Capturing Graded Knowledge and Uncertainty in a Modalized Fragment of OWL

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Abstract: Natural language statements uttered in diagnosis (e.g., in medicine), but more general in daily life are usually graded, i.e., are associated with a degree of uncertainty about the validity of an assessment. This uncertainty is often expressed through specific verbs, adverbs, adjectives, or even phrases in natural language which we will call gradation words (≈ linguistic hedges); e.g., Dr. X suspects that Y suffers from Hepatitis or The patient probably has Hepatitis or (The diagnosis of) Hepatitis is confirmed.

In this paper, we look into a representation of such graded statements by presenting a simple non-standard modal logic which comes with a set of modal operators, directly associated with the words indicating the uncertainty and interpreted through confidence intervals in the model theory. We complement the model theory by a set of RDFS-/OWL 2 RL-like entailment (if-then) rules, acting on the syntactic representation of modalized statements. Our interest in such a formalization is related to the use of OWL as the de facto language in today’s ontologies and its weakness to represent and reason about assertional knowledge that is uncertain or that changes over time.

1 INTRODUCTION

Medical natural language statements uttered by physicians or other health professionals and found in medical examination letters are usually graded, i.e., are associated with a degree of uncertainty about the validity of a medical assessment. This uncertainty is often expressed through specific verbs, adverbs, adjectives, or even phrases in natural language which we will call gradation words (≈ linguistic hedges); e.g., Dr. X suspects that Y suffers from Hepatitis or The patient probably has Hepatitis or (The diagnosis of) Hepatitis is confirmed.

In this paper, we look into a representation of such graded statements by presenting a simple non-standard modal logic which comes with a small set of partially-ordered modal operators, directly associated with the words indicating the uncertainty and interpreted through confidence intervals in the model theory. The work presented here addresses modalized propositional formulae in negation normal form which can be seen as a canonical representation of natural language sentences of the above form (a kind of a controlled natural language).

Our interest in such a formalization is related to the use of OWL in our projects as the de facto standard for (medical) ontologies today (to represent structural/terminological knowledge) and its weakness to represent and reason about assertional knowledge that is uncertain (Schulz et al., 2014) or that changes over time (Krieger, 2012). There are two principled ways to address such a restriction: either by sticking with the existing formalism (viz., OWL) and trying to find an encoding that still enables some useful forms of reasoning (Schulz et al., 2014); or by deviating from a defined standard in order to arrive, at best, at an easier, intuitive, and less error-prone representation (Krieger, 2012).

Here, we follow the latter avenue, but employ and extend the standard entailment rules from (Hayes, 2004; ter Horst, 2005; Motik et al., 2012) for positive binary relation instances in RDFS and OWL towards modalized n-ary relation instances, including negation. These entailment rules talk about, e.g., subsumption, class membership, or transitivity, and have been found useful in many applications. The proposed solution has been implemented for the binary relation case (extended triples, quads) in HFC (Krieger, 2013), a forward chaining engine that builds Herbrand models which are compatible with the open-world view underlying OWL.

Our approach is clearly not restricted to medical statements, but is applicable to graded statements in general, e.g., in technical diagnosis (the engine is probably overheated) or in everyday conversation (I’m pretty sure that Joe has signed a contract with...
In our approach, however, the qualitative information parts from Figure 1 are first class citizens of the object language (the modal operators) and diagnostic statements from the Hepatitis use case are expressed through the binary property suffersFrom between $p$ (patients, people) and $d$ (diseases, diagnoses). The plausible inferences are then simply a byproduct of the instantiation of the entailment rule schemas (G) from Section 5.1, and (S1) and (S0) from Section 5.2 for property suffersFrom (the rule variables are universally quantified; $\top = universal truth; C = confirmed; L = likely; E = excluded; U = unlikely$), e.g.,

(S1) \[ \text{ViralHepatitisB} \sqsubseteq \text{ViralHepatitis} \land \top \text{ViralHepatitisB}(d) \rightarrow \top \text{ViralHepatitis}(d) \]

(G) \[ C\text{suffersFrom}(p,d) \rightarrow L\text{suffersFrom}(p,d) \]

Two things are worth mentioning here. Firstly, not only OWL properties can be graded, such as $C\text{suffersFrom}(p,d)$ (= it is confirmed that $p$ suffers from $d$), but also class membership, e.g., $CV\text{ViralHepatitisB}(d)$ (= it is confirmed that $d \text{ is of type Viral Hepatitis B}$). As the original OWL example from (Schulz et al., 2014) can not make use of any modals, we employ the special modal $\top$ here: $\top \text{ViralHepatitisB}(d)$. Secondly, modal operators are only applied to assertional knowledge (the ABox in OWL)—neither TBox nor RBox axioms are being affected by modals in our approach, as they are supposed to express universal truth.

3 CONFIDENCE AND CONFIDENCE INTERVALS

We address the confidence of an asserted (medical) statement (Schulz et al., 2014) through graded modalities applied to propositional formulae: $E$ (excluded), $U$ (unlikely), $N$ (not excluded), $L$ (likely), and $C$ (confirmed). For various (technical) reasons, we add a wildcard modality $!$ (unknown), a complementary failure modality $\bot$ (error), plus two further modalities to syntactically state definite truth and falsity: $\top$ (true) and $\bot$ (false).

Let $\Delta$ now denotes the set of all modalities:

$$\Delta := \{ ?,!, \top, \bot, E, U, N, L, C\}$$

A measure function

$$\mu : \Delta \rightarrow [0,1] \times [0,1]$$

\[1\] We also call $\top$ and $\bot$ propositional modals as they lift propositional statements to the modal domain. We refer to $?$ and $!$ as completion modals since they complete the modal hierarchy by adding unique most general and most specific elements (see Section 4.3).
is a mapping which returns the associated confidence interval \( \mu(\delta) = [l, h] \) for a modality from \( \delta \in \triangle \) (\( l \leq h \)). We presuppose that

- \( \mu(\top) = [0, 1] \)
- \( \mu(\bot) = [1, 1] \)
- \( \mu(\perp) = [0, 0] \)
- \( \mu(!) = \emptyset^2 \)

In addition, we define two disjoint subsets of \( \triangle \), called

\[ \bot := \{ \top, C, L, N \} \]

and

\[ \top := \{ \bot, E, U \} \]

and again make a presupposition: the confidence intervals for modals from \( \bot \) end in 1, whereas the confidence intervals for \( \top \) modals always start with 0. It is worth noting that we do not make use of \( \mu \) in the syntax of the modal language (for which we employ the modalities from \( \triangle \)), but in the semantics when dealing with the satisfaction relation of the model theory (see Section 4).

We have talked about confidence intervals now several times without saying what we actually mean by this. Suppose that a physician says that it is confirmed (= \( C \)) that patient \( p \) suffers from disease \( d \), for a set of observed symptoms (or evidence) \( S = \{ S_1, \ldots, S_k \} \): \( C \text{suffersFrom}(p, d) \).

Assuming that a different patient \( p' \) shows the same symptoms \( S \) (and only \( S \), and perhaps further symptoms which are, however, independent from \( S \)), we would assume that the same doctor would diagnose \( C \text{suffersFrom}(p', d) \).

Even an other, but similar trained physician is supposed to grade the two patients similarly. This similarity which originates from patients showing the same symptoms and from physicians being taught at the same medical school is addressed by confidence intervals and not through a single (posterior) probability, as there are still variations in diagnostic capacity and daily mental state of the physician. By using intervals (instead of single values), we can usually reach a consensus among people upon the meaning of gradation words, even though the low/high values of the confidence interval for, e.g., confirmed might depend on the context.

Being a bit more theoretic, we define a confidence interval as follows. Assume a Bernoulli experiment (Krengel, 2003) that involves a large set of \( n \) patients

\[ P, \text{sharing the same symptoms } S. \text{ W.r.t. our example, we would like to know whether } \text{suffersFrom}(p, d) \text{ or } \neg \text{suffersFrom}(p, d) \text{ is the case for every patient } p \in P, \text{ sharing } S. \text{ Given a Bernoulli trials sequence } \vec{X} = (X_1, \ldots, X_n) \text{ with indicator random variables } X_i \in \{0, 1\} \text{ for a patient sequence } (p_1, \ldots, p_n), \text{ we can approximate the expected value } E \text{ for } \text{suffersFrom} \text{ being true, given disease } d \text{ and background symptoms } S \text{ by the arithmetic mean } A:\]

\[ E[\vec{X}] = \lim_{n \to \infty} \frac{\sum_{i=1}^{n} X_i}{n} \]

Due to the law of large numbers, we expect that if the number of elements in a trials sequence goes to infinity, the arithmetic mean will coincide with the expected value:

\[ E[\vec{X}] = \frac{\sum_{i=1}^{n} X_i}{n} \]

Clearly, the arithmetic mean for each new finite trials sequence is different, but we can try to locate the expected value within an interval around the arithmetic mean:

\[ E[\vec{X}] \in \left [ A[\vec{X}] - \varepsilon_1, A[\vec{X}] + \varepsilon_2 \right ] \]

For the moment, we assume \( \varepsilon_1 = \varepsilon_2 \), so that \( A[\vec{X}] \) is in the center of this interval which we will call from now on confidence interval.

Coming back to our example and assuming \( \mu(C) = [0.9, 1] \), \( \text{suffersFrom}(p, d) \) can be read as being true in 95% of all cases known to the physician, involving patients \( p \) potentially having disease \( d \) and sharing the same prior symptoms (evidence) \( S_1, \ldots, S_k \):

\[ \sum_{p \in L} \text{Prob}(\text{suffersFrom}(p, d)|S) \approx 0.95 \]

The variance of \( \pm 5\% \) is related to varying diagnostic capabilities between (comparative) physicians, daily mental form, undiscovered important symptoms or examinations which have not been carried out (e.g., lab values), or perhaps even by the physical stature of the patient (crooked vs. upright) which unconsciously affects the final diagnosis, etc, as elaborated above. Thus the individual modals from \( \triangle \) express (via \( \mu \)) different forms of the physician’s confidence, depending on the set of already acquired symptoms as (potential) explanations for a specific disease.

### 4 Model Theory and Negation Normal Form

Let \( C \) denote the set of constants that serve as the arguments of a relation instance. For instance, in
an RDF/OWL setting, C would exclusively consist of XSD atoms, blank nodes, and URIs/IRIs. In order to define basic $n$-ary propositional formulae (ground atoms), let $p(\vec{c})$ abbreviates $p(c_1, \ldots, c_n)$, for $c_1, \ldots, c_n \in C$, given length($\vec{c}$) = $n$. In case the number of arguments does not matter, we sometimes simply write $p$, instead of, e.g., $p(c, d)$ or $p(\vec{c})$. As before, we assume $\Delta = \{ ?, !, \top, \bot, E, U, N, L, C \}$. We inductively define the set of well-formed formulae $\phi$ of our modal language as follows:

$$\phi ::= p(\vec{c}) \mid \neg \phi \mid \phi \land \phi' \mid \phi \lor \phi' \mid \Delta \phi$$

4.1 Simplification and Normal Form

We now syntactically simplify the set of well-formed formulae $\phi$ by restricting the uses of negation and modalities to the level of propositional letters $\pi$:

$$\pi ::= p(\vec{c}) \mid \neg p(\vec{c})$$

$$\phi ::= \pi \mid \Delta \pi \mid \phi \land \phi' \mid \phi \lor \phi'$$

The design of this language is driven by two main reasons: firstly, we want to effectively implement the logic (in our case, in HFC), and secondly, the application of the below semantic-preserving simplification rules in an offline pre-processing step makes the implementation easier and guarantees a more efficient runtime system. To address negation, we first need to represent the notion of a complement modal $\delta^C$ for every $\delta \in \Delta$, where

$$\mu(\delta^C) := \mu(\delta)^C = \mu(\delta^C) = [0, 1] \setminus \mu(\delta)$$

I.e., $\mu(\delta^C)$ is defined as the complementary interval of $\mu(\delta)$ (within the bounds of $[0, 1]$, of course). For example, $E$ and $N$ (excluded, not excluded) or ? and ! (unknown, error) are already existing complementary modals.

We also require mirror modals $\delta^M$ for every $\delta \in \Delta$ whose confidence interval $\mu(\delta^M)$ is derived by "mirroring" $\mu(\delta)$ to the opposite side of the confidence interval, either to the left or to the right:

- If $\mu(\delta) = [l, h]$, then $\mu(\delta^M) := [h, 1 - l]$.
- If $\mu(\delta) = [0, h]$, then $\mu(\delta^M) := [1 - h, 1]$.

It is easy to see that these two equations can be unified and generalized:\footnote{This construction procedure comes in handy when dealing with in-the-middle modals, such as fifty-fifty or perhaps, whose confidence intervals neither touch 0 nor 1. Such modals have a real background in (medical) diagnosis.}

- If $\mu(\delta) = [l, h]$, then $\mu(\delta^M) := [1 - h, 1 - l]$.  

For example, $E$ and $C$ (excluded, confirmed) or $\top$ and $\bot$ (top, bottom) are mirror modals. In order to transform $\phi$ into its negation normal form, we need to apply simplification rules a finite number of times (until rules are no longer applicable). We depict those rules by using the $\vdash$ relation, read as formula $\vdash$ simplified formula ($e$ = empty word):

1. $\vdash \phi \vdash e \quad \% \phi$ is not informative at all
2. $\vdash \neg \phi \vdash \phi$
3. $\vdash \neg(\phi \land \phi') \vdash \neg \phi \lor \neg \phi'$
4. $\vdash \neg(\phi \lor \phi') \vdash \neg \phi \land \neg \phi'$
5. $\vdash \neg \phi \vdash \Delta^C \phi$ (example: $\neg E \phi = E^C \phi = N \phi$)
6. $\vdash \neg \phi \land \Delta^M \phi$ (example: $E \neg \phi = E^M \phi = C \phi$)

Clearly, the mirror modals $\delta^M (\delta \in \Delta)$ are not necessary as long as we explicitly allow for negated statements (which we do), and thus case 6 can, in principle, be dropped.

What is the result of simplifying $\Delta(\phi \land \phi')$ and $\Delta(\phi \lor \phi')$? Let us start with the former case and consider as an example the statement about an engine that a mechanical failure $m$ and an electrical failure $e$ is confirmed: $C(m \land e)$. It seems plausible to simplify this expression to $Cm \land Ce$. Commonsense tells us furthermore that neither $Em$ nor $Ee$ is compatible with this description (we should be alarmed if, e.g., both $Cm$ and $Em$ happen to be the case).

Now consider the "opposite" statement $E(m \land e)$ which must not be rewritten to $E(m \land e)$, as either $Cm$ or $Ce$ is well compatible with $E(m \land e)$. Instead, we rewrite this kind of "negated" statement as $Em \lor Ee$, and this works fine with either $Cm$ or $Ce$.

In order to address the other modal operators, we generalize these plausible inferences by making a distinction between $\bot$ and $\top$ modals (cf. Section 3):

7a. $\bot(\phi \land \phi') \vdash \bot \phi \lor \bot \phi'$
7b. $\top(\phi \land \phi') \vdash \top \phi \land \top \phi'$

Let us now focus on disjunction inside the scope of a modal operator. As we do allow for the full set of Boolean operators, we are allowed to deduce

8a. $\Delta(\phi \lor \phi') \vdash \Delta(\neg(\neg \phi \land \neg \phi')) \vdash \Delta(\neg \phi \land \neg \phi')$

This is, again, a conjunction, so we apply schemas 7a and 7b, giving us

8b. $\Delta(\phi \lor \phi') \vdash \Delta(\neg \phi \lor \neg \phi') \vdash \Delta(\neg \phi \land \neg \phi') \vdash \Delta(\neg \phi \lor \neg \phi') \vdash \Delta(\neg \phi \land \neg \phi')$

Note how the modals from $\bot$ in 7a and 8a act as a kind of negation operator to turn the logical operators into their counterparts, similar to de Morgan’s law.

The final case considers two consecutive modals:
9. $\delta' \delta'' \phi \vdash (\delta' \circ \delta'') \phi$

We interpret the $\circ$ operator as a kind of function composition, leading to a new modal $\delta$ which is the result of $\delta' \circ \delta''$. We take a liberal stance here of what the result is, but indicate that it depends on the domain and, again, plausible inferences we like to capture. The $\circ$ operator will probably be different from the related operation $\odot$ which is used in Section 5.3.4.

4.2 Model Theory

In the following, we extend the standard definition of modal (Kripke) frames and models (Blackburn et al., 2001) for the graded modal operators from $\Delta$ by employing the confidence function $\mu$ and focussing on the minimal definition for $\phi$. A frame $F$ for the probabilistic modal language is a pair

$$F = (W, \Delta)$$

where $W$ is a non-empty set of worlds (or situations, states, points, vertices, etc.) and $\Delta$ a family of binary relations over $W \times W$, called accessibility relations.

In the following, we overload $\delta \in \Delta$ below in that we let $\delta$ both refer to the modal in the syntax as well as to the accessibility relation $R_\delta$ in the semantics.

A model $M$ for the probabilistic modal language is a triple

$$M = (F, \mathcal{V}, \mu)$$

such that $F$ is a frame, $\mathcal{V}$ is a valuation, assigning each proposition $\phi$ a subset of $W$, viz., the set of worlds in which $\phi$ holds, and $\mu$ is a mapping, returning the confidence interval for a given modality from $\Delta$. Note that we only require a definition for $\mu$ in $M$ (the model, but not in the frame), as $F$ represents the relational structure without interpreting the edge labeling (the modal names) of the graph.

The satisfaction relation $\models$, given a model $M$ and a specific world $w$ is inductively defined over the set of well-formed formulae in negation normal form (remember $\pi := p(\bar{c}) \mid \neg p(\bar{c})$):

1. $M, w \models p(\bar{c})$ iff $w \in \mathcal{V}(p(\bar{c}))$ and $w \not\in \mathcal{V}(\neg p(\bar{c}))$
2. $M, w \not\models \neg p(\bar{c})$ iff $w \in \mathcal{V}(\neg p(\bar{c}))$ and $w \not\in \mathcal{V}(p(\bar{c}))$
3. $M, w \models \phi \land \psi$ iff $M, w \models \phi$ and $M, w \models \psi$
4. $M, w \models \phi \lor \psi$ iff $M, w \models \phi$ or $M, w \models \psi$
5. for all $\delta \in \Delta$: $M, w \models \delta \phi$ iff

$$\frac{\# \{ u \mid (w, u) \in \delta \text{ and } M, u \models \pi \} \text{ or } \mu(\delta)}{\# \{ u \mid (w, u) \in \delta' \text{ and } M, u \models \pi \}} \in \mu(\delta)$$

The last case of the satisfaction relation addresses the modals: for a world $w$, we look for the successor states $u$ that are directly reachable via $\delta$ and in which $\pi$ holds, and divide the number of such states ($\# \{ u \mid (w, u) \in \delta \}$) by the number of all worlds that are reachable from $w$ in the denominator. This number, lying between 0 and 1, is then required to be an element of the confidence interval $\mu(\delta)$ of $\delta$ in order to satisfy $\delta \pi$, given $M, w$.

It is worth noting that the satisfaction relation above differs from the standard definition in its handling of $M, w \models \neg p(\bar{c})$, as negation is not interpreted through the absence of $p(\bar{c})$ ($M, w \not\models \neg p(\bar{c})$), but through the existence of $\neg p(\bar{c})$. This treatment addresses the open-world nature in OWL and the evolvement of a (medical) domain over time.

We also note that the definition of the satisfaction relation for modalities (last clause) is related to the possibility operators $M_k = (\equiv \circ^{\Delta k}; k \in \mathbb{N})$ introduced by (Fine, 1972) and counting modalities $k \geq n$ (Areces et al., 2010), used in modal logic characterizations of description logics with cardinality restrictions.

4.3 Well-behaved Frames

As we will see later, it is handy to assume that the graded modals are arranged in a kind of hierarchy—the more we move along the arrows in the hierarchy, the more a statement $\phi$ in the scope of a modal $\delta \in \Delta$ becomes uncertain. In order to address this, we slightly extend the notion of a frame by a third component $\preceq \subseteq \Delta \times \Delta$, a partial order (i.e., a reflexive, antisymmetric, and transitive binary relation) between modalities:

$$F = (W, \Delta, \preceq)$$

Let us consider the following modal hierarchy that we build from the set $\Delta$ of already introduced modals (cf. Figure 1):

$$\top \longrightarrow C \longrightarrow L \longrightarrow N \rightarrow ? \downarrow \longrightarrow E \longrightarrow U$$

This graphical representation is just a compact way to specify a set of 33 binary relation instances over $\Delta \times \Delta$, such as $\top \preceq \top$, $\top \preceq N$, $C \preceq N$, $\bot \preceq ?$, or $\top \preceq ?$. The above mentioned form of uncertainty is expressed by the measure function $\mu$ in that the associated confidence intervals become larger:

$$\text{if } \delta \preceq \delta' \text{ then } \mu(\delta) \subseteq \mu(\delta')$$

In order to arrive at a proper and intuitive model-theoretic semantics which mirrors intuitions such as If $\phi$ is confirmed (C$\phi$) then $\phi$ is likely (L$\phi$), we will
focus here on well-behaved frames \( F \) which enforce the existence of edges in \( W \), given \( \preceq \) and \( \delta, \delta' \in \Delta \):
\[
\text{if } (w, u) \in \delta \text{ and } \delta \preceq \delta' \text{ then } (w, u) \in \delta'.
\]

However, by imposing this constraint, we also need to adapt the last case of the satisfiability relation from Section 4.2 above:

5. for all \( \delta \in \Delta \): \( M, w \models \delta \pi \) iff
\[
\# \cup_{\delta \preceq \delta} \{ u \mid (w, u) \in \delta \} \text{ and } \# \cup_{\delta' \in \Delta} \{ u \mid (w, u) \in \delta' \} \in \mu(\delta).
\]

Not only are we scanning for edges \( (w, u) \) labeled with \( \delta \) and for successor states \( u \) of \( w \) in which \( \pi \) holds in the numerator (original definition), but also take into account edges marked with more general modals \( \delta' \): \( \delta' \preceq \delta \). This mechanism implements a kind of built-in model completion that is not necessary in ordinary modal logics as they deal with only a single relation (viz., unlabeled arcs).

5 ENTAILMENT RULES

We now turn our attention, again, to the syntax of our language and to the syntactic consequence relation. This section addresses a restricted subset of entailment rules which will unveil new (or implicit) knowledge from already existing graded statements. Recall that these kind of statements (in negation normal form) are a consequence of the application of simplification rules as depicted in Section 4.1. Thus, we assume a pre-processing step here that “massages” more complex statements that arise from a representation of graded (medical) statements in natural language. The entailments which we will present in a moment can either be directly implemented in a tuple-based reasoner, such as \( HFC \) (Krieger, 2013), or in triple-based engines (e.g., Jena (Carroll et al., 2004) or \( OWLIM \) (Bishop et al., 2011)) which need to reify the medical statements in order to be compliant with the RDF triple model.

5.1 Modal Entailments

The entailments presented in this section deal with plausible inference centered around modals \( \delta, \delta' \in \Delta \) which are, in part, also addressed in (Schulz et al., 2014) in a pure OWL setting. We use the implication sign \( \to \) to depict the entailment rules
\[
\text{lhs } \to \text{ rhs}
\]
which act as completion (or materialization) rules the way as described in, e.g., (Hayes, 2004) and (ter Horst, 2005), and used in today’s semantic repositories (e.g., \( OWLIM \)). We sometimes even use the bi-conditional \( \leftrightarrow \) to address that the LHS and the RHS are semantically equivalent, but will indicate the direction that should be used in a practical setting. As before, we define
\[
\pi := p(\bar{c}) \mid \neg p(\bar{c})
\]
We furthermore assume that for every modal \( \delta \in \Delta \), a complement modal \( \delta^c \) and a mirror modal \( \delta^M \) exist (cf. Section 4.1).

5.1.1 Lift

\[
\text{(L) } \pi \leftrightarrow \top \pi
\]
This rule interprets propositional statements as special modal formulae. It might be dropped and can be seen as a pre-processing step. We have used it in the Hepatitis example above. Usage: left-to-right direction.

5.1.2 Generalize

\[
\text{(G) } \delta \pi \land \delta \preceq \delta' \to \delta' \pi
\]
This rule schema can be instantiated in various ways, using the modal hierarchy from Section 4.3, e.g., \( \top \pi \to C \pi, C \pi \to L \pi, \) or \( E \pi \to U \pi \). It has been used in the Hepatitis example.

5.1.3 Complement

\[
\text{(C) } \neg \delta \pi \leftrightarrow \delta^c \pi
\]
In principle, \( (C) \) is not needed in case the statement is already in negation normal form. This schema might be useful for natural language paraphrasing (explanation). Given \( \Delta \), there are four possible instantiations: \( E \pi \leftrightarrow \neg N \pi, N \pi \leftrightarrow \neg E \pi, \top \pi \leftrightarrow \neg \top \pi, \) and \( \bot \pi \leftrightarrow \neg \bot \pi \).

5.1.4 Mirror

\[
\text{(M) } \delta \pi \leftrightarrow \delta^M \pi
\]
Again, \( (D) \) is in principle not needed as long as the modal proposition is in negation normal form, since we do allow for negated propositional statements \( \neg p(\bar{c}) \). This schema might be useful for natural language paraphrasing (explanation). For \( \Delta \), there are six possible instantiations, viz., \( E \pi \leftrightarrow C \neg \pi, C \pi \leftrightarrow E \neg \pi, L \pi \leftrightarrow U \neg \pi, U \pi \leftrightarrow L \neg \pi, \top \pi \leftrightarrow \bot \neg \pi, \) and \( \bot \pi \leftrightarrow \top \neg \pi \).

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5.1.5 Uncertainty

(U) $\delta_\pi \land \neg \delta_\pi \leftrightarrow \delta_\pi \land \delta^{C_\pi} \leftrightarrow \neg \pi$

The co-occurrence of $\delta_\pi$ and $\neg \delta_\pi$ does not imply logical inconsistency (propositional case: $\pi \land \neg \pi$), but leads to complete uncertainty about the validity of $\pi$. Remember that $\mu(\neg ?) = \mu(\delta) \cup \mu(\delta^{C}) = [0, 1]$: $\mu: \{\neg \delta^{C}, \neg \delta\} \rightarrow [0, 1]$

Usage: left-to-right direction.

5.1.6 Negation

(N) $\delta(\pi \land \neg \pi) \leftrightarrow \delta_\pi \land \neg \pi \leftrightarrow \delta_\pi \land \delta^{M_\pi} \leftrightarrow \delta^{M_\pi} \land \pi \land \neg \pi$

(N) shows that $\delta(\pi \land \neg \pi)$ can be formulated equivalently by using the mirror modal $\delta^{M}$: $\mu: \{\neg \delta^{M}, \neg \delta\} \rightarrow [0, 1]$

Usage: left-to-right direction.

5.1.7 Error

(E) $\top(\pi \land \neg \pi) \leftrightarrow \bot(\pi \land \neg \pi) \leftrightarrow !\top(\pi \land \neg \pi) \leftrightarrow !\pi$

(E) is the modal counterpart of the law of non-contradiction (note: $\bot^M = \top, \top^M = \bot, \bot^M = \bot$). For this reason and by definition, the error (or failure) modal $!$ from Section 3 comes into play here. The modal $!$ can serve as a hint to either stop a computation the first time it occurs, or to continue reasoning and to syntactically memorize the ground literal $\pi$. Usage: left-to-right direction.

5.2 Subsumption Entailments

As before, we define two subsets of $\Delta$, called $\{ \top, C, L, N \}$ and $\{ \bot, E, U \}$, thus $\bot$ and $\top$ effectively become $\{ \top, C, L, N, U^C \}$ and $\{ \bot, E, U, C^C, L^C, N^M \}$ due to the use of complement modals $\delta^C$ and mirror modals $\delta^M$ for every base modal $\delta \in \Delta$ and by assuming that $E = N^C$, $E = C^M$, $U = M^C$, and $\bot = M^M$, together with the four “opposite” cases.

Now, let $\sqsubseteq$ abbreviate relation subsumption as known from description logics and realized through $\text{rdfs:subClassOf}$ and $\text{rdfs:subPropertyOf}$. Given this, we define two further very practical and plausible modal entailments which can be seen as the modal extension of the entailment rules ($\text{rdfs8}$) and ($\text{rdfs7}$) for classes and properties in RDFS (Hayes, 2004):

$$\begin{align*}
(S1) & \quad \neg p(c) \land p \sqsubseteq q \rightarrow \neg q(c) \\
(S0) & \quad \neg q(c) \land p \sqsubseteq q \rightarrow q(c)
\end{align*}$$

Note how the use of $p$ and $q$ switches in the antecedent and the consequent, even though $p \sqsubseteq q$ holds in both cases. Note further that propositional statements $\pi$ are restricted to the positive case $p(c)$ and $q(c)$, as their negation in the antecedent will not lead to any valid entailments.

Here are four instantiations of (S0) and (S1) for the unary and binary case (remember, $C \in \Delta$ and $E \in \{ \} )$:

$$\begin{align*}
\text{ViralHepatitisB} \quad & \text{ViralHepatitis} \\
\text{CViralHepatitisB(x)} \quad & \text{CViralHepatitis(x)} \\
\text{ViralHepatitis} \quad & \text{Hepatitis} \\
\text{EHepatitis(x)} \quad & \text{EViralHepatitis(x)}
\end{align*}$$

$$\begin{align*}
\text{deeplyEnclosedIn} \sqsubseteq \text{containedIn} \\
\text{CdeeplyEnclosedIn}(x,y) \quad & \text{CcontainedIn}(x,y) \\
\text{superficiallyLocatedIn} \sqsubseteq \text{containedIn} \\
\text{EcontainsIn}(x,y) \quad & \text{EsuperficiallyLocatedIn}(x,y)
\end{align*}$$

5.3 Extended RDFS & OWL Entailments

In this section, we will consider further entailment rules for RDFS (Hayes, 2004) and a restricted subset of OWL (ter Horst, 2005; Motik et al., 2012). Remember that modals only head positive and negative propositional letters $\pi$, not TBox or RBox axioms. Concerning the original entailment rules, we will distinguish four principal cases to which the extended rules belong (we will only consider the unary and binary case here as used in description logics/OWL):

1. TBox and RBox axiom schemas will not undergo a modal extension;
2. rules get extended in the antecedent;
3. rules take over modals from the antecedent to the consequent;
4. rules aggregate several modals from the antecedent in the consequent.

We will illustrate the individual cases in the following subsections with examples by using a kind of description logic rule syntax. Clearly, the set of extended entailments depicted here is not complete.
5.3.1 Case-1: No Modals

Entailment rule (rdfs11) from (Hayes, 2004) deals with class subsumption: \( C \sqsubseteq D \land D \sqsubseteq E \rightarrow C \sqsubseteq E \). As this is a terminological axiom schema, the rule stays constant in the modal domain. Example rule instantiation:

\[
\begin{align*}
\text{ViralHepatitisB} & \sqsubseteq \text{ViralHepatitis} \\
\text{ViralHepatitis} & \sqsubseteq \text{Hepatitis} \\
\text{ViralHepatitisB} & \sqsubseteq \text{Hepatitis}
\end{align*}
\]

5.3.2 Case-2: Modals on LHS, No Modals on RHS

The following original rule (rdfs3) from (Hayes, 2004) imposes a range restriction on objects of binary ABox relation instances: \( \forall P.C \land P(x, y) \rightarrow C(y) \). The extended version needs to address the ABox proposition in the antecedent (don’t care modal \( \delta \)), but must not change the consequent (even though we always use the \( \top \) modality here—the range restriction \( C(y) \) is always true, independent of the uncertainty of \( P(x, y) \); cf. Section 2 example):

\[(\text{Mrdfp3}) \quad \forall P.C \land \delta P(x, y) \rightarrow \top C(y)\]

Example rule instantiation:

\[
\forall \text{suffersFrom.Disease} \land L\text{suffersFrom}(x, y) \rightarrow \top \text{Disease}(y)
\]

5.3.3 Case-3: Keeping LHS Modals on RHS

Inverse properties switch their arguments (ter Horst, 2005) as described by (rdfp8):

\[P \equiv Q^- \land P(x, y) \rightarrow Q(y, x)\]

The extended version simply keeps the modal operator:

\[(\text{Mrdfp8}) \quad P \equiv Q^- \land \delta P(x, y) \rightarrow \delta Q(y, x)\]

Example rule instantiation:

\[\text{containedIn} \equiv \text{contains}^- \land C\text{containedIn}(x, y) \rightarrow C\text{contains}(x, y)\]

5.3.4 Case-4: Aggregating LHS Modals on RHS

Now comes the most interesting case of modalized RDFS & OWL entailment rules, that offers several possibilities on a varying scale between skeptical and credulous entailments, depending on the degree of uncertainty, as expressed by the measuring function \( \mu \) of the modal operator. Consider the original rule (rdfp4) from (ter Horst, 2005) for transitive properties:

\[P^+ \sqsubseteq P \land P(x, y) \land P(y, z) \rightarrow P(x, z)\]

Now, how does the modal on the RHS of the extended rule look like, depending on the two LHS modals? There are several possibilities. By operating directly on the modal hierarchy, we are allowed to talk about, e.g., the least upper bound or the greatest lower bound of \( \delta^L \) and \( \delta^U \). When taking the associated confidence intervals into account, we might play with the low and high numbers of the intervals, say, by applying min/max, the arithmetic mean or even by multiplying the corresponding numbers.

Let us first consider the general rule from which more specialized versions can be derived, simply by instantiating the combination operator \( \circ \):

\[(\text{Mrdfp4}) \quad P^+ \sqsubseteq P \land \delta P(x, y) \land \delta' P(y, z) \rightarrow (\delta \circ \delta') P(x, z)\]

Here is an instantiation of Mrdfp4, dealing with the transitive relation contains from above, assuming that \( \circ \) reduces to the least upper bound (i.e., \( C \circ L = L \)):

\[C\text{contains}(x, y) \land L\text{contains}(y, z) \rightarrow L\text{contains}(x, z)\]

What is the general result of \( \delta \circ \delta'^U \)? It depends, probably both on the application domain and the epistemic commitment one is willing to accept about the “meaning” of gradation words/modal operators. To enforce that \( \circ \) is at least both commutative and associative (as is the least upper bound) is probably a good idea, making the sequence of modal clauses order independent. And to work on the modal hierarchy instead of combining low/high numbers of the corresponding intervals is probably a good decision for forward chaining engines, as the latter strategy might introduce new individuals through operations such as multiplication, thus posing a problem for the implementation of the generalization schema (\( G \)) (see Section 5.1.2).

5.4 Custom Entailments: An Example from the Medical Domain

Consider that Hepatitis B is an infectious disease

\[\text{ViralHepatitisB} \sqsubseteq \text{InfectiousDisease} \sqsubseteq \text{Disease}\]

and note that there exist vaccines against it. Assume that the liver l of patient p quite hurts

\[\text{ChasPain}(p, l)\]

but p has been definitely vaccinated against Hepatitis B before:

\[\top \text{vaccinatedAgainst}(p, \text{ViralHepatitisB})\]

We apply OWL2-like punning here when using the class ViralHepatitisB (not an instance), as the second argument of vaccinatedAgainst; cf. (Golbreich and Wallace, 2012).

Given that p received a vaccination, the following custom rule will not fire (\( x, y \) below are now
universally-quantified variables; \( z \) an existentially-
quantified RHS-only variable):
\[
\top \text{Patient}(x) \land \top \text{Liver}(y) \land \text{ChasPain}(x,y) \land \\
\text{VaccinatedAgainst}(x, \text{ViralHepatitisB}) \rightarrow \\
\text{ViralHepatitisB}(z) \land \text{NSuffersFrom}(x,z)
\]
Now assume another person \( p' \) that is pretty sure (s)he
was never vaccinated:
\[
\text{EvaccinatedAgainst}(p', \text{ViralHepatitisB})
\]
Given the above custom rule, we are allowed to infer
that (h instantiation of \( z \))
\[
\text{ViralHepatitisB}(h) \land \text{NSuffersFrom}(p', h)
\]
The subclass axiom from above thus assigns
\[
\text{INfectiousDisease}(h)
\]
so that we can query for patients for whom an infectious
disease is not excluded (\( \equiv N \)), in order to initiate
appropriate methods (e.g., further medical investigations).

5.5 Implementing Modal Entailments

The negation normal form from Section 4.1 makes
it relatively easy to implement entailment rules in-
volving modalized propositional letters of the form
\( \delta \pm p(\vec{z}) \), \( \pm \) is a polarity value as known from situa-
tion theory (Devlin, 2006) in order to make negative
property assertions available in the object language.

We have implemented a modalized extension of
the RDFS and OWL rule sets (Hayes, 2004; ter Horst,
2005) by employing the tuple-based rule engine HFC
(Krieger, 2012; Krieger, 2013). Without loss of gen-
erality, let us focus here on the positive case for the
three binary entailment schemas from Section 5.3.2,
5.3.3, and 5.3.4 and their HFC rule representation, as
negation inside the scope of a modal can be rewritten
using the mirror modal, thus turning the quintuple
into a quad (rule variables start with a \( ? \)):

\( \text{(Mrdfs3)} \forall P.C \land \delta P(x,y) \rightarrow \top C(y) \)
\( ?p \text{rdfs.range } ?c \)
\( ?\text{modal } ?x \ ?p \ ?y \\
\rightarrow \\
\text{mod:T } ?y \text{ rdf:type } ?c \\
\)

\( \text{(Mrdfp8)} \ P \equiv Q^\top \land \delta P(x,y) \rightarrow \delta Q(y,x) \)
\( ?p \text{owl:inverseOf } ?q \\
?\text{modal } ?x \ ?p \ ?y \\
\rightarrow \\
?\text{modal } ?y \ ?q \ ?x \\
\)

\( \text{(Mrdfp4)} \ P^\top \sqcap P \land \delta P(x,y) \land \delta P(y,z) \rightarrow \\
(\delta \circ \delta')P(x,z) \)
\( ?p \text{rdf:type owl:TransitiveProperty} \\
?\text{modal1 } ?x \ ?p \ ?y \\
?\text{modal2 } ?y \ ?p \ ?z \\
\rightarrow \\
?\text{modal } ?x \ ?p \ ?z \\
@\text{action} \\
?\text{modal } = \text{CombineModals } ?\text{modal1 } ?\text{modal2} \\
\)

Triple-based engines, such as OWLIM clearly
need to reify such extended descriptions (expensive;
no termination guarantee). Even more important, addi-
tional tests going beyond simple symbol matching
and function calls, such as \text{CombineModals} (the
equivalent to \( \circ \) in the abstract syntax) in the HFC
version of (Mrdfp4) above, are rarely available in to-
day’s RDFS/OWL reasoning engines, thus making it
impossible for them to implement such modal entail-
ments.

We finally describe how the implementation of the
generalization schema \((G)\) (Section 5.1.2) works. As
explained in Section 4.3, the modal operators \( \delta \) are
arranged in a modal hierarchy that is based on the in-
clusion of their confidence intervals \( \mu(\delta) \). This hierar-
chy is realized in OWL through a subclass hierarchy,
using \text{rdfs:subClassOf} to implement \( \leq \):

\( \text{(G)} \delta P(x,y) \land \delta \leq \delta' \rightarrow \delta P(x,y) \)
\( ?\text{modal1 } ?x \ ?p \ ?y \\
?\text{modal1 } \text{rdfs:subClassOf } ?\text{modal2} \\
\rightarrow \\
?\text{modal2 } ?x \ ?p \ ?z \\
\)

6 A FOURTH KIND OF MODALS

The two modalities \( \Box \) and \( \Diamond \) from standard modal
logic are often called dual as they can be defined in terms of each other:
\( \Box\phi \equiv \neg \Diamond \neg \phi \) and \( \Diamond \phi \equiv \neg \Box \neg \phi \),
resp. At first sight, it seems that our non-standard
modal logic is missing a similar property, as we origi-
nally dealt with five modal operators, extended by the
propositional modals \( \top \) and \( \bot \), and the completion
modals \( ? \) and \( ! \). For every such modal \( \delta \), we can fur-
thermore think of additional complement modal \( \delta^C \)
and additional mirror modal \( \delta^M \) whose confidence
intervals \( \mu(\delta^C) \) and \( \mu(\delta^M) \) can be derived from \( \mu(\delta) \)
(cf. Section 4.1). Some of these modal coincide with
original modals from \( \Delta \), others do not have a direct
counterpart. However, the confidence intervals for
the “anonymous” modals can be trivially computed by
applying the two equations from Section 4.1.

Coming back to the question of whether dual
modal exist for every \( \delta \in \Delta \), we need to simplify
\( \neg \delta \phi \) by applying the schemas from Section 4.1. We
can either start with the inner or with the outer nega-
tion, resulting in either mirror modals or complement
modals. Interestingly, the resulting confidence intervals at which we reach in the end are the same, and this is clearly a good point and desirable, as simplification is supposed to be an order-independent process:

\[
\delta^C \phi \quad \delta^M \phi \\
= \delta^C \phi \quad \delta^M \phi
\]

Thus, \(\delta^C \equiv \delta^M\), for every \(\delta \in \Delta\) which can be shown by applying the definitions for complement and mirror modals from Section 4.1. The deeper reason why this is so is related to the inherent properties of the two operations complementation and mirroring. Contrary to complement and mirror modals, dual modals \(\delta^2\) are either supersets or subsets of \(\mu(\delta)\), i.e., if \(\delta\) is a \(1\)- or \(0\)-modal, so is \(\delta^2\).

7 RELATED WORK & REMARKS

It is worth noting to state that this paper is interested in the representation of and reasoning with uncertain assertional knowledge, and neither in dealing with vagueness/fuzziness found in natural language (very small, hot), nor in handling defaults and exceptions in terminological knowledge (penguins can’t fly).

To the best of our knowledge, the modal logic presented in this paper uses for the first time modal operators for expressing the degree of (un)certainty of propositions. These modal operators are interpreted in the model theory through confidence intervals via measure function \(\mu\). From a model point of view, our modal operators are related to counting modalities \(\delta \geq k\) (Fine, 1972; Areces et al., 2010). However, for \(M, w \models \delta \pi\) to be the case, we do not require a fixed number \(k \in \mathbb{N}\) of reachable successor states (absolute frequency), but instead divide the number of worlds reached through label \(\delta \in \Delta\) and in which \(\pi\) holds by the number of all directly reachable worlds, yielding fraction \(0 \leq p \leq 1\). This number then is further constrained by requiring \(p \in \mu(\delta)\) (relative frequency), as defined in case 5 of the satisfaction relation in Section 4.2 and extended in Section 4.3.

As (Wikipedia, 2015) precisely put it: “... what axioms and rules must be added to the propositional calculus to create a usable system of modal logic is a matter of philosophical opinion, often driven by the theorems one wishes to prove ...”. Clearly, the logic presented here is no exception and its design is driven by commonsense knowledge and plausible inferences, we try to capture and generalize. In a strict sense, it is a non-standard modal logic in that it is not an instance of the normal modal logic \(K = (N) + (K)\)

\[
(N) p \rightarrow □p
\]

\[
(K) □(p \rightarrow q) \rightarrow (□p \rightarrow □q)
\]

as the necessitation rule \((N)\) and the distribution axiom \((K)\) does not hold for every \(\delta \in \Delta\). However, we can show that restricted generalized forms of these axioms are in fact the case for our logic \((1 \geq 0.5\) are \(1\)-modals whose low value is \(\geq 0.5\) and \(0 \leq \delta 0.5\) are \(0\)-modals whose high value is \(\leq 0.5\)):

\[
(N1) p \rightarrow \top p
\]

\[
(N0) \neg p \rightarrow \bot p
\]

\[
(K1 \geq 0.5) (p \rightarrow q) \rightarrow (\frac{1}{2} p \rightarrow \frac{1}{2} q)
\]

\[
(K0 \geq 0.5) (p \rightarrow q) \rightarrow (\frac{0}{2} p \rightarrow \frac{0}{2} q)
\]

In addition, the well-behaved frames condition (Section 4.3) generalizes the seriality condition \((D)\) on frames and a kind of forward monotonicity, we would like to keep for an evolving domain, is directly related to transitivity \((4)\) of the accessibility relations from \(\Delta\) in \(F\):

\[
(D) \delta p \land \delta \leq \delta' \rightarrow \delta' p
\]

\[
(4) \delta p \rightarrow \delta' p
\]

Several approaches to representing and reasoning with uncertainty have been investigated in Artificial Intelligence, see (Halpern, 2003) for a (biased) overview. (Halpern, 1990) was probably the first attempt of a first-order logic which unifies probability distributions over classes and individuals. Weaker decidable propositional formalisms such as Bayesian Networks (Pearl, 1988) and related probabilistic graphical models (Koller and Friedman, 2009) have found their way into causal (medical) reasoning (Lucas et al., 2004). Programming languages for these kind of models exist; e.g., Alchemy for Markov Logic Networks (Richardson and Domingos, 2006). In Markov Logic, first-order formulae are associated with a numerical value which softens hard first-order constraints and a violation makes a possible world not impossible, but less probable (the higher the weight, the stronger the rule). For example, the Markov Logic rule smoking causes cancer with weight 1.5 (Richardson and Domingos, 2006, p. 111)

\[
1.5 : \forall x. \text{smokes}(x) \rightarrow \text{hasCancer}(x)
\]

might be approximated in our approach through the use of modals:

\[
\top \text{smokes}(x) \rightarrow \top \text{hasCancer}(x)
\]

Very less so has been researched in the Description Logic community (as it is smaller) and little or nothing of this research has find its way into implemented description logic systems. As we focus in this paper on a modalized extension of OWL, let us
Capturing Graded Knowledge and Uncertainty in a Modalized Fragment of OWL

<table>
<thead>
<tr>
<th>△</th>
<th>meaning</th>
<th>confidence</th>
<th>belief</th>
<th>disbelief</th>
<th>uncertainty</th>
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<tr>
<td>!</td>
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<td>0.5</td>
<td>0.5</td>
<td>0</td>
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<tr>
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<td>[0, 0]</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
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<td>[0, 0.1]</td>
<td>0</td>
<td>0.9</td>
<td>0.1</td>
</tr>
<tr>
<td>U</td>
<td>unlikely</td>
<td>[0, 0.3]</td>
<td>0</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>PN</td>
<td>perhaps not</td>
<td>[0.4, 0.5]</td>
<td>0.4</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>FF</td>
<td>fifty-fifty</td>
<td>[0.45, 0.55]</td>
<td>0.45</td>
<td>0.45</td>
<td>0.1</td>
</tr>
<tr>
<td>P</td>
<td>perhaps</td>
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<td>0.5</td>
<td>0.4</td>
<td>0.1</td>
</tr>
<tr>
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<td>[0.1, 1]</td>
<td>0.3</td>
<td>0</td>
<td>0.7</td>
</tr>
<tr>
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<td>likely</td>
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<td>0.7</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
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<td>[0.9, 1]</td>
<td>0.9</td>
<td>0</td>
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</tr>
<tr>
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<td>0</td>
</tr>
<tr>
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<td>unknown</td>
<td>[0, 1]</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 2: Representation of modal operators from △ (incl. three in-the-middle modals) in terms of opinions in Subjective Logic. The confidence intervals for the five initial modals roughly coincide with the numbers depicted in Figure 1.

review here some of the work carried out in description logics. (Heinsohn, 1993) and (Jaeger, 1994) consider uncertainty in ALC concept hierarchies, plus concept typing of individuals (unary relations) in different ways (probability values vs. intervals; conditional probabilities in TBox vs. TBox+ABox). They do not address uncertain binary (or even n-ary) relations. (Tresp and Molitor, 1998) investigates vague-ness in ALC concept descriptions to address statements, such as the patient’s temperature is high, but also for determining membership degree (FN ⊤ N). This is achieved through membership manipulators which are functions, returning a truth value between 0 and 1, thus deviating from a two-valued logic. (Straccia, 2001) defines a fuzzy extension of ALC, based on Zadeh’s Fuzzy Logic. As in (Tresp and Molitor, 1998), the truth value of an assertion is replaced by a membership value from [0, 1]. ALC assertions α in (Straccia, 2001) are made fuzzy by writing, e.g., (α ≥ n), thus taking a single truth value from [0, 1]. An even more expressive theoretical description logic, Fuzzy OWL, based on OWL DL, is investigat- ed in (Stoilos et al., 2005).

Our work might also be viewed as a modalized version of a restricted fragment of Subjective Logic (Josang, 1997; Josang, 2001), a probabilistic logic that can be seen as an extension of Dempster-Shafer belief theory (Wilson, 2000). Subjective Logic addresses subjective believes by requiring numerical values for believe b, disbelieve d, and uncertainty u, called (subjective) opinions. For each proposition, it is required that $b + d + u = 1$.

The translation from modal letters $\delta$ to $(b, d, u)$ is determined by the length of the confidence interval $\mu(\delta) = [l, h]$ and its starting/ending numbers, viz., $u := h - 1, b := l$, and $d := 1 - h$ (cf. Figure 2).

These definitions also address in-the-middle modals (cf. footnote 3). Such modals even do not need to be symmetrical, i.e., being around the center of the confidence interval. The definitions are clearly not applicable to the error modal $!$ (cf. Section 5.1.7) and it makes perfect sense to assume $u = 0$ here (remember, $\mu(!) = 0$), and thus bisecting the belief mass for this corner case, i.e., $b = 0.5$ and $d = 0.5$.

The simplification and entailment rules of the formalism (Sections 4.1 and 5) allow rule-based (forward) engines to easily implement this conservative extension of OWL. Through these rules, the formalism is compositional by nature and thus afflicted with all the problems, reviewers have already noted on the interplay between logic and uncertainty (Dubois and Prade, 1994). Due to the finite number of modal operators, the approach is only able to approximately compute the degree of uncertainty of new knowledge instead of giving more precise estimations, by combining the low/high numbers of the confidence intervals through min/max, multiplication, addition, etc. Contrary to other approaches, we do not talk about the uncertainty of complex propositions (conjunction, disjunction) or sets of beliefs, but instead focus merely on the uncertainty of atomic ABox propositions.

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