Parametric Study of Liquid Flow in Five- and Six-stage Centrifugal Pumps

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Keywords: Multistage Centrifugal Pump, Impeller, Diffuser, Computational Fluid Dynamics (CFD), Modeling and Simulation.

Abstract: Two models of a multistage centrifugal pump using a five- and six-stage centrifugal pumps were developed and numerically investigated. The continuity and Navier-Stokes equations with the k-ε turbulence model and standard wall functions were used by means of the ANSYS-CFX code. To enhance the design of the multistage pump, the concept consisting of varying three parameters at a time was used. Thus, the combined effects of the impeller blade angle, the impeller blade number and the impeller blade width on the performance of the five- and six-stage centrifugal pumps was analyzed. The results obtained reveal, among other things, that the highest pump efficiency was reached for the outlet impeller blade angle of 25.38°, the number of impeller blades of 7, and the impeller blade width of 7 mm.

1 INTRODUCTION

Multistage centrifugal pumps are widely used in industrial and mining enterprises (Gülich, 2010). For a more performing multistage pump, its design parameters, such as the number of stages, impeller blades, diffuser vanes and diffuser return vanes, angle of the impeller blade, height of the impeller blade and diffuser vane, the width of the impeller blade and diffuser vane, the impeller and diffuser diameter, the rotating speed of the impeller and the casing geometry must be determined accurately. Many experimental and numerical studies have been conducted on the liquid flow through a multistage centrifugal pump varying one key parameter at a time (Among other things, La Roche-Carrier et al., 2013; Miyano et al., 2008; Kawashima et al., 2008, Gantar et al., 2002). In this study, a concept consisting of varying three key parameters of multistage centrifugal pumps at a time was used to identify parameters to lead to the best design and performances of multistage centrifugal pumps. Thus, the following parameters were varied at a time for the five- and six-stage centrifugal pumps: a) the numbers of impeller blades (6, 7 and 8), b) the outlet angle of the impeller blade (19.81°, 21.24° et 25.38°), and c) the impeller width blade (4 mm, 5 mm and 7 mm).

2 GOVERNING EQUATIONS

Fig. 1 shows the domain fluids of five- and six-stage centrifugal pumps considered in this research work to run the numerical simulations.

![Figure 1: Domain fluids of five- and six-stage pumps.](image)

The following assumptions were made for the governing equations for liquid flow in the five- and six-stage centrifugal pumps: (i) a steady state, three-dimensional and turbulence flow using the k-ε model was assumed; (ii) it was an incompressible liquid; (iii) it was a Newtonian liquid; and (iv) the liquid’s thermophysical properties were constant with the temperature.

To account for these assumptions, the theoretical analysis of the liquid flow in the impeller passages, diffuser vane passages and diffuser return vane passages was based on the continuity and Navier-Stokes equations (Ansyl inc., 2011). For the three-dimensional liquid flow through these five- and six-stage centrifugal pumps as shown in Fig. 1,
the continuity equations are expressed by:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  

(1)

and the Navier–Stokes equations are given by:

\[
\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = \mu_{\text{eff}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\partial p}{\partial x} + B_x
\]

\[
\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = \mu_{\text{eff}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\partial p}{\partial y} + B_y
\]

\[
\rho \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = \mu_{\text{eff}} \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{\partial p}{\partial z} + B_z
\]  

(2)

where \( p \) is the pressure, \( \rho \) is the density, \( \mu_{\text{eff}} \) is the effective viscosity accounting for turbulence, and \( B_x, B_y, \) and \( B_z \) are the source terms. They can be expressed by:

\[
B_x = \rho \left( \frac{1}{2} \frac{\partial^2 \rho}{\partial x^2} + 2 \frac{\partial \rho \omega_x}{\partial x} \right)
\]

\[
B_y = \rho \left( \frac{1}{2} \frac{\partial^2 \rho}{\partial y^2} + 2 \frac{\partial \rho \omega_y}{\partial y} \right)
\]

\[
B_z = \rho \left( \frac{1}{2} \frac{\partial^2 \rho}{\partial z^2} + 2 \frac{\partial \rho \omega_z}{\partial z} \right)
\]  

(3)

Furthermore, \( \mu_{\text{eff}} \) is defined as \( \mu_{\text{eff}} = \mu + \mu_t \), where \( \mu \) is the dynamic viscosity and \( \mu_t \) is the turbulence viscosity, it is linked to turbulence kinetic energy \( k \) and dissipation \( \varepsilon \) via the relationship:

\[
\mu_t = C_{\mu} \rho \kappa \varepsilon
\]

where \( C_{\mu} \) is a constant. The values of \( k \) and \( \varepsilon \) tem directly from the differential transport equations for turbulence kinetic energy and turbulence dissipation (Ansys inc., 2011). Additionally, for the flow modeling near the wall, the logarithme wall function is used to model the viscous sub-layer (Ansys inc., 2011).

To solve equations 1 and 2 numerically while accounting for the boundary conditions and turbulence model \( k-\varepsilon \), the computational fluid dynamics ANSYS-CFX code, based on the finite volume method, was used to obtain the liquid flow velocity and pressure distributions. Pressure velocity coupling is calculated in ANSYS-CFX code using the Rhie Chow algorithm (Ansys inc., 2011).

In the cases examined involving the five- and six-stage centrifugal pumps, the boundary conditions were formulated as follows (figure 2): the static pressure provided was given at the pump inlet, while the flow rate provided was specified at the pump outlet. The frozen rotor condition was set for the impeller-diffuser interfaces. A no-slip condition was set for the flow at the wall boundaries.

The pump head is determined as follows:

\[
H = \frac{p_{\text{out}} - p_{\text{in}}}{\rho g}
\]  

(4)

where \( p_{\text{in}} \) is the total pressure at the pump inlet and \( p_{\text{out}} \) the total pressure at the pump outlet. They are expressed as:

\[
p_{\text{in}} = p_{\text{ti}} + \frac{1}{2} \rho v_{\text{ti}}^2
\]

\[
p_{\text{out}} = p_{\text{to}} + \frac{1}{2} \rho v_{\text{to}}^2
\]  

(5)

Moreover, the hydraulic power of the pump is given by \( P_{\text{h}} = \rho Q g H \), where \( Q \) is the flow rate and \( H \) is the pump head.

Furthermore, the brake horsepower of the pump stage is expressed as \( \omega C \), where \( \omega \) is the angular velocity and \( C \) is the impeller torque.

From the hydraulic power and the brake horsepower, the efficiency of the pump can be written as

\[
\eta = \frac{P_{\text{h}}}{\omega C}
\]  

(6)

3 RESULTS AND DISCUSSION

The main reference data used for the impeller were 89 mm for the inner diameter, 175 mm for the outer diameter and 2800 rpm for the rotating speed. For the diffuser, the main reference data were 176 mm for the inner diameter, 234 mm for the outer diameter, 6 for the number of vanes and 8 for the number of return vanes. The numerical simulation results presented in this work were obtained with the highest accuracy by conducting mesh-independent solution tests in each case study using different numbers of mesh elements.
3.1 Combined Effect of the Impeller Blade Angle, Impeller Blade Number and Impeller Blade Width

To investigate the influence of the combined effect of the impeller blade angle, the impeller blade number and the impeller blade width on the pump head, brake horsepower and efficiency, the values of the blade angle, the blade number and the blade width were varied at a time while the other parameters were kept constant, as indicated in Tab. 1 for the five- and six-stage centrifugal pumps.

Table 1: Selected parameters for simulations.

<table>
<thead>
<tr>
<th>Case</th>
<th>Impeller blade number; Impeller blade angle [°]; Impeller blade width [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>6; 21.24; 7</td>
</tr>
<tr>
<td>Case 2</td>
<td>7; 19.81; 4</td>
</tr>
<tr>
<td>Case 3</td>
<td>8; 25.38; 5</td>
</tr>
</tbody>
</table>

Figs. 3 and 4 represent the variation of the head as a function of the flow rate for the five- and six-stage centrifugal pumps respectively. There, it is observed that the lowest head was achieved with the combined of the values of the blade number of 6, the blade angle of 25.34° and the blade width of 4 mm.

Additionally, as shown in Figs. 5 and 6, the lowest brake horsepower was reached using the blade number of 7, the blade angle of 25.38° and the blade width of 7 mm for the five- and six-stage centrifugal pumps.

Figure 3: Head versus volume flow rate (five-stage pump).

Figure 4: Head versus volume flow rate (six-stage pump).

Figure 5: Brake horsepower versus volume flow rate (five-stage pump).

Figure 6: Brake horsepower versus volume flow rate (six-stage pump).

Furthermore, Figs. 7 and 8 represent the efficiency curves, showing that the highest efficiency was achieved for the flow rate about of 33 m³/h.

Figure 7: Efficiency versus volume flow rate (five-stage pump).

Figure 8: Efficiency versus volume flow rate (six-stage pump).
3.2 Parameters of Improved Five- and Six-centrifugal Pumps

Taking account of the accomplished simulations in section 3.1, an analysis was performed to identify the parameters which improve the efficiency of the five- and six-centrifugal pumps. Thus, the following values were found for the five- and the six-stage centrifugal pumps:

- Impeller blade number: 7
- Impeller blade angle: 25.38°
- Impeller blade width: 7 mm

4 CONCLUSIONS

In this study, a liquid flow in a five- and six-stage centrifugal pumps was numerically investigated using a ANSYS- CFX code. The combined effects of the outlet blade angle, the impeller blade number and impeller width on the on the five- and six centrifugal pump head, efficiency and the brake horsepower were performed. The results obtained demonstrate, among other things, that a multistage centrifugal pump can be improved selecting adequately three key parameters. Further research work is planned to complete this study finding a general appropriated to enhance the performances of a multistage centrifugal pump of any size.

NOMENCLATURE

B    source term (Nm⁻³)
C    torque (Nm)
g    acceleration of gravity (ms⁻²)
H    head (m)
P    power (W)
p    pressure (Nm⁻²)
p_v turbulence production due to viscous and buoyancy forces
Q    flow rate (m³s⁻¹)
r    radial coordinate (m)
V    velocity (ms⁻¹)
u    flow velocity in x direction (ms⁻¹)
v    flow velocity in y direction (ms⁻¹)
w    flow velocity in z direction (ms⁻¹)
x    x-coordinate (m)
y    y-coordinate (m)
z    z-coordinate (m)

Greek symbols

Δ    difference
ε    turbulence dissipation (m² s⁻³),
η    efficiency
κ    turbulence kinetic energy (kg m² s⁻²)
ρ    fluid density (kg m⁻³)
μ    dynamic viscosity (Pa s)
μ_e effective viscosity (Pa s)
μ_t    turbulence viscosity (Pa s)
ω    angular velocity (rad s⁻¹)

Subscripts

1    inlet
2    outlet
h    hydraulic
i    inlet
m    mechanical
o    outlet
s    shaft
st    total
v    volumetric
ve    velocity

ACKNOWLEDGEMENTS

The authors are grateful to the Foundation of University of Quebec in Abitibi-Temiscamingue (FUQAT) and the Company Technosub inc.

REFERENCES


