ΔL_1\) and \(ΔL_2\) were calculated, these value were sent to the optical model, then the wave-front parameters, such as Zernike coefficients and Peak-to-valley (PV) of the wave-front can be generated by the software. The comparison before and after compensation by adjusting of \(\kappa, ΔL_1\) and \(ΔL_2\) for four cases of input low order aberrations are listed in Table 1.

It shows that the adjusting parameters calculated in case1~case3 are very well to compensate the low order aberrations in the input plane, and the PV value after compensation is below \(1λ\), which is suitable for later wave-front corrections by a higher order deformable mirror, as shown in Fig. 1.

When the aberrations consists of both defocus and 45-Deg astigmatism, the first step of adjusting values of \(ΔL_1\) and \(ΔL_2\) are less effective, and the PV value after compensation is still larger than \(1λ\), as shown in Fig.2(h). It is because the adjusting of 45-Deg astigmatism can introduce small defocus, as illuminated in Eq. (27) and Eq. (28). So two steps compensation are necessary to solve this problem. That is, after the first step compensation, the normalized radius and Zernike coefficients are renewed to serve as the calculating parameters for second step compensation, as listed in case 4b in Table 1. Then the second adjusting parameters are obtained, and the final compensation result is satisfactory with a wave-front PV of \(0.31λ\).

<table>
<thead>
<tr>
<th>Table 1: Compensation results for 4 cases.</th>
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<tbody>
<tr>
<td>Optical parameters on (S_{\text{out}}) before compensation</td>
</tr>
<tr>
<td>(t_0)</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4a</td>
</tr>
<tr>
<td>4b</td>
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<tr>
<td>4b</td>
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</tbody>
</table>

Figure 2: Wave-front distribution before compensation (a) PV=4.18λ, in case 1, (b) PV=8.53λ, in case 2, (c) PV=2.53λ, in case 3, (d) PV=13.3λ, in case4 and after low order aberration compensation (e) PV=0.31λ, (f) PV=0.24λ, (g) PV=0.43λ, (h) PV=1.13λ, (i) 0.31λ after two steps compensation.

4 Conclusions

Based on the relationships between the optical layout parameters of a reflective beam shaper and Zernike coefficients on the output plane, the low order aberrations can be well compensated by directly adjusting the parameters. And the PV value after compensation is below \(1λ\), which can be further corrected by deformable mirrors. With this direct compensation method, low order aberration with large PV value in slab laser could be compensated both efficiently and quickly.

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