Using Genetic Algorithm with Combinational Crossover to Solve Travelling Salesman Problem

Ammar Al-Dallal

Computer Engineering Department, Ahlia University, Manama, Kingdom of Bahrain

Keywords: Genetic Algorithm, Travelling Salesman Problem (TSP), Crossover.

Abstract: This paper proposes a new solution for Traveling Salesman Problem (TSP) using genetic algorithm. A combinatorial crossover technique is employed in the search for optimal or near-optimal TSP solutions. It is based upon chromosomes that utilise the concept of heritable building blocks. Moreover, generation of a single offspring, rather than two, per pair of parents, allows the system to generate high performance chromosomes. This solution is compared with the well performing Ordered Crossover (OX). Experimental results demonstrate that, due to the well structured crossover technique, has enhanced performance.

1 INTRODUCTION

Genetic algorithm is a sort of adaptive worldwide searching probabilistic optimization algorithm, which often simulates the procedure associated with organisms’ inheritance and evolution. It's been widely applied to the areas of neural network, combinatorial optimization, economical forecasts and pattern recognition. The principle concept of the genetic algorithm is derived from inheritance and evolution. It is mainly based on mechanism of biological evolution and genetics principle, in accordance with natural selection and survival of the fittest principle, using simple coding technology in order to resolve complex problems.

Travelling salesman problem (TSP) is one of the typical NP-completeness problems in combinatorial optimization (Yuan et al, 2009). It is described as follows. A salesman has to visit a number of cities only once and then returns home. The distances between any pair of cities are given at first and it is required to find the shortest path. In which order should the cities be visited to minimize the distance travelled?

There are two kinds of TSP, symmetric TSP and asymmetric TSP. In symmetric TSP, the distance travelled from city \( i \) to city \( j \) is equal to the distance travelled from city \( j \) to city \( i \). In asymmetric TSP, the distance travelled from city \( i \) to city \( j \) is not equal to the distance travelled from city \( j \) to city \( i \). In this paper, symmetric TSP is considered.

1.1 The Mathematical Representation of TSP

In math, the problem may be stated as follows (Yuan et al, 2009). Given \( n \) cities, if \( V \) denotes the set of cities as \( V = \{v_1, v_2, v_3, \ldots, v_n\} \), and \( T \) denotes a permutation as \( T = (t_1, t_2, t_3, \ldots, t_n) \), where \( t_i \in V \), \( i=1,2,3,\ldots,n \), and denoted \( t_{n+1} = t_1 \), then TSP is given as:

\[
\min L = \sum_{i=1}^{n} d_{t_i, t_{i+1}}
\]  

Although this formulation is simple, TSP is hard to solve. For a symmetric TSP with \( n \) cities the number of possible solution is \((n-1)!/2\), which is a very large number even for a relatively small \( n \) (number of cities).

1.2 Genetic Algorithm Representation of TSP

The search space \( S \) is a set consisting of all cities provided in the dataset. Each path \( P \) consists of all cities of the search space \( S \) selected in a random order. Each chromosome in GA represents a possible path, and each path is evaluated using a fitness function \( f \) that accumulates the summation of distances between these cities (more about the fitness function is explained in Section III). A set of possible paths (chromosomes) associated with their
fitness value forms one generation. Finally, a GA engine tries to output a set of paths that minimize the value of f. The optimal solution is a path or set of paths that have the minimum score (distance) returned by the function f.

It is found that such an optimization problem can be solved efficiently using Genetic Algorithm (GA) (Yuan et al, 2009; Vahdati et al, 2009, Wang and Zha, 2009; Zhao et al, 2008; Yu et al, 2011). In addition, GA demonstrates near global optimality, implicit parallelism, adaptability, and high robustness (Yu et al, 2011).

In its simplest form, a GA is a probabilistic algorithm used to simulate the mechanism of natural selection of living organisms, and it is often used to solve problems having expensive solutions in terms of time and computation complexity. This is due to the principles of selection and evolution employed to produce several solutions for a given problem. Generally speaking, GA’s search space is composed of candidate solutions to the problem (represent possible paths in TSP). Each chromosome has an objective function value known as fitness value that is the path length joining the cities. This measure is used to favour selection of successful parents for producing new offspring. Offspring solutions are produced from parent solutions by the application of selection, crossover and mutation operators (Lianshuan, Zengyan, 2009). The Offspring forms the next generation of possible solution. Following generations are then created by applying these three operators until one of the two following criteria is satisfied. Either the maximum predefined number of generations is created, or no further enhancement to chromosomes (in terms of overall fitness value) is observed, as compared to the previous generation. In all cases, the system will return the best solution from the last generation.

The rest of this paper is organized as follows: Section two lists related work of TSP that is implemented using GA. Section three describes the proposed GA model to improve the process of solving the symmetric TSP. Section four describes the experiments and results. Finally, Section five concludes this paper.

2 RELATED WORK

Many approaches and results on evolutionary optimization for TSP have been published. Samples of these approaches include the following:

Authors of (Yuan et al, 2009) proposed GA based on good character breed. The searching space is divided into segments based on the fine seed. At first, GA runs several times and gets a perfect individual every time to gather a fine seed set. To strengthen the local search, every segment is treated as a small size TSP to be optimum. This approach demonstrates a slight improvement when applied to a small dataset of size 150.

Vahdati et al proposed in (Vahdati et al, 2009) a new solution for TSP using genetic algorithm. This technique is based on applying heuristic crossover and mutation operation to GA in order to prevent premature convergence.

The heuristic crossover considers each chromosome as a closed circle. Two pointers are used for each parent. The decision whether to select the clockwise or anticlockwise pointer is based on the fitness. This technique is able to accelerate the speed of convergence by reducing the number of generations.

An improved combinational genetic algorithm for travelling salesman problem is applied in (Wang and Zhao, 2009). This approach adds a local search procedure in the standard genetic algorithm. Local search is performed on the best individual of each generation. This method increases the stability of the algorithm in addition to improving the precision.

The authors of (Yu et al, 2011) solved the TSP using GA in which the algorithm employed a roulette wheel based selection mechanism, the use of a survival-of-the-fittest strategy, a heuristic crossover operator, and an inversion operator. This approach was applied in TSP with 50 cities, and it was able to quickly obtain an optimal solution to TSP from a huge search space. However, this study is not provided with clear figures to demonstrate the level of improvement over other techniques as stated by the authors.

A multi-objective TSP that is implemented using GA is presented in (Lianshuan and Zengyan, 2009). It creates an initial population which satisfies the basic qualification, then it calculates the two objective-values: distance and cost. The aim is to find the minimal of both the distance and the cost. The objective value is then used to rank the chromosomes with Pareto function. However, this approach is not provided with clear figures to demonstrate the level of accuracy and how quickly the result is obtained.

Takahashi (Takahashi, 2011) suggested solving TSP through a Combinational method called: iterative Extended Changing Crossover Operators. This method combines Edge Assembly Crossover (EAX) and Ant Colony Optimization (ACO). At the first stage, ACO tries to search for provisional
solutions by using initial circuits generated by uniformly random numbers with a certain random number called seed. Next EAX follow the provisional solutions generated by ACO and continuously searches for the better solutions and generates individuals having higher fitness. The provisional solutions compose the chromosomes. The next solutions are generated by EAX and merged with new solutions generated by ACO. With this merged file, EAX is executed again in order to improve the solutions to create the next optimum solutions. This approach is applied on medium-sized TSP dataset and the results shows that it is effective to produce well performing offspring.

Another approach developed by (Kuroda et al, 2010) is called Z-crossover. The Z-Cross works as follows: first is to set a zone in the travelling area according to some rules. Secondly, the edges connecting cities between inside and outside the zone are cut. Thirdly, the edges inside the zone of one parent and those of the other parent are exchanged. Finally, the sub-tours and isolated cities produced by the third step mentioned above are reconnected to construct a new tour of TSP.

Razali and Geraghty in (Razali and Geraghty, 2011) examined the performance of TSP through analysing three selection techniques. These techniques are: Tournament Selection, Proportional Roulette Wheel Selection and Rank-based Roulette Wheel Selection. The experiments are performed on datasets with sizes between 10 and 51 cities. The results show that the tournament selection outperforms the others where it achieved the best solution quality with low computing times.

The authors of (Sallabi and El-Haddad, 2009) proposed an improved genetic algorithm to solve the TSP. They suggested two crossover techniques. The first one is the swapped inverted crossover. It can be applied to a one or two-point crossover. After partitioning the chromosomes, the genes within the each partition are flipped in their order and then swapped between parents. The second suggested crossover is the rearrangement crossover. It works by finding the greatest or maximum value of a city distance among all the adjacent cities on the tour, and then swap city i with three other cities, one at a time. This method may not achieve improvement after several iterations but may take a big jump and improve the result.

A comparative study is done by Otman and Abouchabaka (Otman and Abouchabaka, 2012) to investigate the performance of six crossover techniques that are applied to solve TSP. These crossover techniques include: Uniform Crossover Operator, the Cycle Crossover, the Partially Mapped Crossover, the Uniform Partially-Mapped Crossover, the Non-Wrapping Ordered Crossover and the Ordered Crossover (OX). The experiments were done on one instance of TSPLIB that is BERLIN52. The results confirm that the best known solution for the TSP instance was obtained by using the OX.

(Osaba et al, 2013) applied multi-crossover and adaptive island based population algorithm to solve routing problems such as TSP. In this technique, the entire population is divided into subpopulations, each with a different crossover function, which can be switched according to the efficiency. Each subpopulation begins with a very low value of crossover probability and then varies with the change of the current generation number and the search performance of recent generations.

As mentioned above, several approaches are applied to solve TSP. The main contribution of this work is to develop two new crossover techniques: Two-Point Crossover with Replacement and Combinational Crossover as well as adopting specific GA operators in order to investigate their performance compared to well-performing techniques.

3 THE PROPOSED GA FOR SOLVING TSP

This section describes the proposed GA model by explaining its components and operators.

3.1 Chromosome Representation

As a matter of fact, chromosomes in GA are used to represent the possible solution for the problem under consideration. The chromosome consists of genes that when integrated together form the complete solution. For TSP, the best representation is to use integers to refer to cities numbers. Each chromosome consists of a sequence of integers to represent a possible path that the salesperson may visit. The order of these cities within the chromosome represents the order of cities to be visited. A problem with 7 cities is represented as a chromosome of 7 genes as shown in Figure 1. This chromosome forms a possible solution that is a path starts from city number 2 passes through cities 5 - 1 – 4 - 6 - 7 -3 and ends at 2 again, since the path must be closed path.
### 3.2 Initial Generation Creation

When looking at the methods of creating initial generation, there is a trade-off between creating an initial generation in a fast way with low quality or slower way but with higher quality. Fast creation is done by selecting individuals randomly without any selection criteria. However, this method may stick at a local optimum solution causing the results to be less effective due to fast convergence (Aly, 1990). Since there is no specific criterion that favours any city, a pure random selection is applied to select the genes (cities) of the initial generation.

#### 3.3 Parent Selection

Once the first generation is created, the operators of GA are applied. The first operator of GA is parent selection. Based on the results obtained by (Razali et al., 2009), the tournament selection is adopted in this work.

In tournament selection (Yeh, 2007), a group of $i$ individuals are randomly chosen from the population. This group takes part in a tournament and an individual with highest fitness value wins. In many cases $i$ is chosen to be two, and this method is called binary tournament selection. To further enhance this selection, $i$ is selected to be five. And the best of these five is selected.

In order to speed up convergence and avoid creating large number of generation, elitism technique can be applied as a selection technique (Asllani and Lari, 2007). In this technique best $l$ members from the current generation are selected to form the mating pool for next generation. It is applied to ensure gradual improvement of the solution. In this paper, $l$ is selected to have best two thirds of the current generation, i.e. the best 67% of the current population.

A validation process, before applying crossover, is applied to insure that the two selected parents are different from each other, i.e. $P1 \neq P2$.

#### 3.4 Fitness Function

The fitness function $f$ is used to evaluate each individual (chromosome) to determine the best 67% chromosomes of the population.

It is evaluated by accumulating the sum of distances between the adjacent cities forming a path. Assume that a chromosome has a length of $n$ cities, then the fitness function is calculated as follows (Vahdati et al., 2009):

$$ f = \sum_{i=1}^{n} \text{dist}(c_{i,i+1}) $$

And $\text{dist}(c_{i,i+1})$ is the distance between city $c_i$ and $c_{i+1}$.

If both cities have $x$ and $y$ coordinates, then the distance is calculated as:

$$ \text{dist}(c_{i,i+1}) = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2} $$

#### 3.5 Crossover Operator

One of the main operators that heavily affects the performance of GA system is crossover. In literature, many techniques of crossover have been developed and analysed to study their effect on the output from several perspectives. In all cases, crossover is performed such that the order of the genes, hence the developed path, is changed in a systematic manner to produce a new visible solution.

While many approaches are developed by researchers to enhance the performance of GA, this work is featured by introducing two new crossover techniques that are expected to improve the performance of the proposed system. The first proposed crossover technique is called Two Point Crossover with Replacement (TPXwR). The second proposed crossover technique is called Combinational Crossover Technique (CXT). These two crossover techniques will be investigated in terms of improvement of finding the shortest path.

#### 3.5.1 The Design of Two Point Crossover with Replacement (TPXwR)

The technique proposed in TPXwR is derived from the two-point crossover, and it works like this:

Two chromosomes (parents) are selected from best 67% (two-thirds) of the current generation using binary tournament selection. These parents are named $p1$ and $p2$. Then, two crossover points $cp1$ and $cp2$ are selected randomly. These two points will divide each parent into three portions. Let $a$, $b$ and $c$ be the portions of $p1$ and $d$, $e$ and $f$ be the portions of $p2$ as shown in Figure 2.

Two offsprings $O_1$ and $O_2$ are formed from these parents. Offspring one is created according to the following steps. The first and last portions $a$ and $c$ are copied to the corresponding positions of $O_1$. Then portion $e$ of $p2$ is copied to $O_1$. If a gene of
portion e is already existed in \( O_1 \), then it will be replaced by a gene from portion d of p2. If genes of d are again exist in \( O_1 \), then it will be replaced by a gene from portion f.

To create the second offspring \( O_2 \), similar steps are applied. The first and last portions \( d \) and \( f \) are copied to the corresponding positions of \( O_2 \). Then portion b of p1 is copied to \( O_2 \). If a gene of portion b is already existed in \( O_2 \), then it will be replaced by a gene from portion a. If genes of a are again exist in \( O_2 \), then it will be replaced by a gene from portion c.

During this process, the uniqueness of genes (cities) within the newly created chromosome is preserved. Figure 2 provides an example of this technique where the steps to create the offspring are as follow:

1. Genes of portions a and c are copied directly to the corresponding position of the first offspring \( O_1 \) since these are the first genes added and no need to check their uniqueness.
2. Genes of portion e are copied to the corresponding position of the first offspring after checking the uniqueness of genes. 9 and 7 are added from e.
3. If the added gene to the offspring is already existed in \( O_1 \) (8 from e in this example) then it is replaced by one from portion d, having that it does not already exist in this offspring (3 from portion d). Otherwise, it is replaced with one from portion f.
4. Similar steps are applied when creating the second offspring with reversing rules.

This technique is considered to be an example of simple two-point crossover technique but with replacement when the newly added gene is already existed in the constructed chromosome. Apparently, it could produce better offspring compared to their parents. However, a good generated chromosome (path) could be disrupted or broken if the crosspoints happen to be somewhere within a good block causing it to break an optimal path.

### 3.5.2 The Design of Combinational Crossover Technique (CXT)

The second proposed crossover technique is the Combinational Crossover Technique (CXT). This technique is applied to a semi-ordered chromosome. In the semi-ordered chromosome, the first two genes (cities) are ordered such that these genes have the shortest path among the paths between all genes of this chromosome. The closest cities (shortest paths) appear to the left of the chromosome. The process of reordering of genes is done after creating the chromosome.

The proposed CXT also applies the fusion crossover (Vrajitoru, 1998), where only one offspring is generated from two selected parents. In this technique, the offspring inherits the genes from one of the parents with a probability according to its performance. The advantage of this technique is that the good genes of both parents are inherited simultaneously to the offspring, producing high-quality offspring and increasing the speed of convergence.

![Figure 2: Example to illustrate the TPXwR crossover technique.](image)

![Figure 3: Example to illustrate the CXT crossover technique.](image)
1. Select two parents $x$ and $y$ randomly from the best 67% chromosomes of the current generation.
2. Select one crosspoint randomly. This crosspoint will divide $x$ into two portions $a$ and $b$. Also, $y$ will be divided into two portions $c$ and $d$.
3. Calculate the fitness of the first gene of each parent. That is the first gene of portions $a$ and $c$.
4. The one that has the lower fitness values is selected as parent 1. In this example, $y$ is selected as parent 1 since the fitness of the first gene is 1 which is less than 2 for parent $x$.
5. Copy the genes from the left portion (portion $c$ in this example) to the left portion of the offspring $O$.
6. Continue filling the offspring with the genes from the left portion of the second parent to the offspring (portion $a$). Need to maintain the length of the offspring equal to the parent’s length. That is the genes from the second parent will be copied until the length of the offspring is equal to the parent’s length.
7. If the length of the offspring is still smaller than the parent’s after the previous step, then more genes need to be copied from either parent.
8. The fitness values of genes of portions $b$ and $d$ are compared starting from location $cp+1$. The portion with the gene that has a lower fitness value will contribute to the offspring $O$. This is done in order to generate offspring with appropriate genes from each parent and to guarantee that the length of $O$ is equal to the length of the parent.
9. Through the process of copying the remaining genes from the parents, the uniqueness of the copied genes must be considered, i.e., each gene can occur only once in the new offspring.
10. Last step is to order the genes based on their fitness value. The order is done in ascending order as preparation for the next crossover process.

The rationale behind selecting the portion with the lower fitness to start with is the need to inherit the good order of cities and maintain the good building blocks while passing them to the resulting offspring.

4 EXPERIMENTS AND RESULTS

This section describes the data set used to test the performance of the proposed technique. Then it analyses the obtained results.

4.1 Dataset Description

In order to investigate the performance of the proposed techniques, several experiments need to be conducted on a benchmark dataset. 10 instances are chosen from TSPLIB (Reinelt, 1991) to perform the needed experiments. These 10 sets differ in size and ranging from 22 cities to 318 cities.

The proposed GA system starts by creating initial generation consisting of 50 chromosomes. Recall that each chromosome forms a possible solution (travelling path). Hence, the length of the chromosome is equal to the number of cities of the applied dataset. The stopping condition of the proposed system is one of the following: either the maximum predefined number of generations is created, or no further enhancement to chromosomes (in terms of overall fitness value) is observed, as compared to the previous generation. The crossover probability is chosen to be 1. These parameters are listed in Table 1.

4.2 Analysis of the Results

In order to evaluate the proposed crossover techniques, they are compared with the results obtained by applying the algorithm explained in (Oman and Abouchabaka, 2012). As mentioned in Section II, OX has obtained best results among six options of crossover techniques.

The considered results of the experiments are the chromosomes of the last generation. These results are illustrated in Table 2. The first column is the name of the dataset. The number associated to the dataset name reflects the size of the set. The next three columns are the results obtained from TPXwR, CXT and OX techniques. The last two columns provide the improvement in the fitness value of CXT compared to OX and TPXwR.

When comparing the two proposed crossovers, the CXT which produces one offspring in each crossover process, shows better performance in general in terms of the shortest path (fitness value). For all sets, CXT achieves shorter path than TPXwR except for berlin52 dataset. The improvement of CXT over TPXwR ranges between 1.3% and 17.19%. The reason behind this improvement is that in CXT the offspring inherits the well-performing building blocks from parents that include adjacent cities with minimal distance. Repeating this process in every generation provides a great chance to achieve better results, having that both techniques are producing the same number of generations.
By analysing the results obtained by the two proposed techniques and the results obtained by the well-known OX technique, it can be noticed that CXT outperforms OX by percentage starting from 2.86% for only one dataset, i.e., lin318 dataset and increases to reach maximum of 22.38% for the eil76.

When looking at the results of the last generations, it ascertains that the proposed crossover is successful in enhancing the chromosome performance since the aim of this process is to produce better offspring from the parents.

The main reason behind the improvement in performance is the way of implementing CXT. The main contribution to the high improvement is coming from the method of adding first genes to the offspring. The first added genes are added based on the fitness value (path length). The cities with lower path length form a “good” building block. One of the objectives when developing a crossover technique is not to destruct these well performing building blocks. And this is what is achieved in this technique. This “good” building block is inherited unchanged to the offspring. Hence, each crossover process adds two “good” building blocks to the created offspring causing an enhancement in the fitness value.

Table 1: Parameter setting of GABIR.

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>50</td>
</tr>
<tr>
<td>Maximum number of generations</td>
<td>200</td>
</tr>
<tr>
<td>Chromosome length</td>
<td>No. of cities of the running set</td>
</tr>
<tr>
<td>Crossover rate</td>
<td>1</td>
</tr>
<tr>
<td>The number of best individuals forming the crossover mate (Elitism)</td>
<td>2/3 of the population</td>
</tr>
<tr>
<td>Mutation rate</td>
<td>0.1</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

One of the permanent challenges for TSP problem is to find the optimal solution (shortest path) using a simple solution.

This paper applied a GA-based system to solve it by developing two crossover techniques. The first one is the 2-point crossover with replacement TPXwR that produces two offspring in each crossover operation. However, this technique is defective, where using multiple crosspoints causes many breaks in good behavioural building blocks. This in turn delays the convergence process. In order to overcome this drawback, another crossover technique is proposed. This crossover is the Combinational Crossover Technique CXT. It works by applying one-point crossover on an ordered chromosome to produce only one offspring. The experiment was applied on 10 datasets ranging in size from 22 and 318 cities and compared to well-performing OX crossover technique.

The results obtained show that the CXT outperforms both TPXwR and OX. The improvement of CXT ranges between 1.3% to around 17% compared to TPXwR and ranges between 2% and 22.4% compared to OX.

To generalize the results and further demonstrate its efficiency, the proposed techniques need to be compared with additional crossover techniques such as the Uniform crossover, Edge Assembly Crossover and Ant Colony Optimization. In addition, they need to be applied on bigger TSP datasets.

Table 2: The fitness of each dataset after last generation.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>CXT</th>
<th>TPXwR</th>
<th>OX</th>
<th>% Impr. of CXT over TPXwR</th>
</tr>
</thead>
<tbody>
<tr>
<td>ulysses22</td>
<td>1310</td>
<td>1480</td>
<td>1582</td>
<td>11.49%</td>
</tr>
<tr>
<td>att48</td>
<td>1211</td>
<td>1461</td>
<td>15356</td>
<td>17.14%</td>
</tr>
<tr>
<td>eil51</td>
<td>1370</td>
<td>1407</td>
<td>1530</td>
<td>2.63%</td>
</tr>
<tr>
<td>berlin52</td>
<td>2625</td>
<td>2506</td>
<td>2951</td>
<td>-4.75%</td>
</tr>
<tr>
<td>st70</td>
<td>3129</td>
<td>3182</td>
<td>3730</td>
<td>1.67%</td>
</tr>
<tr>
<td>eil76</td>
<td>1971</td>
<td>2285</td>
<td>2539</td>
<td>13.74%</td>
</tr>
<tr>
<td>rat99</td>
<td>7617</td>
<td>8054</td>
<td>8333</td>
<td>5.43%</td>
</tr>
<tr>
<td>kroA100</td>
<td>1483</td>
<td>1522</td>
<td>1686</td>
<td>2.53%</td>
</tr>
<tr>
<td>gr120</td>
<td>9389</td>
<td>10182</td>
<td>10886</td>
<td>7.79%</td>
</tr>
<tr>
<td>lin318</td>
<td>5686</td>
<td>5762</td>
<td>5854</td>
<td>1.31%</td>
</tr>
</tbody>
</table>

REFERENCES
