

Effect of Dispersive Reflectivity on the Stability of Gap Solitons in Systems with Separated Bragg Grating and Nonlinearity

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Abstract: The existence and stability of quiescent solitons in a dual core optical medium, where the one core has only Kerr nonlinearity while the other has Bragg grating and dispersive reflectivity are investigated. Three spectral gaps are identified in the systems linear spectrum, in which both lower and upper band gaps overlap with one branch of the continuous spectrum for all values of normalized group velocity c in the linear core; and, the central band gap remains a genuine bandgap. Soliton solutions exist only in the lower and upper gaps. In the absence of dispersive reflectivity, stable solitons are only found in the upper bandgap. However, introduction of dispersive reflectivity significantly alters the stability characteristics of solitons and results in the stabilization of solitons in a portion of the lower bandgap.

1 INTRODUCTION

One of the main characteristics of fiber Bragg gratings (FBGs) is that cross-coupling between forward and backward propagating waves leads to a strong effective dispersion which may be 6 orders of magnitude greater than the dispersion of silica (de Sterke and Sipe, 1994; Eggleton et al., 1997). At sufficiently high intensities this effective dispersion can be balanced by nonlinearity giving rise to the formation of gap solitons (de Sterke and Sipe, 1994). Over the past two decades, gap solitons have received much interest and have been studied extensively both theoretically (Aceves and Wabnitz, 1989; Christodoulides and Joseph, 1989; Malomed and Tasgal, 1994; Barashenkov et al., 1998; De Rossi et al., 1998; Neill and Atai, 2006) and experimentally (Eggleton et al., 1996; de Sterke et al., 1997; Eggleton et al., 1999).

One of the main features of gap solitons is that they can have any velocity between zero and the speed of light in the medium. Zero velocity or very slow solitons may be used as optical switches, optical memory or buffer elements (Krauss, 2008). As a result, much of the experimental activity in this area has focused on the generation of zero velocity (quiescent) solitons. Thus far, gap solitons with a velocity of 23% of the speed of light in the medium have been observed (Mok et al., 2006).

Gap solitons have been studied in a variety of

structures such as planar photonic crystal waveguides (Monat et al., 2010), dual core fibers (Atai and Malomed, 2000; Atai and Malomed, 2001; Mak et al., 1998; Atai and Baratali, 2012), waveguide arrays (Mandelik et al., 2004; Tan et al., 2009; Dong et al., 2011), and nonuniform Bragg gratings with dispersive reflectivity (Atai and Malomed, 2005; Neill et al., 2008). Since dual core fibers made of dissimilar cores can exhibit better performance and switching characteristics than the ones with identical cores (Atai and Chen, 1992; Atai and Chen, 1993; Bertolotti et al., 1995; Nistazakis et al., 2002), one may anticipate rich dynamics and interesting applications if such couplers are also equipped with Bragg gratings. Moreover, in the case of Bragg gratings with dispersive reflectivity, it has been demonstrated that the presence of dispersive reflectivity has a stabilizing effect (Atai and Malomed, 2005).

The objective of this work is to analyze the existence and stability of gap solitons in a dual core optical system where one core is linear and contains a nonuniform Bragg grating with dispersive reflectivity and the other core has only Kerr nonlinearity.

2 THE MODEL

Starting with the model of (Atai and Malomed, 2001) and following a similar procedure as described in (Atai and Malomed, 2005), one can derive the fol-

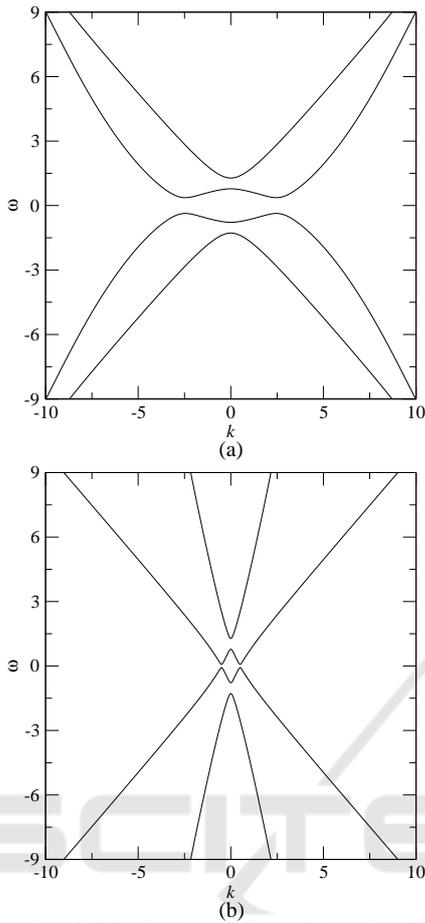


Figure 1: Typical examples of the linear spectrum for (a) $\lambda = 0.5$, $c = 0.0$ and $m = 0.5$ and (b) $\lambda = 0.5$, $c = 4.0$ and $m = 0.5$.

lowing normalized model which describes the propagation of light in a linearly coupled dual core system with one core having Kerr nonlinearity and the other one being linear and equipped with Bragg grating with dispersive reflectivity:

$$\begin{aligned}
 iu_t + iu_x + \left[|v|^2 + \frac{1}{2}|u|^2 \right] u + \phi &= 0, \\
 iv_t - iv_x + \left[|u|^2 + \frac{1}{2}|v|^2 \right] v + \psi &= 0, \\
 i\phi_t + ic\phi_x + u + \lambda\psi + m\psi_{xx} &= 0, \\
 i\psi_t - ic\psi_x + v + \lambda\phi + m\phi_{xx} &= 0,
 \end{aligned} \tag{1}$$

where the forward- and backward- propagating waves are u and v in the nonlinear core, ϕ and ψ in the grating aided core, function evolution time is t and x is the transverse coordinate, the coefficient of linear coupling between the cores is normalized to be 1, Bragg grating induced linear coupling coefficient is represented as $\lambda > 0$ between the left- and right- propagating waves. The group velocity in the nonlinear core is

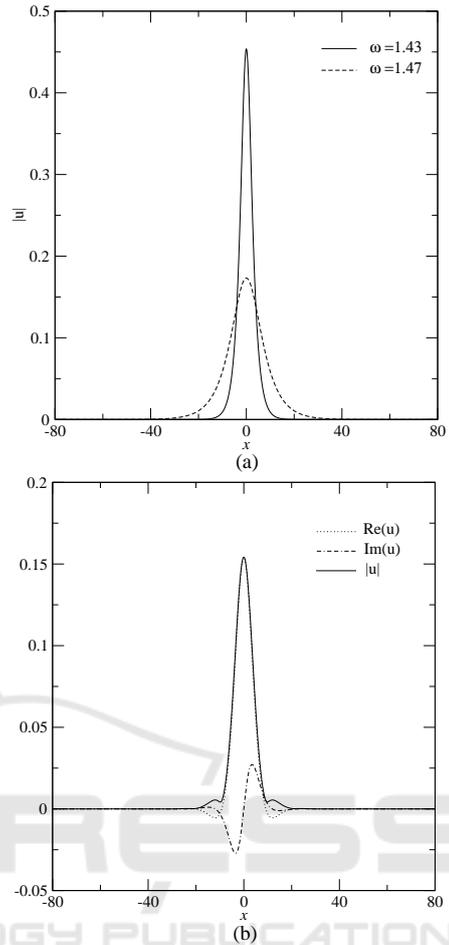


Figure 2: Examples of typical quiescent soliton solutions in upper gap with different parameter values (a) $\lambda = 0.8$, $c = 0.0$ and $m = 0.1$ (b) sidelobes formation with its real and imaginary parts for $\lambda = 0.8$, $c = 0.1$, $\omega = 1.47$ and $m = 0.5$.

set equal to 1 and c represents the relative group velocity in the linear core. Real parameter $m > 0$ is the dispersive reflectivity strength. Our analysis is limited to $0 < m < 0.5$ as there is no practical importance for $m > 0.5$ (Atai and Malomed, 2005).

Spectrum analysis is an important feature in order to find the existence of the gap soliton in the spectral gap (de Sterke and Sipe, 1994). Substituting $u, v, \phi, \psi \sim \exp(ikx - i\omega t)$ into Eqs. (1) and linearizing we arrive at the following dispersion relation:

$$\begin{aligned}
 \omega^4 - \left(2 + (\lambda - mk^2)^2 + (1 + c^2)k^2 \right) \omega^2 + \\
 (\lambda k - mk^3)^2 + (ck^2 - 1)^2 = 0.
 \end{aligned} \tag{2}$$

The dispersion relation gives rise to a central genuine gap and two gaps (one in the upper half and one in the lower half of the spectrum). The upper and lower gaps overlap with one branch of the continuous spectrum and therefore are not genuine gaps (Atai

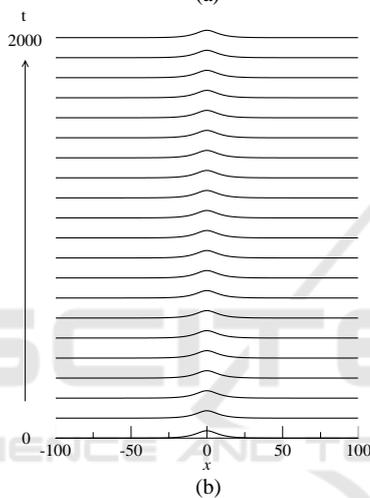
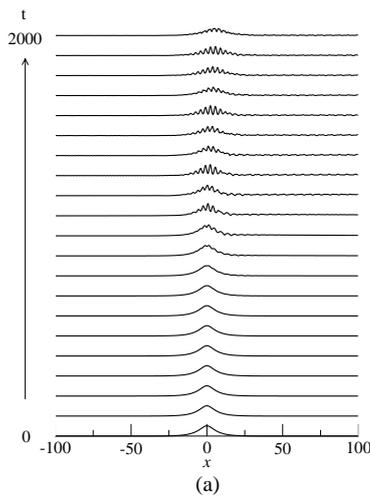


Figure 3: Typical examples of the stationary solitons evolution in the upper band gap (a) Unstable soliton for $\lambda = 0.2$, $c = 0.2$, $\omega = 1.02$ and $m = 0.1$ and (b) stable soliton for $\lambda = 0.2$, $c = 0.2$, $\omega = 1.06$ and $m = 0.1$. Only u components shown here.

and Malomed, 2001). For fixed values of c and λ , the width of the central gap varies with m . Typical examples of the bandgap structures are shown in Fig. 1.

3 STABILITY ANALYSIS

The soliton solutions are sought numerically by means of relaxation algorithm as there are no exact analytical solutions for $m \neq 0$. Stationary soliton solutions are found both in the upper and lower spectral gaps. No stationary solutions are found in the central band gap. With higher values of λ sidelobes are formed in the stationary soliton both in the upper and lower band gaps over a certain value of dispersive reflectivity. Typical soliton solutions with and without

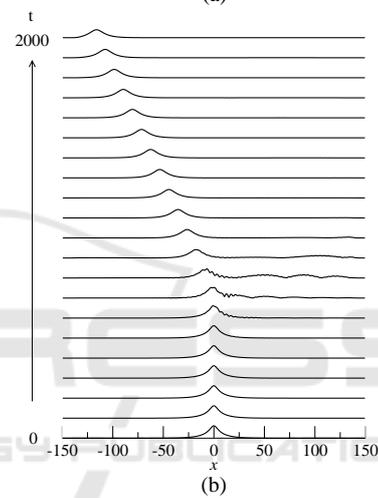
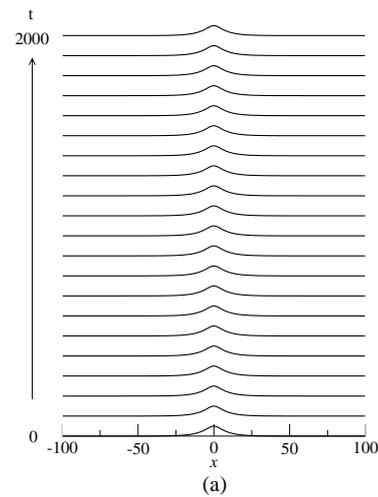


Figure 4: Typical examples of the stationary solitons evolution in the lower band gap (a) stable soliton for $\lambda = 0.2$, $c = 0.5$, $\omega = -1.01$ and $m = 0.1$ and (b) unstable soliton for $\lambda = 0.2$, $c = 0.5$, $\omega = -1.07$ and $m = 0.1$. Only u components shown here.

sidelobes are shown in Fig. 2.

We have investigated the stability of the stationary soliton solutions by means of systematic numerical simulations. Figs. 3 and 4 display examples of the evolution of stable and unstable solitons in the upper and lower bandgaps for various values of c , λ , m and ω . As is shown in Figs. 3(a) and 4(b) unstable solitons shed some energy in the form of radiation and subsequently they are either destroyed or evolve to a moving soliton. This demonstrates that stable moving solitons exist in this model.

Fig. 5 summarizes the outcomes of the stability analysis for $\lambda = 0.2$ and $m = 0.2$ in the (c, ω) plane. A noteworthy feature of the stability diagram is that there are large regions within the upper and lower bandgaps where solitons are stable. Additionally, it is found that in both lower and upper bandgaps, when

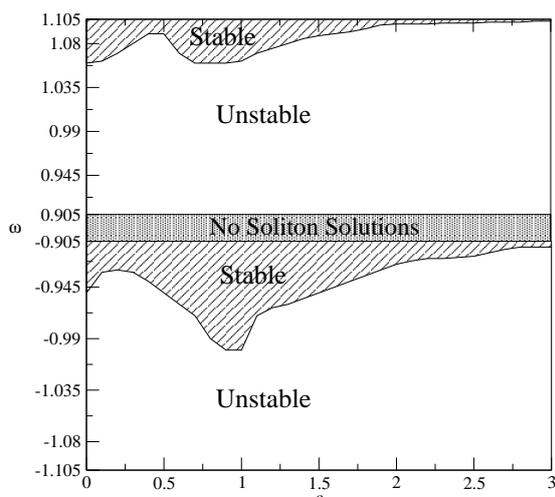


Figure 5: Stability diagram for $\lambda = 0.2$, $m = 0.2$ in the (c, ω) plane.

$c > 1$, the stable region shrinks as c increases. On the other hand, in the range $0.2 \leq c < 1$, increasing c results in the expansion of the stable region in the lower bandgap.

4 CONCLUSIONS

Bragg grating solitons are investigated numerically in a systematic way in a dual core coupled nonlinear medium where one core is nonlinear that contains Kerr nonlinearity and another core is linear and has a Bragg grating with dispersive reflectivity. The linear spectrum of the system has three spectral gaps: a genuine central gap and an upper and a lower gap each overlapping with one branch of the continuous spectrum for all values of c . Stationary soliton solutions are found only in the lower and upper band gaps. Above a certain value of dispersive reflectivity parameter m , solitons develop sidelobes. Sidelobes are dominant in the the soliton profile for lower values of c but for higher values of c no sidelobes are generated.

As for the stability of solitons, unlike the model without dispersive reflectivity, stable solitons are found in both upper and lower gaps. We have also identified nontrivial stability borders in the plane of (c, ω) .

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