# Design of Robust Control Strategy for Non-linear Multivariable Systems with Delay, Parametric Uncertainty and External Disturbances

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# **1 RESEARCH PROBLEM**

Historically, the process industry has recognized the important work of automatic control in the proper functioning of the production process. Although the preferred control strategy in most applications is the implementation of simple PID control loops (Proportional-Integral-Derivative), there are а number of characteristics which sometimes are not considered explicitly in the design of these PID controllers, such as delays, unmeasurable variables, parameter uncertainty, time variant systems. nonlinearities. constraints and multivariable interactions. Many developments of modern control theory are designed to face up these features, but the industry has been conservative in applying these tools. This has led many critics to say that there is a gap between theory and practice of control.

In industry, many processes are behind in their dynamic behavior. Although these delays are due primarily to dynamic characteristics of some systems, they may also be made by processing time or the accumulation of time delays in a number of simple dynamical systems connected in series. Typical applications in the presence of delays are communication systems, chemical processes, transportation systems, power systems, teleoperation systems and bio-systems.

From classical control perspective, the presence of delays in a system helps to reduce the phase margin and hence profit margins, achieving even destabilize the closed loop response. However, the introduction of a delay may be beneficial to achieve stability in an unstable system (Stépán, 1989), which explains the five decades of interest in the stability and control of these systems (Stépán, 1989), (Bellm, 1963), (Datko, 1978), (Hale, 1993), (Diekmann et al., 1995). (Niculescu, 2007), (Niculescu, 2001).

Furthermore, due to the difficulty of accurately model a complex process, there are always modeling errors. The development of methods to address the problem of model uncertainty is a big challenge and today there have been different approaches to tackle it. Sometimes, in an attempt to take into account all relevant dynamics and reduce modeling error, it comes to the development of increasingly complex models. However, this maneuver can lead to models that are too difficult for mathematical analysis and design of controllers.

Another common problem in the control systems is due to external disturbances. Such disturbances bring harm to the system performance, so rejection is one of the key objectives in the design of the controller. In control community of Industrial processes - like oil and metal industries - the production processes are usually influenced by external disturbances such as variations in raw material quality, production load fluctuations, and variations of complicated production environments. In the regulation of blood glucose in diabetic patients, for example, external disturbances are related to food intake, physical activity conducting and stress, among others.

In this work, a new control strategy that combines the virtues of control techniques MPC (Model Predictive Control), QFT (Quantitative Feedback Theory) and Disturbances Observers (DOB) is proposed, in order to address delays, parametric uncertainty in the model and external disturbances of nonlinear multivariable systems. It is intended that the proposed scheme be as simple and practical as possible and that is validated in at least two cases of multivariable systems, which can be active power control in a wind turbine, the automatic regulation of glucose levels in patients with type 1 diabetes mellitus (T1DM) and / or control of various variables of quality in a crude distillation process.

### **2** OUTLINE OF OBJECTIVES

The overall objective of the PhD work is to improve the dynamic performance of multivariable nonlinear systems in the presence of delays, external

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disturbances and parametric uncertainty, by designing a new control strategy that combines techniques such as MPC and QFT, and observer disturbances.

To meet the overall goal above the implementation of the following specific objectives are planned:

- Solve the problem of designing robust controllers to parametric uncertainty and external disturbances from the use of QFT techniques and disturbance observer.
- Improve dynamic performance of multivariable control systems delays, through an approach to predictive control (MPC).
- Integrate the capabilities of QFT, MPC and observer design techniques to obtain a robust control strategy to parametric uncertainty, delays and external shocks.
- To validate the proposed control strategy in at least two cases of nonlinear multivariable systems where problems arise in the delay time, external disturbances and parametric uncertainty. Some of these cases may be automatic regulation of glucose in patients with type I diabetes mellitus, power control in a wind turbine and / or control various variables in a crude distillation process.

# **3** STATE OF THE ART

With the purpose of studying the possibilities that can address the problems of control, a review of the results in the control of systems with delays, parameter uncertainty and external disturbances is carried out. Then, a review of robust predictive control as a promising solution is done and the basis of the alternative solution proposed in this research is explained afterwards.

### 3.1 Dead-time

Dead time is the property of a physical system by which the response to an applied force (action) is delayed in its effect (Stépán, 1989). Whenever the material, information or energy are physically transmitted from one place to another, there is a delay associated with the transmission. The control of these delays has been of great interest because they are the main cause of instability and poor performance in control systems, such as chemical processes, long transmission lines in pneumatic systems, among others (Camacho, 2007).

There are various sources of delay; one of these sources is the nature of the system, in other words the way it works. For example, in chemical reactors there is a finite time reaction and, in an internal combustion engine a time period is required to mix air and fuel. Another source of dead-time is the delay of transport due to the fact of carrying material through heat or mass transfer systems, as in a heating system where the transport delay occurs because of hot air. A delay could also be present in the communication between the parts of the system, for example, it takes time for signals to travel between controllers, sensors and actuators in any closed-loop system characteristic, particularly in control systems network and high availability systems (Figure 1).



Figure 1: Delay in a feedback system.

A system with multiple delays in the state vector can be represented as:

$$\frac{dx(t)}{dt} = A_0 x(t) + \sum_{i=1}^{N} A_i x(t - \tau_i)$$
(1)

Where x(t) is the n-dimensional state variable,  $A_i$ , with i=0,1,...,N, is an nxn matrix size and N is a positive integer.  $\tau_i$  is the delay, which causes  $\dot{x}(t)$  not only depends on x(t) at time t but also of the time instants  $t - \tau_i$ .

Moreover, the characteristic equation of Equation (1) is given by:

$$f(s;\tau_1,\tau_2,...\tau_N) = det \left[ sI - A_0 - \sum_{i=1}^N A_i e^{-s\tau_i} \right] = 0$$
(2)

Due to the presence of exponential terms, Equation (2) is a quasi-polynomial and a transcendental equation, which has an infinite number of roots in the complex plane  $\mathbb{C}$ . Therefore, Equation (1) is asymptotically stable if and only if all the roots of the above equation are in the right half of  $j\omega$  axis. Verifying the asymptotic stability of Equation (2) can be difficult since it has an infinite number of characteristic equations.

#### 3.1.1 Synthesis of Controllers

It is becoming increasingly clear that delays are a major cause of instability and poor performance of

dynamic systems, added to which are frequently found in various engineering and physics systems. The stability analysis and control design of systems with time delay have attracted the attention of many researchers (Kolmanovskii et al., 1999), (Silva and Datta, 2005), (Senthilkumar, 2010). The difficulty of controlling these processes is due to the fact that downtime causes a phase delay which decreases the phase margin deteriorating both performance and system stability.

#### 3.1.2 PID Controllers

Due to the low cost and easy implementation, most of controllers used in industry are based on classical control schemes (Hägglund, 2009), (Takatsu and Itoh, 1998). In this approach the idea is to compute the parameters of the controllers considering the inherent delay to the process. The design problem is to reduce the design conditions too conservative (Silva and Datta, 2005). In (Oliveira et al., 2009), (Hohenbichler., 2009), (Termeh., 2011) can be reviewed contributions in the design and implementation of PID controllers in systems with time delays; specifically in (Yuan-Jay et al., 2011) we can study the case for variable delays, and in (Sala and Cuenca, 2009) Application of PID controllers are presented with dynamic adaptation of its parameters based on the measured delay.

Although many processes can be controlled by PID controllers, they have many limitations. Consider, for example, unity feedback system of Figure 2, with transfer function of the plant as:

Figure 2: Feedback control scheme.

And PID control as:

$$K(s) = K_P \left( 1 + T_d s + \frac{1}{T_i s} \right) \tag{4}$$

The transfer function in closed loop would be given by:

$$T(s) = \frac{K(s)G(s)}{1 + K(s)G(s)} = \frac{KK_P(T_dT_is^2 + T_is + 1)e^{-\tau s}}{(Ts + 1)T_is + KK_P(T_dT_is^2 + T_is + 1)e^{-\tau s}}$$
(5)

and the characteristic equation of the closed loop system is:

$$(Ts+1)T_is + KK_P(T_dT_is^2 + T_is + 1)e^{-\tau s} = 0 \qquad (6)$$

Because it is a transcendental equation becomes difficult to analyze the stability of the system or design a controller to ensure stability. To reduce the analysis, we assume that it is a PI controller with  $T_i = T$ . Then, the transfer function of the closed loop becomes:

$$T(s) = \frac{KK_P e^{-\tau s}}{Ts + KK_P e^{-\tau s}}$$
(7)

This system is stable only when

$$0 < K_P < \frac{\pi T}{2\tau K} \tag{8}$$

Therefore, the controller gain is limited by the length of the delay: the greater the delay, the lower the maximum allowable gain and thus a slower response is obtained.

#### 3.1.3 Dead Time Compensation (DTC)

Control schemes for dead time compensation can be classified into two types: the Smith Predictor and Finite Spectrum Assignment (FSA, for its acronym in English). In 1959 Smith Predictor (Smith., 1959) is proposed in order to design controllers that allow isolate the feedback loop delay, thereby enabling to obtain significant simplifications in the system analysis and design of the controller.

Figure 3 shows the diagram of a control system based on the Smith Predictor. C(s) represents the controller  $G(s) = P(s)e^{-\tau s}$  is the plant or process being controlled and  $Z(s) = P(s)(1 - e^{-\tau s})$  Smith predictor.

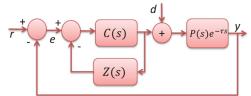


Figure 3: Control system based on the Smith predictor.

Assuming no perturbations in the system, the transfer function of the closed loop is given by:

$$T(s) = \frac{C(s)P(s)}{1 + C(s)P(s)}e^{-\tau s}$$
(9)

Although you can see that this configuration allows controller design regardless of the delay has some significant limitations, such as not being able to be applied to unstable processes, it has great sensitivity to modeling errors and external disturbances and only applies to systems with constant delay and entry (i.e., not applicable to systems with retarded state) (Zhong, 2006).

In order to overcome some limitations of Smith Predictor in 1974 the technique of finite spectrum allocation was developed. This new approach is not only useful for the design of controllers for unstable systems with delay but also such delays can be of input or in the states (Manitius and Olbrot, 1979).

This technique is based on the transformation of the state vector of the process in order to eliminate delays of the characteristic equation of the system so that the closed-loop poles can be allocated from the required specifications of design (Furutani, 1998). This method requires no prior knowledge of the spectrum of the plant, only requires to be assigned n spectral points while the others left are automatically deleted (Artstein, 1982).

Consider a system described in state space as:

$$\dot{x}(t) = Ax(t) + Bu(t - \tau);$$

$$y(t) = Cx(t)$$
(10)

Then, the transfer function of the plant is:

$$G(s) = P(s)e^{-\tau s} = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} e^{-\tau s}$$
(11)

Finite Spectrum Assignment (FSA) adopts the following feedback control law:

$$u(t) = F x_P(t) \tag{12}$$

The state predicted  $x_P(t)$  is given by:

$$x_P(t) = e^{A\tau} x(t) + \int_0^\tau e^{A\lambda} Bu(t-\lambda) d\lambda \qquad (13)$$

Similar to the control scheme based on predictor, the delay term is removed from the design process. The resulting closed-loop system is stable if si A + BF is stable. However, the resulting controller cannot be expressed in the form of Equation (13), to require additional terms, increasing the controller implementation effort.

Although DTC structures are more complex and require greater knowledge for tuning than traditional PIDs, these have a better compensation for delays, especially when downtime of process is dominant (Camacho, 2007). However, because the state prediction is made from the model, these techniques have a high sensitivity to modeling errors, especially when the delay is very large. If a high order model with delay to describe the dynamics of a process is needed, both a primary controller of higher order in the DTC as a traditional (different PID) controller are needed. In these cases it is clear that the limitations on the performance of PID are due to model order and not to delay (Camacho, 2007).

#### 3.1.4 Sliding Mode Control

The technique of sliding mode control (SMC) is a good alternative for robust stability to uncertainty in model parameters, non-linearities and external shocks. This approach provides rapid response and asymptotic stability and has two main advantages: a) when the state is limited by the sliding surface can completely reject SMC uncertainties; b) high possibility of stabilizing some nonlinear complex systems which are difficult to stabilize by feedback law states. Due to these advantages the theory of sliding mode control has been used in countless applications (Yu and Kaynak, 2009).

This led to the design studio SMC controllers for systems with delays at the entrance and / or states (Richard and Gouaisbaut, 2001). In (Wu et al., 2002) a control structure on slippery for convergence in finite systems with input delay time modes is proposed. In (Roh and Oh, 1999) a sliding surface based on a predictor that minimizes the effects of delays system input, and derives a robust control law that guarantees the existence of a sliding mode and overcome the delay and uncertainty of the system is exposed.

As is the case with other conventional control laws, if when making the design does not take into account the delay, the system may become unstable or aggravate the effect of chattering (Sorribes, Octubre 2011). In (Gouaisbaut et al., 2002) proposes a methodology to design controllers in sliding mode based on LMI (Linear Matrix Inequalities) for systems containing both a delay and multiple delays and constant or variable delays. The conditions for the existence of the sliding regime are studied by using the Lyapunov-Krasovskii functions and Lyapunov-Razumikhin and LMIs scheme is used in the optimization procedure.

#### 3.1.5 Model Predictive Control (MPC)

MPC is a powerful technique of control that has found great acceptance in industrial applications such as in academia. This success is due perhaps to the fact that systems be useful both in single variable and multivariable systems, considering the restrictions of the control system and inherently compensate for delays in the process (Ramírez, 2002).

Because of the predictive nature of the MPC controllers, time delays are considered internally, property that allows them to be compared with DTC

algorithms (Camacho, 2007), (Bordons, 2007). It is possible to say that each linear MPC can be stated as a DTC two degrees of freedom when the primary controller is calculated using an optimization process. The optimization structure of the internal DTC is defined as much by the process model as the model of disturbances and is not dependent of the optimization procedure even when considering the restrictions. Figure 4 illustrates this idea.

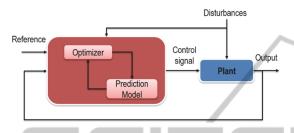


Figure 4: Outline of model predictive control.

Figure 5 shows the general structure of an MPC controller predictor for a process with time delay. It can be seen that the prediction  $y_P(t)$  consists of the addition of the output of the delay free ideal model  $\hat{y}(t + d|t)$ , and a correction based on the current plant output y(t) and the predicted output  $\hat{y}(t|t)$ , passing through a filter.

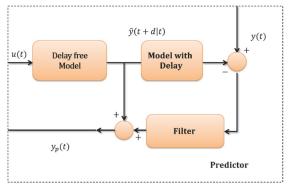


Figure 5: General structure of predictor MPC.

Although the reference tracking is not dependent on 1 disturbance rejection and robustness of the closed loop system are directly related to the predictor filter block MPC. Therefore, these two characteristics are affected by the dead time of the process, and in some applications greater scheme is required.

#### 3.2 Uncertainty

The design of a control system depends significantly on the dynamic model of the plant or process. As a real process may be too complex to be described so absolutely precise by a mathematical model, they always have modeling errors. The origins and causes of this discrepancy are many and control theory is referred collectively as uncertainty in model: parametric uncertainty, little knowledge of the dynamics of the process, unknown entries and dynamic despised and simplifications in the model, among others (Rodríguez, 1996), (Diederich, 2005).

For example, if a model based on the linearized about a nominal operating point of a nonlinear system controller is designed, the nonlinearities are presented as modeling uncertainties.

In Figure 6 the general outline of feedback control system in the presence of uncertainty arises. E(s) is the uncertainty associated with the model and  $G^*(s)$  represents the actual plant model.

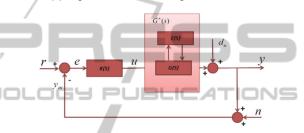


Figure 6: Control system with uncertainty in the plant model (Rodríguez, 1996).

Items with uncertainty can be classified as structured uncertainties and unstructured uncertainties. In the first, sources of uncertainty of systems are localized, obtaining with this a tighter or structured modeling errors description. In the Unstructured uncertainties what is commonly known is a dimension of the magnitude of the uncertainty, usually frequency dependent. These complex uncertainties generally occur in the high frequency range and may include not modeled time delays, coupling, hysteresis parasitic and other nonlinearities. An example of this kind of uncertainty is presented in the linearization of a nonlinear plant. If the actual plant is nonlinear and its model is linear, the difference acts as unstructured uncertainty (Matušů, 2007).

#### 3.2.1 Control of Systems with Uncertainty

Currently there are two main approaches that try to overcome the uncertainty in the model: adaptive control (Slotine and Weiping, 1991), (Bodson, 1989) based on online identification process and adjustment of the slider to the desired conditions; and robust control (Sidi, 2001), (Horowitz, 1992), (Ortega and Rubio, 2004), which guarantees the preservation of certain properties of the control loop for the whole family of controlled plants.

Various strategies of adaptive control have been proposed considering uncertainty SISO and MIMO systems. Such controllers often involve some type of functions to approximate the unknown dynamics. However, the approximation error and disturbanceinternal or external-can impair controller performance or even destabilize the system of closed loop control. Therefore, in order to ensure the performance of the controller, various robust components are incorporated in the design of adaptive controllers, resulting in robust adaptive controllers (Ioannou, 1996), (Moheimani, 2001), (Wenjie, 2005).

### 3.3 Disturbances

The problem of disturbance rejection is an eternal subject of investigation since the introduction of control theory and applications. From direct design, the interference rejection, traditional control methods such as proportional integral derivative (PID) and linear quadratic regulator controllers (LQR), may be unable to comply with the specifications of high precision control in strong disturbances and uncertainties. The rationale for this is that these traditional methods do not take into account explicitly the attenuation of uncertainty or disruption when controllers are designed.

The typical characteristics of the main disturbance mitigation methods are summarized as follows:

Adaptive Control (AC): The idea of adaptive control is that first the model parameters controlled online system are identified, and then the controller parameters are tuned based on that estimate. This control technique is very effective in the treatment of model uncertainties and has gained wide applications in engineering practice. Successful applications of adaptive control usually rely heavily on design ID laws or estimate model parameters variation in time. When these key parameters are difficult to identify or estimate online, these methods are not valid.

*Robust Control (RC)*: The robust controller design considers the worst case of disturbances and model uncertainties. The robustness of robust control is usually obtained by sacrificing the transient performance of other highlights. Therefore, the robust control is often criticized for being sometimes very conservative.

*Sliding Mode Control (SMC):* SMC has fine skills in suppressing the effects of parameter variations and external disturbances. However, the discontinuous

switching controller makes it prone to induce chattering (chattering) of high frequency mechanical systems. Although the use of some modification methods such as the method of saturation function could effectively reduce the chattering problem, the performance advantage of the disturbance rejection is sacrificed. These disadvantages significantly limit SMC applications.

Internal Model Control (IMC): Since the 1980s, the IMC approach proposed by Garcia and Morari has been used to mitigate the effects of external shocks in the control systems. This technique has received much attention in control theory and various applications due to its simple concept and intuitive design philosophy. However, BMI is generally available for linear systems. Furthermore, the application of BMI algorithm for high-dimensional system is quite sophisticated because of the need to calculate the inverse of a matrix high dimensional transfer function.

The motivation of the aforementioned approaches of control is to reject control disturbances of feedback rather than control feedforward compensation. These control methods generally achieve the goal of disturbance rejection through feedback regulation based on the tracking error between the measured outputs and set-points (Hohenbichler., 2009). Therefore, controllers designed can't react fast enough and directly in strong disturbances through feedback regulation of a relatively slow manner. To this end, these control approaches are generally recognized as passive anti disturbs control methods (PADC).

To overcome the limitations of the PADC methods in handling the disturbs, it has been proposed the approach called active anti disturbs control (AADC). Generally speaking, the idea behind the AADC is directly counteract disturbances by the feedforward control of compensation based on measurements or estimates disturbances.

#### 3.3.1 Control based on Disturbance Observer

Unlike Passive Anti Disturbs Control (PADC), the Disturbance Observer Control (DOBC) provides an active and effective way to deal disturbances and improves the robustness of the control system in closed loop. Feedback control and feedforward control based on disturbance observer: Figure 7 shows a basic control scheme based on the observation of the disturbance, the control structure consists of two parts. Feedback control is generally employed to ensure monitoring and stabilizing the dynamics of the controlled nominal plant. At this stage the disturbances and uncertainties not necessarily need to be considered. These can be estimated by a disturbance observer to be compensated by a feedforward controller.

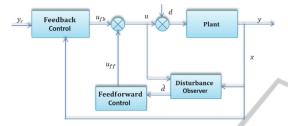


Figure 7: Basic structure of a controller based on disturbance observer.

The greatest merit of this design approach is that the combination of feedback control and feedforward control allows isolating control performance monitoring with the rejection of disturbances.

#### 3.4 Robust Predictive Control

From the analysis previously developed, one can sense that the predictive nature of the MPC controllers inherently considers time delays, so that they can be compared with control algorithms for dead time compensation (DTC). These controllers have good performance in systems with time delays; reason derives its widespread use in systems which perform chemical processes.

Moreover, in plants having model uncertainty, researchs strongly favor the use of robust controller. From this point of view, it is not illogical to think that the combination of two control strategies could provide the benefits of both systems having different open problems as nonlinearities, model uncertainty, delay, external shocks, variance in time, among others.

In recent years the use of these mixed Control strategies presented considerable growth for their welcome in increasingly complex systems. Therefore, for processes containing exhibit delays and uncertainty in the model parameters, a control strategy robust MPC promise satisfactory results (Ramírez, 2002), (Raimondo et al., 2009), (Maciejowski, 2000), (Bemporad and Morari, Robust model predictive control: A survey, 2007).

The MPC control technique MPC Min-Max is a robust control technique useful for solving problems caused by the discrepancies in the prediction model and the real process. Not only minimizes a criterion that considers the nominal value of the process output, but also minimizes the maximum value that can take the objective function from the consideration of uncertainty in the model. In other words, the optimal sequence of actions is calculated as:

$$u^* = \arg\min_{u \in U} \max_{\theta \in \Theta} J(u, \theta) \tag{14}$$

where  $\theta$  represents uncertainty and  $\Theta$  describes the set of values considered in the uncertainty.

Although the advantages of MPC Min-Max on the Control MPC nominal technique lie better control when the dynamic is not described well enough by the prediction model, this technique has problems because of the high computational cost (Bemporad and Morari, 2007).

# 3.5 Proposed Solution: Control MPC / QFT based on Disturbance Observer

As mentioned in the previous point, in the MPC control area there is a need to seek predictive control algorithms that besides being robust to model uncertainties and external disturbances are computationally efficient on-line. To that end, in this thesis a scheme of MPC / QFT control based on observer for multivariable plants is proposed, as shown in Figure 8.

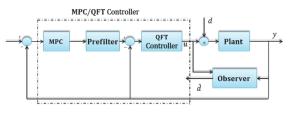


Figure 8: MPC/QFT controller based on observer.

The scheme consists of an internal QFT controller that reduces the uncertainty in the plants family and therefore increases the stability of the external controller robust MPC. The latter facilitates the management of dynamic constraints and delays in the system.

Thanks to QFT uncertainties are taken into account in a systematic way in order to contain results without conservatism- property that promises better results than Robust-Predictive and Adaptive-Robust techniques. Given these considerations, and coupled with the difficulty of QFT robust control technique to manage restrictions, the union of the QFT and MPC techniques provides a new approach that turns out to be less sensitive to disruption and uncertainty in the process model, with less computational load and taking into account the constraints of the process since MPC has the ability to include systematically.

In addition, the MPC algorithm control through the prediction feature allows QFT go forward when the reference signal is known. The inclusion of disturbance observer to MPC/QFT scheme contributes to external disturbance estimation allowing the control system a better accuracy and robustness.

# 4 METHODOLOGY

The development of the research will take place in the following stages:

# 4.1 Bibliography Review

At this stage is carefully reviewed the relevant bibliography on QFT and MPC design control and disturbance observer applied in multivariable systems. Besides courses, materials and videos related to the control approach in order to solve control problems studied.

# 4.2 Tackling the Problem of Parametric Uncertainty in Multivariable Systems

At this stage the current strengths and limitations of Quantitative Control Theory QFT in the regulation and control of multivariable systems with uncertainty in the model parameters are studied and will plan an approach to reduce the difficulties.

# 4.3 Solving the Problem of External Disturbances

At this stage we study the different types of observers to cope with external disturbances present in multivariable processes. Such observers may be linear, non-linear or non-linear advanced, so the adequate structure should be carefully reviewed to solve the problem of estimation and disturbance rejection.

#### 4.4 Tackling the Problem of Time Delays

At this stage the representation is obtained in state space in discrete time of the internal control loop (consisting of the pre-controller and observer) to solve the problem of time delays through an approach based on Model Predictive Control (MPC), and taking into account systematically the constraints of the system. Also are adjusted some parameters of the cost function of the predictive controller as the forecast horizon, the weighting factors of the control effort and mean square error and the sampling period.

# 4.5 Integration of QFT, MPC and Observer of Disturbances

At this stage the control QFT and MPC and disturbance observer approaches are gathered, and the problem of global optimization is solved by using techniques based on Lyapunov stability as Linear Matrix Inequalities (LMI) or heuristics optimization techniques such as genetic algorithms, among others.

# 4.6 Validation of MPC / QFT Control Strategy in Cases 1 and 2

At this stage is validated the design methodology of controllers in the regulation of glucose in patients with diabetes mellitus type I and in another case such as the power control in a wind turbine or in controlling various process variables in a crude distillation.

# 4.7 Dissemination of Results

The dissemination to the scientific community is continuously from the time the methodology of design of controllers is developed. Such disclosure is for attendance at international events and publications in recognized journals.

### 4.8 Final Report

Finally a complete report of the contribution originated-doctorate level -in this research is made.

# **5 EXPECTED OUTCOME**

Taking advantage of the virtues of robust control technique and predictive control, the merger of the two strategies will enable optimum results in nonlinear systems with parametric uncertainty, and in the presence of delays and restrictions. Robust MPC Min-Max controllers are useful for troubleshooting in the discrepancy of the model parameters and systems with time delays. However, due to the high computational cost the number of applications of this technique is relatively small, so you should delve in the study of robust MPC different approaches in order to solve this problem.

This research proposed a novel method that combines the virtues of MPC control techniques proposed nonlinear QFT and observers of disturbances (DOB), to address the delays, uncertainty in the model and external shocks of multivariable systems. Cascade structure combining an inner loop containing the nonlinear QFT with an outer loop controller where a predictive controller provides the appropriate reference to inner loop is proposed. This fusion of drivers considered the estimate made by a disturbance observer to mitigate the impact of external disturbance.

The results to be obtained in this thesis are:

- 1. Approach of an alternative solution to the problem of parametric uncertainty nonlinear multivariable processes, using the virtues of QFT nonlinear and making important contributions in this area.
- 2. Proposal for an alternative solution to the problem of external disturbances in multivariable systems using observer's theory design.
- 3. Formulation of a control scheme to work with time delays and constraints multivariable systems using predictive control strategy and discrete delta transform.
- 4. Approach to technical integration of nonlinear QFT and MPC Control and Disturbance Observer to solve the problems of temporary delay, parameter uncertainty, constraints and external disturbances in multivariable systems.
- 5. A comparison and analysis of the results obtained with the proposed control approach applied to two cases that have the control problems studied.

#### 6 **STAGE OF THE RESEARCH**

Since most physical systems are characterized by uncertain nonlinear models, it is natural to apply a linearized approximation of the system because it replaces the nonlinear uncertain plant by a set of uncertain LTI plants. However, for operating points far from the vicinity this procedure may fail.

To work with uncertainty in the model and nonlinear systems, (Baños and Bailey, 2001) proposes a non-linear approach QFT. In that

approach an equivalent LTI system is defined by replacing the nonlinear plant by a set of LTI plants  $P_e$  and a set of attached disturbances De. The replacement has to be done in such a way that the LTI equivalent problem has a solution for the compensators F and G -which are the system controller, respectively-; and with the goal that this solution be valid for the original nonlinear problem. The basic idea is to make both control problems, the nonlinear and the LTI problem, equivalent with respect to a particular sets of acceptable outputs  $A_{rd}$ , depending on each particular combination of references and disturbances.

The thesis currently working on using this technique nonlinear QFT control to initially validate their results in regulating glucose in patients with diabetes. To this end Bergman minimal model which was proposed for to represent glucose concentrations and plasma insulin test after intravenous glucose tolerance (IVGTT). Bergman used to represent these three compartments concentrations: Plasma insulin I (t) (mU / L), remote insulin X (t) (mU / L) and plasma glucose G (t) (mg /dL or mmol/L), and raised the following differential equations:

$$\dot{G}(t) = -p_1(G(t) - G_B) - G(t) * (X(t) - X_B) + D(t)$$
(15)

$$\dot{X}(t) = -p_2(X(t) - X_B) + p_3(I(t) - I_B)$$
(16)

$$\dot{I}(t) = -nI(t) + \frac{u(t)}{V_1}$$
(17)

Where:

J

G(t)[mg/dL]: is the blood glucose concentration at time t [min];

 $I(t)[\mu U/mL]$ : is the blood insulin concentration at time t [min];

 $X(t)[min^{-1}]$ : is proportional to the plasma insulin concentration in a remote compartment function.

 $G_B[mg/dL]$ : is the basal glucose level of the patient;  $I_B[\mu U/mL]$ : is the patient's basal insulin level.

With this research is expected to use the nonlinear QFT control scheme to work with nonlinear systems in the presence of dynamic delays, as presented in the regulation of glucose and estimating external disturbances (which for this system would be glucose intake) through the design of nonlinear observers. In the outer loop an MPC controller that deals with the restrictions of the plant and the existing delay will be designed. In order to do so, it will obtain the representation in state space in discrete time of internal loop control using the delta transformation and the uncertainty of the system will be considered when solving the problem of global

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optimization through techniques based on Lyapunov stability as Linear Matrix Inequalities (LMI).

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