Guaranteed Control of a Robotic Excavator During Digging Process

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Abstract: Automation of excavators offers a promise for increasing productivity of digging. At the same time, it’s a highly difficult issue due to presence of various nonlinearities and uncertainties in excavator mechanical structures and hydraulic actuators, disturbance when a bucket contacting the ground etc. This paper concerns the problem of robust trajectory tracking control of an excavator arm. To solve this problem, the computed torque control with the guaranteed cost control is considered. The mathematical tool of R-functions as an alternative to the linear matrix inequality approach to constructing information sets of an excavator arm state is used. Simulation results and functional ability analysis for the proposed control system are given.

1 INTRODUCTION

Hydraulic excavators are used at a wide variety of sites from civil construction to disaster elimination, therefore efficiency and productivity increase of these machines is a highly important problem. One of the ways to solve the problem is to design a robotic excavator. In addition to the increase of productivity, the automation of excavators reduces loads on an operator, improves his safety and makes it possible to work in places that are inaccessible for humans.

However, robotic excavators are created extremely slowly due to high dynamic loads during the bucket and soil interaction, which is difficult to predict, and other uncertainties such as backlashes between machine parts, variability of a fluid viscosity in hydraulic actuators, oil leaks, etc.

There are a lot of papers focused on the robotic excavator design and creation of digging process control system. For example, some works (Koivo et al, 1996; Gao et al., 2009; Gu et al., 2012) describe PD and PID controllers application to control a robotic excavator arm movement. Besides, in one of the papers (Gu et al., 2012) a proportional-integral-plus (PIP) controller and a nonlinear PIP controller based on a state-depended parameter model structure were proposed.

In one of the works (Yokota et al., 1996) a disturbance observer in addition to PI-controller to control a mini excavator arm was proposed. Along with the computed torque control, the adaptive and robust controls of the excavator arm were designed in (Yu et al., 2010).

In (Bo et al.) a fuzzy plus PI controller with fuzzy rules based on the soft-switch method was developed. In (Zhang et al., 2010) an adaptive fuzzy sliding mode control to realize the trajectory tracking control of an automatic excavator was designed. Two controllers based on fuzzy logic, including the fuzzy PID controller and fuzzy self tuning with neural network, were developed in (Le Hanh et al., 2009) to control the electro hydraulic mini excavator. In (Choi, 2012) the Time-Varying Sliding Mode Controller with fuzzy algorithm was applied to the tracking control system of the hydraulic excavator. Time-delay controllers were proposed for motion control of a hydraulic excavator arm in (Chang and Lee, 2002; Vidolov, 2012).

All these works have made a valuable contribution to solve the problem of robotic excavator creating, but a commercial fully robotic excavator will probably appear not soon due to the mentioned above factors.

In this paper we propose the guaranteed cost control for the trajectory tracking control of the
excavator arm during digging operation. The control guarantees the robustness against uncertainties of modelling and unexpected disturbances due to, for instance, the bucket and soil interaction.

2 EXCAVATOR MODELLING

2.1 Modelling of an Excavator Arm

The dynamic model of an excavator arm can be obtained using the Lagrange equation and can be expressed concisely in matrix form as the well-known equations for a rigid-link manipulator (Spong et al, 2006):

\[ D(\theta) \ddot{\theta} + C(\dot{\theta}, \dot{\theta}) + G(\theta) + B(\dot{\theta}) = \tau - \tau_L, \]

where \( \dot{\theta} \) and \( \ddot{\theta} \) are the 4x1 vectors of the measured joint position, velocity and acceleration angles as shown in Figure 1; \( D(\theta) \) is the 4x4 symmetric, positive-definite inertia matrix; \( C(\dot{\theta}, \dot{\theta}) \) is the 4x4 Coriolis and centripetal matrix; \( G(\theta) \) is the 4x1 vector of gravity terms; \( B(\dot{\theta}) \) is the 4x1 vector of frictions; \( \tau \) is the 4x1 vector specifying the torques acting on the joint shafts; and \( \tau_L \) is the 4x1 vector representing the interactive torques between the links and environment during the digging operation.

For the convenience, dynamic equation (1) can be rewritten as follows:

\[ D(\theta) \ddot{\theta} + N(\theta, \dot{\theta}) + \tau_L = \tau. \]  

where \( N(\theta, \dot{\theta}) = C(\dot{\theta}, \dot{\theta}) + G(\theta) + B(\dot{\theta}) \).

2.2 Digging Resistance Force

Digging by an excavator is performed due to the bucket movement in two directions. The main movement, named lifting, cuts a slice of soil. The second movement (penetration) is perpendicular to the main movement and regulates the thickness of the cut slice of the soil.

During digging of soil by an excavator there acts a resistance force \( F_r \) at the cutting edge of the bucket teeth (Figure 2). \( F_r \) is a resultant reaction force of the tangential \( F_t \) and the normal \( F_n \) forces. According to M.G. Dombrovskij (Alekseeva et al. 1985), the tangential force can simplistically be determined as

\[ F_t = k_c bh, \]

where \( k_c \) is the specific cutting force in N/m² that takes into account soil resistance to cutting as well as all other forces (frictional resistance of the bucket with the ground, resistance to the movement of the prism of soil etc.); \( h \) and \( b \) are the thickness and width of the cut slice of soil.

The normal component \( F_n \) is calculated as:

\[ F_n = \psi F_t, \]

where \( \psi \) is a dimensionless factor depending on the digging angle, digging conditions and the cutting edge where \( \psi = 0.1–0.45 \). Higher values of \( \psi \) corresponds to more dulling of the bucket teeth edge.

Thus, the torques of resistance forces for each link of an excavator arm can be calculated as:

\[ \tau_L = \begin{bmatrix} 1 & 1 & 1 & \Delta \tau_{L2} \\ 0 & 1 & 1 & \Delta \tau_{L3} \\ 0 & 0 & 1 & \Delta \tau_{L4} \end{bmatrix}, \]
where $\Delta t_{L4}=l_4(F_i\sin\theta_b-F_n\cos\theta_b)$;
$\Delta t_{L3}=-l_2(F_i\sin(\theta_3-\theta_b)+F_n\cos(\theta_4-\theta_b))$;
$\Delta t_{L2}=l_2(F_i\sin(\theta_3-\theta_b)+F_n\cos(\theta_3-\theta_b))$;
$\theta_b$ is the angle between the axes $x_4$ and the direction of the force $F_i$ (Figure 2); $\theta_{34}=\theta_3+\theta_4$ (Figure 1); $l_j, j=2,4$ are the lengths of the excavator arm links.

It is obvious that using more accurate models of a bucket and soil interaction, for example given in (Luengo, 1998), still possible improve the performance of proposed control system.

### 2.3 Controller Model

In classical case of manipulator control, the computed-torque control (CTC) and computed-torque-like controls are widely used.

The equation for the CTC is given by Spong (Spong et al., 2006)

$$u=D(\theta)a+N(\theta,\dot{\theta})+\tau_L,$$  \hspace{1cm} (6)

where $u$ is the control vector; $a=\dot{\theta}^T+K_v\dot{\theta}+K_p\dot{\theta}$; $K_v$ and $K_p$ are symmetric positive-definite matrices; $\epsilon=\dot{\theta}^T-\dot{\theta}$ is the position error vector; $\dot{e}=\dot{\theta}^T-\dot{\theta}$ is the velocity error error vector; and superscript “$d$” means “desired”.

As far as the values of the parameters in (2) are not known exactly due to the uncertainties in the system, we have to rewrite the control (6) as

$$u=\hat{D}(\theta)a+\hat{N}(\theta,\dot{\theta})+\dot{\tau}_L,$$ \hspace{1cm} (7)

where the notation $\hat{\cdot}$ represents the estimates of the terms in the dynamic model.

Having substituted (7) in (2), we can obtain $\hat{\dot{\theta}}=a-\eta$, where $\eta$ is the uncertainty. Hence, $\dot{e}=\dot{\theta}^T-a+\eta$. We can set the outer loop control as $\dot{\eta}=a+\delta\alpha$ , where $\delta\alpha$ is to be chosen to guarantee robustness to the uncertainty effects $\eta$. By taking $x=[e^T \dot{e}^T]^T$ as the state system, the following first-order differential matrix equation is obtained:

$$\dot{x}=Ax+B(\eta-\delta\alpha),$$ \hspace{1cm} (8)

where $A$ and $B$ are the block matrices of the dimensions $(6\times 6)$ and $(6\times 3)$ respectively:

$$A=\begin{bmatrix} 0 & E \\ -K_p & -K_v & \end{bmatrix}; B=\begin{bmatrix} 0 \\ E \end{bmatrix}.$$  

Thus, the issue of the control of an excavator arm movement is reduced to finding an additional control input $\delta\alpha$ to overcome the influence of the uncertainty $\eta$ in the nonlinear time-varying system (7) and to guarantee ultimate boundedness of the state trajectory $x$ in (8).

### 3 CONTROLLER DESIGN

#### 3.1 Kinematic Control

Previously to development of control system as subject to improve an excavator dynamics, it is necessary to solve the problem of its kinematic control. In (Sergiyenko et al., 2013) it was considered an optimal solution of inverse kinematics task for robotic excavator that provides bucket teeth movement along the desired path. As optimality criterion the minimizing of quadratic function (9) of joint angles associated with the respective weights was accepted:

$$J_0=\sum_{j=2}^{4}\gamma_j(\theta_j^0-\theta_j)^2,$$ \hspace{1cm} (9)

where $\theta_j^0$ and $\theta_j$ are the initial and the final values of the angles $\theta_j, j=2,4$, respectively (Figure 1); $\gamma_j$ are the weighting factors, that prioritize the angles changing $\theta_j$; $\Omega_j$ is the given subset.

To solve the problem (9) it is necessary to solve the matrix equation (10):

$$H\Theta_i=F_i,$$ \hspace{1cm} (10)

where $\Theta_i=[\Delta\theta_2^T \Delta\theta_4^T \Delta\theta_2^T]^T$; $\Delta\theta_j^i$ are increments of the joint angles of an excavator arm at each step $i$ in time domain; $F_i=[-\Delta\theta_2^0 \Delta\theta_4^0]^T$; $\Delta\chi_2^i$ and $\Delta\chi_4^i$ are increments of a bucket teeth coordinates in a Cartesian frame at each $i$-th step in time domain;

$$H_i=\begin{bmatrix} \sum_{j=2}^{4}l_j\sin\alpha_j^{-1} & \sum_{j=3}^{4}l_j\sin\alpha_j^{-1} & l_4\sin\alpha_4^{-1} \\ \sum_{j=2}^{4}l_j\cos\alpha_j^{-1} & \sum_{j=3}^{4}l_j\cos\alpha_j^{-1} & l_4\cos\alpha_4^{-1} \end{bmatrix};$$

$$\alpha_j=\sum_{k=2}^{j}\theta_k, j=2,4.$$  

Using the Tikhonov's regularization method we can write the original equation (10) in the next form...
\[
(H_I^T H_f + \lambda \Gamma)^{-1} \Theta_i = H_I^T F_i,
\]
where \(\lambda\) is arbitrary small positive parameter that provides stability of the matrix \((H_I^T H_f + \lambda \Gamma)^{-1}\) computation; \(\Gamma\) is the square 3×3 matrix.

In classical problems the matrix \(\Gamma\) has equal diagonal elements. Taking into account the specifics of the vector \(\Theta_i\), we will use the diagonal matrix \(\Gamma\) which non-zero elements are defined as:

\[
(\Gamma)_{j-1,j-1} = \gamma_j, \quad j = 2, 4.
\]

If the values of \(\gamma_i\) are known, solution of (12) is trivial.

The weighting coefficients \(\gamma_i\) we propose to define in next way. The value of \(\gamma_2\) is selected wittingly large to minimize the boom motion. Values \(\gamma_2\) and \(\gamma_4\) depend on the method of digging:
- when digging with the bucket \(\gamma_2 >> \gamma_4\);
- when digging with the stick \(\gamma_4 >> \gamma_2\);
- when excavator digs simultaneously with the stick and with the bucket, the \(\gamma_2\) and \(\gamma_4\) ratio is chosen to equate the maximum angular acceleration of \(\varepsilon_2\) and \(\varepsilon_4\). Accelerations \(\varepsilon_j\) are calculated by the well-known formula:

\[
\varepsilon_j = \frac{\theta_{j+1} - 2\theta_j + \theta_{j-1}}{\Delta t^2}.
\]

### 3.2 Robust Control

For an additional control \(\delta\) determining, we propose the optimal guaranteed cost control approach. According to this approach, it is assumed that uncertainties in the system are known with accuracy to a certain guaranteed bounded set. During the control system operation the new sets representing the estimates of the system state are built. The advantage of this approach is in providing an upper bound on a given performance index and thus, the system performance degradation incurred by the uncertainties is guaranteed to be less than this bound (Gurko et al., 2012).

Let’s derive the digital version of the equation (8) for the digital control system implementation:

\[
x_{k+1} = A_d x_k + B_d [\eta_k - \delta \omega_k] \quad (k = 0, 1, \ldots, n-1),
\]

where \(A_d\) and \(B_d\) are the digital versions of the matrices \(A\) and \(B\) in (8); the uncertainty \(\eta_k\) is bounded by the known set \(\Omega^\eta_k\); \(k\) – moments of quantization.

Control is formed on the basis of joint angles \(\theta\) measurements are represented in the form of the vector \(y_k\):

\[
y_k = C_d (x_k + \nu_k), \quad (k = 1, 2, \ldots, n-1).
\]

where \(C_d\) is the output matrix; \(\nu_k\) is the vector of measurement noises bounded by the known set \(\Omega^\nu_k\).

As the aim of the control we assume the minimizing of the following cost function:

\[
J_k (x_k, \delta \omega_k) = V_k (x_{k+1}) + o_k (x_k, \delta \omega_k),
\]

where \(V_k\) is Lyapunov function that allows estimating the quality of the further excavator arm motion in the absence of perturbations; \(o_k\) is the given function, which defines the control costs and assigns limitations on their value.

For the well-posed task (16) formulation, information about the uncertainty \(\eta_k\) has to be redefined. As far as the \(\eta_k\) can take on any value inside the set \(\Omega^\eta_k\), we have to consider the values maximizing the cost function (16).

Moreover, the fact that \(\eta_k\) and \(v_k\) belong to the proper sets \(\Omega^\eta_k\) and \(\Omega^\nu_k\) enables to suppose that as a result of measurement (15) of the excavator arm joint angles \(\theta\), information about the current state is obtained in the form of the set \(x_k \in \Omega^x_k\). For the additional control \(\delta\) determining the point estimation of \(x_k \in \Omega^x_k\) is required. For this purpose we will consider the point maximizing the cost function (16). So, the objective of the additional control \(\delta\) is to solve the following task:

\[
\min_{\delta \omega_k \in \Omega^\eta_k} \max_{\nu_k \in \Omega^\nu_k} \max_{x_k \in \Omega^x_k} J_k (x_k, \delta \omega_k).
\]

It’s obvious that the task (17) solution guarantees the proper excavator control system performance that depends on \(J_k\) at any allowed \(\eta_k\) and \(v_k\).

The description of the sets of the possible states of the excavator arm we will carry out according to following algorithm (Gurko et al., 2012).

1. Let at an arbitrary moment of quantization \(k\) there is an estimate of the excavator arm state as \(x_k \in \Omega^x_k\). The transformation (18) should be realised to find the set of states \(\Omega^f_{k,k+1}\)

\[
\Omega^f_{k,k+1} = A_d \Omega^x_k,
\]

where \(\Omega^f_{k,k+1}\) is a prediction of possible system states \(x^f_{k+1} \in \Omega^f_{k,k+1}\) at the \([k+1]\) th moment to which
it must transit moving freely from the state \( x_k \in \Omega_k \).

2. A new set \( \Omega_{k+1}^w \) of possible system states is developed by transformation (blurring) of the set of states \( \Omega_{k+1}^\ell \):

\[
\Omega_{k+1}^w = \Omega_{k+1}^\ell \cup B \partial \Omega_k^n,
\]

where \( \partial \Omega_k^n \) is the aggregate of boundary elements of the set \( \Omega_k^n \).

Thus, the set \( \Omega_{k+1}^w \) is a prediction of the excavator arm state at the \([k+1]\)th moment with allowance for the influence exerted by uncertainties \( \eta_k \) on values of parameters of the vector \( x_{k+1}^\ell \).

3. A value \( x_{k+1}^w \in \Omega_{k+1}^w \) of the system state is found. The \( x_{k+1}^w \) is used for an additional control \( \eta_k \) determined to solve the task (16).

4. The moving of the set \( \Omega_{k+1}^w \) by the additional control \( \eta_k \) is provided and a new set \( \Omega_{k+1}^u \) is constructed. The set \( \Omega_{k+1}^u \) is an estimation of the system state to which it must transit at the \([k+1]\)th moment under \( \eta_k \) action.

5. The new measurement of joint angles \( \theta_j \) is carried out to find a posteriori estimate \( x_{k+1}^r \in \Omega_{k+1}^r \) of the system state at the \([k+1]\)th moment:

\[
\Omega_{k+1}^r = \Omega_{k+1}^u \cap \Omega_{k+1}^r.
\]

Further, the mentioned procedure is repeated iteratively.

4 DETERMINING A SET OF POSSIBLE STATES

Until recently linear matrix inequalities have been used to construct sets of control system possible states. In (Gurko and Kolodyazhny, 2013) we proposed to use \( R \)-functions for this purpose. This significantly simplifies the estimation of a control system state.

The \( R \)-function \( \varphi(x_k) \) of the set \( \Omega_k \) has the following properties:

\[
\begin{align*}
\varphi(x_k) &> 0, \text{ when } x_k \in \Omega_k, \\
\varphi(x_k) &< 0, \text{ when } x_k \in \partial \Omega_k, \\
\varphi(x_k) &= 0, \text{ when } x_k \notin \Omega_k \cup \partial \Omega_k,
\end{align*}
\]

where \( \partial \Omega_k \) is the aggregate of boundary elements of the set \( \Omega_k \).

Let’s denote \( R \)-functions of the sets \( \Omega_k^r \), \( \Omega_{k+1}^r \), \( \Omega_{k+1}^w \), \( \Omega_{k+1}^u \), \( \Omega_{k+1}^v \) as \( \varphi^r(x_k) \), \( \varphi^w(x_k) \), \( \varphi^v(x_k) \), \( \varphi^u(x_k) \) and \( \varphi^v(x_k) \). For instance, the set \( \Omega_k^r \) is constructed using the following \( R \)-function:

\[
\varphi^r(x_k) = \varphi^w(x_k) \land \varphi^v(x_k),
\]

where \( \land \) is the \( R \)-operation of conjunction:

\[
\varphi^r(\varphi^r + \varphi^v - \sqrt{(\varphi^r)^2 + (\varphi^v)^2}).
\]

5 SIMULATIONS

A simulation study of the excavator arm motion with the numerical values given in Table 1 (Koivo et al, 1996) was performed in MATLAB.

Table 1: Excavator parameters.

<table>
<thead>
<tr>
<th>Link</th>
<th>Mass, kg</th>
<th>Inertia, kg( \cdot )m(^2 )</th>
<th>Length, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boom</td>
<td>1566</td>
<td>14250.6</td>
<td>5.16</td>
</tr>
<tr>
<td>Stick</td>
<td>735</td>
<td>727.7</td>
<td>2.59</td>
</tr>
<tr>
<td>Bucket</td>
<td>432</td>
<td>224.6</td>
<td>1.33</td>
</tr>
</tbody>
</table>

A bucket desired trajectory is presented in Figure 3.
\[ |\psi| \leq 0.5 \text{deg} \text{ and is subject to the uniform distribution law. The resistance forces experienced when the bucket penetrates into the soil are calculated by (3)-(4).} \]

![Figure 4: Desired angles.](image)

Loam as the type of soil has been considered; the loam density varied arbitrarily in the range \(1600 \leq \rho_s \leq 1900 \text{ kg/m}^3\). The exact value of the force \(k_c\) in (3) was considered to be unknown except for the fact that it belongs to the set \(117600 \leq k_c \leq 245000 \text{ N/m}^2\). The value of the factor \(\Psi\) in (4) was assumed to be 0.25. Changing of the bucket mass has been also taken into account. The true load torques \(\tau_L\) acted at the links are shown in Figure 5.

![Figure 5: Load torques \(\tau_L\) acted at the links.](image)

As the aim of the control the task (16)-(17) solution has been assumed, where \(V_k = x_{k+1}^TPx_{k+1}\); \(\omega_k = \delta a_k^TR \delta a_k\); \(R = \text{diag}\{0.7, 0.5, 0.2\}\) and

\[
P = \begin{bmatrix} 3.2 & 0 & 0 & 1.12 & 0 & 0 \\ 0 & 3.2 & 0 & 0 & 1.12 & 0 \\ 0 & 0 & 3.2 & 0 & 0 & 1.12 \\ 1.12 & 0 & 0 & 1.86 & 0 & 0 \\ 0 & 1.12 & 0 & 0 & 1.86 & 0 \\ 0 & 0 & 1.12 & 0 & 0 & 1.86 \end{bmatrix}
\]

Sampling time was \(T_s = 0.1 \text{ s}\). For the sets of the system possible states \(R\)-functions have been used.

The simulation results are presented in Figures 6-8. As depicted in Figure 6, the joint angles tracking errors are less than 0.1, 0.2, and 1 degrees for the boom, stick and bucket, respectively.

![Figure 6: Joint angles tracking errors versus time.](image)

In Figure 7 the predicted sets of possible states \(\Omega^r\) vs. the true system states \(x_t\) at \(t = 4 \text{ s}\) are shown. It corresponds to the maximum value of the bucket tracking error. For the sets \(\Omega^r\) determine the expressions (18) - (20) have been used.

![Figure 7: Predicted sets \(\Omega^r\) and the true system states \(X_t\).](image)
6 CONCLUSIONS

The work presented in this article investigates a new controller to do digging trajectory tracking for a robotic excavator. The controller requires two circuits: the first circuit calculates the main control using the CTC, and the aim of the second one is to provide an additional control to compensate effect of uncertain factors on the basis of differential games with quadratic cost.

The mathematical tool of $R$-functions as the alternative of the linear matrix inequality approach to constructing information sets of the excavator arm state is used.

The practical value of the proposed controller is in providing an upper bound on a given performance index at any uncertainties from the given bounded set, as well as in requiring a relatively low computational capability compared to other reviewed methods.

Since the uncertainties do not always tend to maximize the cost function, the implementation of an additional circuit of adaptation which adjusts the bounds of sets of uncertain parameters is desirable. Our future work will investigate this aspect.

REFERENCES


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