Thinning based Antialiasing Approach for Visual Saliency of Digital Images

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Abstract: A thinning based approach for spatial antialiasing (TAA) has been proposed for visual saliency of digital images. This TAA approach is based on edge-matting and digital compositing strategies. Prior to edge-matting the image edges are detected using ant colony optimization (ACO) algorithm and then thinned using a fast parallel algorithm. After the edge-matting, a composite image is created between the edge-matted and non-antialiasing image. Motivations for adopting the ACO and fast parallel algorithm in lieu of others found in the literature are also extensively addressed in this paper. Preliminary TAA experimental outcomes are more promising but with debatable smoothness to some extent of the original size of the images in comparison.

1 INTRODUCTION

Reproducing faithfully a continuous signal from digital samples remains a main concern in digital signal processing – especially in cases where the samples have regions with strong spatial discontinuities or features (Rukundo, 2012; Rukundo and Maharaj, 2014; Rukundo, Cao and Huang, 2012; Rukundo, Wu and Cao, 2011; Rukundo and Cao, 2012). Failure to accurately reproduce a continuous signal leads results in artefacts known as aliasing (Crow, 1981). In computer vision and graphics, aliasing effect creates visually distracting artefacts and loss of salient details in an image. Visual saliency is a broad term that refers to the idea that certain parts of a scene are pre-attentively distinctive and create some form of immediate significant visual arousal within the early stage of human visual system (Timor and Michael, 2001). Early approaches used edge detectors to extract such pre-attentively distinctive parts of an object. However, today such an extraction is increasingly also being performed by saliency detectors, for example, for salient map construction for assessments applications in still image and video processing (Tong, et al, 2010; Itti, et al, 1998; Tong, et al, 2011; Timor and Michael, 2001). Given that salient regions of an image attract our attention, it is very important to improve the visual quality of an aliased salient map or contour in an image. Therefore, much effort has been expended in attempts to attain near-realism by developing advanced antialiasing techniques. Near-realism is a word used in this paper to signify the best image approximation of reality ever achieved digitally. With a few computationally-expensive exceptions, most approaches based on filtering before sampling have focused on weakening (if not to entirely remove) the effects of image aliasing artefacts using strategies based on creating new pixels or using artificial or non-original information. There has been reluctance to use non-artificial or original information of the yet-to-be antialiased image, even despite the well-known blurriness artefacts (which reduce sharpness) associated with creating new or non-original pixel values or using weighted functions. However, an edge inferring and smoothing antialiasing technique - defining jags as a sequence of corner points separated by a one-pixel width – has been proposed in (Bloomenthal, 1983) and remains among the rarest techniques for tackling aliasing issues by paying attention to edge information, but this technique remains prone to potential errors, especially at the edge vertices. It is important to note that aliasing artefacts are easily visible on the edges or highly discontinuous features of an (aliased) image rendered, not on the main

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texture or foreground. Another edge-directed antialiasing approach has been proposed in (Iourcha, Yang and Pomianowski, 2009) that is technically based on the computation of the gradients of edge information prior to deriving filtered colors. However, it does not scan larger edged areas or refine potential edge pixels. Following from these earlier attempts, a new approach based on digital thick-edge thinning is studied and presented in this paper. This approach begins with the premise that the simplest way of achieving an overlay image is via compositing – but, in this regard, the question becomes what kind of image is to be composited or blended for antialiasing purposes? A practical answer to this question is very important if the concern is the minimization or removal of the visual aliasing artefacts via simultaneous elimination of blurring artefacts in order to increase the sharpness (of specific salient features) of the processed images.

Figure 1: (a) shows an aliased image of the size 256 × 256; (b) shows an antialiased image of the size 256x256 using the Snowbound Software.

Therefore, this paper takes care to address these issues extensively throughout the second and third sections, before moving on to discuss compositing in detail. Compositing is performed between the image with thinned edges (i.e. after the edge-matting) and the original image or the image with aliased artefacts. It is important to note that in order to prevent the aliasing artefacts from occurring it is also necessary to review the need for sampling continuous signals as well as isotropic usage of regular grid of pixels, but this is necessarily beyond the scope of this paper. Figure 1 shows two images, the aliased or original image (Figure 1 (a)) and the image antialiased by a Snowbound Software's Virtual Viewer Alias/Anti-alias tool (Figure 1 (b)); in this paper, this is referred to as a traditional anti-aliasing technique. As it can be seen in Figure 1, the shapes of the boundaries or edges of Figure 1 (a) are harsh; this perception of harshness (by human eyes) is believed or inferred to be falsely represented or aliased. In the same figure, Figure 1 (b) will appear less harsh or smoothed at a lower resolution (i.e. the resolution lower that 256 × 256 shown in Figure 1(b)) - compared to Figure 1 (a). Readers can use their computer's photo viewer to change the Figure 1 (a) and Figure 1 (b) image resolutions by zooming out up to 10% to see how the smoothness perceived in both cases are different or simply see how at 10% of the Figure 1 (a) there are still image boundaries with jagged parts, or how at 10%, the Figure 1 (b) therefore looks smoother than Figure 1 (a)). This is the simplest and most rapid efficiency assessment adopted for the experimental part of this paper. By considering the blurriness of Figure 1 (b), it becomes apparent that surrounding stair steps or jagged edges with intermediate shades of gray is not the most promising way to achieve visual salience of an image without further introducing blurriness in the rendered image. A combination of strategies that involve refining the actual sampled image signals via detecting the strongest discontinuity zone (or area) first, and then applying the thinning technique to achieve a less harsh look, before producing the overlay image – have been developed and applied, and include further blurriness removal advantages. The preliminary outcomes of this technique are presented and discussed in this paper. Because the history and literature around antialiasing, visual saliency, edge detection and thinning techniques and algorithms are already well-known and widely documented, they will not be replicated in this paper.

This paper is organized as follows: Part II introduces ACO edge detection; Part III presents the thinning and algorithm adopted; Part IV presents the proposed TAA, Part V presents experimental discussions, Part VI presents further discussions and Part VII offers a conclusion.

2 ACO EDGE DETECTION

Detecting thick edges – or large image's zones with stronger discontinuities – is crucial in the strategy of edge-matting. The tool adopted for this process is the ACO (Dorigo and Stutzle, 2004; Baterina and Oppus, 2010; Rezaee, 2008; Lu and Chen, 2008). The traditional edge detection methods – such as Sobel or Canny edge filter/detector – act like step detectors in their operations, and consequently are unable to adequately cover large enough areas of discontinuities (Canny, 1986; Gonzalez and Woods, 2008). Unlike the ACO edge detector results shown in Figure 3 (a), after applying these step detectors to the image in Figure 1 (a), the obtained results (shown in Figure 2 (a) and Figure 2 (b)) are evidence that Sobel and Canny edge detectors cover a smaller area of image discontinuities. As it can be observed, the boundaries of Figure 2 (a) do not have a discernable difference in shape compared to Figure 1 (a). Under such circumstances, there is no need for thinning – given that the single-pixel-width ridgeline has been achieved – but without considering the influence of potentially larger zones of image discontinuities surrounding or adjacent to the step edge strip detected. Also, as shown in Figure 2 (b), the outcome of the Canny step detector technique is unsatisfactory when applied to larger zones of image feature discontinuities – which are crucial for achieving antialiasing of images without introducing new pixels or using non-artificial pixel values. Figure 3 (a) shows the results obtained after detecting edges of Figure 1 (a) using the ACO edge detection method.

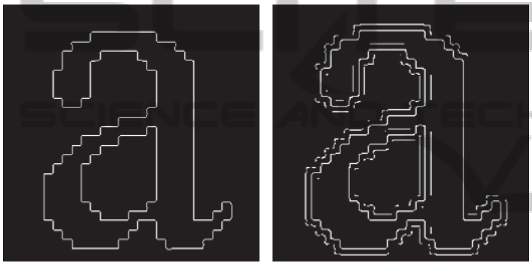

Figure 2: (a) edge detection using Sobel and (b) edge detection using Canny method.

As it can be observed, the boundaries are thicker and visibly different in shape than in Figure 2 (a) and Figure 2 (b). This demonstrates the utility of the ACO edge detection method for the purpose of studying larger zones of image discontinuities (without the need for redefining the edge or the algorithmic parameters). Following from this, the question becomes how to reduce the large zones covered by ACO to a ridgeline of single pixel width? It is important to note that the objective is to achieve an average shape that uses only the original pixels

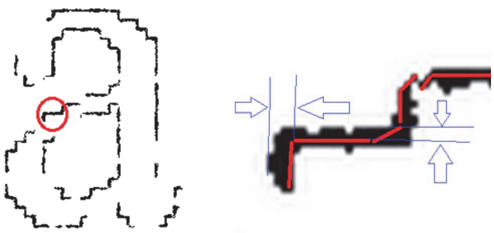

Figure 3: (a) shows the edges detected by ACO; (b) shows a zoomed-in portion of the detected edge with a red line showing the approximation of the average shape; and the estimate location of the thinned edge ridgeline.

and reduces the visible harshness possibly induced by the aliasing artefacts. It is from this that the need for application of the thinning concept emerges. In order to reduce detected edges to a single pixel width size, it is critical to thin the ACO detected edges. As high computational cost is another issue in developing digital image processors, a fast thinning algorithm has been used in this process.

3 THINNING

A process of reducing an object in a digital image to the minimum size necessary for machine recognition of that object is referred to as thinning (Abe, Mizutani and Wang, 1994; Wang and Zhang, 1989; Zhang and Suen, 1984). This consists of converting binary shapes obtained from edge/boundary detection to one-pixel width lines. This is based on deleting pixels iteratively inside the shape to shrink it without shortening it or breaking it apart. A fast parallel algorithm based on the work done by Zhang and Suen in (Zhang and Suen, 1984) has been used for this purpose and the result is shown in Figure 4. The matlab source code of this algorithm is available at the website of matworks. As can be observed in Figure 4, the right-angledness of the stair-steps' vertices has been changed.

Figure 4: A portion of the edge thinned using fast parallel algorithm for thinning developed by Zhang and Suen, in 1984.

The new image appears much more near-realistic and with reduced harshness, an outcome achieved via inducing these changes to every vertex. Therefore, thinning the image edges detected by ACO is the final step in the edge-matting strategy and is also the most computationally expensive part of the TAA.

4 TAA

A process of extracting the object from the original image is often referred to as matting, while the process of inserting the object into another image is called compositing (Szeliski, 2010). In this paper, the word matting has been used in connection with the process of extracting the thinnest ridgelines and therefore named 'edge-matting'. A corresponding and specifically extracted ridgeline or shape has been referred to as 'edge-matted'. In Figure 5, the light blue color indicates the thickest ridgeline detected by the ACO – or 'thick-ridgeline' – while a dark blue color (crossing into the center of the thick-ridgeline) indicates the potential final location of the 'edge-matted' with reference to this thick-ridgeline.

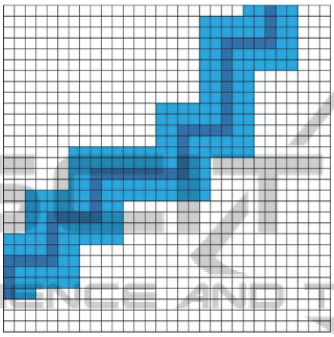

Figure 5: Example of the thick-ridgeline and edge-matted in light and dark blue, respectively – with similar thick-ridgelines boundaries.

As can be observed, the thick–ridgeline shape is similar to the estimate edge-matted shape. The edge-matted shape is inferred according to the concept of iterative deleting of pixels side-by-side adopted by many image thinning and skeletonization algorithms. Referring to the results shown in Figure 2 (a) and Figure 2 (b) and the inferred result in Figure 5, either in light or dark blue, there is a strong similarity in shapes. This indicates that when the thick-ridgelines boundaries are similar in shape (or evolve at the same frequency) the resulting edge-matted will have exactly the same shape, thus

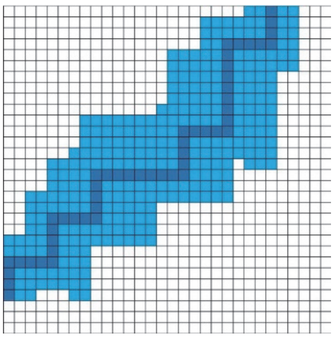

Figure 6: Example of the thick-ridgeline and 'copied' edge-matted in light and dark blue, respectively – with dissimilar thick-ridgelines boundaries.

making it difficult to antialias using the TAA approach. However, when the thick–ridgelines boundaries (see Figure 6) are dissimilar in shape, consequently the inferred edge-matted will also be dissimilar to those thick-ridgelines shapes (see Figure 7). This phenomenon demonstrates the importance of not ignoring larger image discontinuity zones, especially when the concern is to achieve the most reasonable shape of edge-matted.

It should be noted that the ACO edge detector was chosen due to its potential for overcoming the limitations of traditional edge detection methods and its ability to extract the thickest ridgeline in a more efficient and reliable way (Baterina and Oppus, 2010).

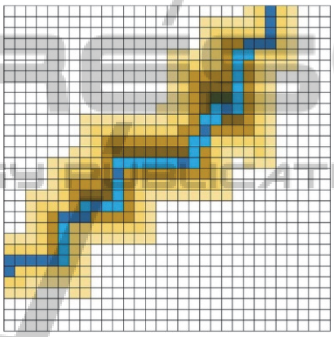

Figure 7: Example showing approximately how iterative deletion of spurious pixels is performed. The blueish lines represent the edge-matted.

Figure 7 uses various colors to show approximately how the iterative deletion of spurious pixels is performed (from light yellow color to dark yellow color and then to a mix of light and dark blue colors) – and provides evidence that, under such circumstances, the final inferred edge-matted shape is the average shape of (and is thereby dependent upon) the shapes of the thick-ridgeline boundaries. This dependence is outstanding finding of this paper and could serve to initiate a revolutionary step forward in developing further applications. Figure 8 shows the inferred edge-matted according to the shape of the thick-ridgeline boundaries. In Figure 8, the shape of edge-matted is different from that in Figure 5 and with less harsh boundaries compared to the thick-ridgelines shapes – while in Figure 5, edge-matted and thick-ridgelines have no dissimilarities in shape. In addition, comparing the edge-matted shape to that of any of the boundaries of the thick-edge, the former looks 'smoothly' much better than the latter. As these ideal cases demonstrate, the result is that the edge-matted inferred in Figure 5 and Figure 8 have different outcomes although the thinning processes (or iteratively deletion performed side–by–side) were the same. For antialiasing purposes, the final compositing strategy has been divided into two important steps, namely alpha compositing and thresholding.

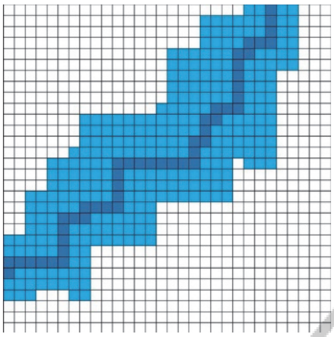

Figure 8: Example of the resulting edge-matted.

Alpha compositing was adopted as the easiest operation of combining a non-antialiasing image with an edge-matting image in order to create a new image or appearance. Thresholding has been introduced and applied to the overlay image in order to extract (or output) the refined composite image. Referring to compositing algebra, the over-operator has been adopted - as it is suitable for placing an image on the top of the other (or simply blending two images) – and accomplished by applying Equation (1).

$$C_o = C_a\alpha_a + C_b\alpha_b(1-\alpha_a) \quad (1)$$

where C_o is the result of the operation or composite image, C_a is the colour of the pixel in edge-matted image, C_b is the colour of the pixel in the original image – and, α_a and α_b are the alpha of the pixels in the edge-matted and original image, respectively (Porter and Duff, 1984). Figure 9 recapitulates the TAA and briefly illustrates the edge-matting as a combination of edge detection and thinning. The compositing is presented as the final strategic step in which the overlay image is formed before applying the threshold and outputting the results. This relative simplicity makes the TAA approach interesting to apply for antialiasing purposes as well as the further development of robust antialiasing techniques. In this approach, the original image is input and its edges are simultaneously detected using the algorithm developed by Tian, Yu and Xie in (Tian,Yu and Xie, 2008) and whose matlab source code is available on the website of matworks. This is followed by the edge thinning process using a fast parallel algorithm for thinning developed by Zhang and Suen (Zhang and Suen, 1984).

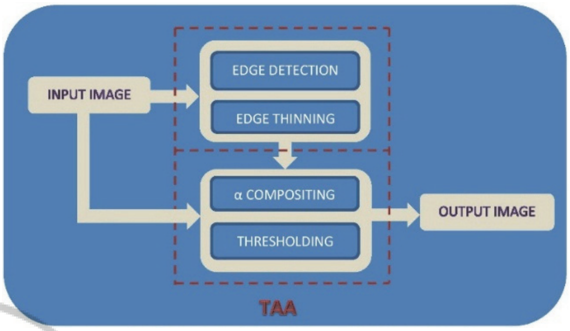

Figure 9: Recapitulation of the TAA.

After, the resulting edge-matted image is blended with the original image thus forming a composite image. It should be noted that in Matlab a similar process of blending two images can be achieved with the help of the *imfuse* function. The obtained blended images (see Figure 13) are then refined by applying a threshold to the results of the compositing operation, according to the Equation (2) and Equation (3).

$$C_o(C_o > 124) = 255 \quad (2)$$

$$C_o(C_o \leq 124) = 0 \quad (3)$$

The outcomes of preliminary experiments and those of the Snowbound Software's Virtual Viewer Alias/Anti-alias tool are presented and discussed in the following part.

5 EXPERIMENTAL RESULTS AND DISCUSSIONS

The TAA preliminary results are displayed at a higher resolution in Figure 12 to clearly show the visual difference output between the new TAA and old concept based antialiasing. Figure 10 (a) shows an original or non-anti-aliasing image with very harsh edges. The thinnest skeleton (or edge-matted) is shown in Figure 10 (b) with less harsh edges compared to the former. It is important to note that what can be seen as disconnections of edges is not a flaw specific to the TAA method, and additionally is not a concern of this experiment, since the TAA output is an overlay image of two images (see Figure 13 (a) and Figure 13 (b)).

In addition, it is evident that it is not necessary for all the edge-lines/boundaries of a

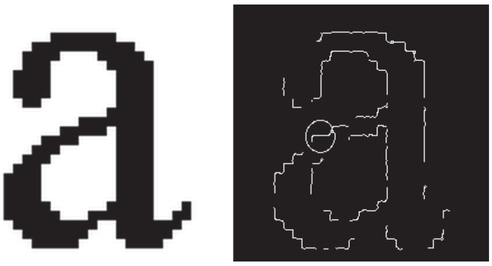

Figure 10 : (a) non-anti-aliasing and (b) thinned edge image.

non-antialiasing image to undergo antialiasing (see and compare Figure 11 (a) and Figure 11 (b)).

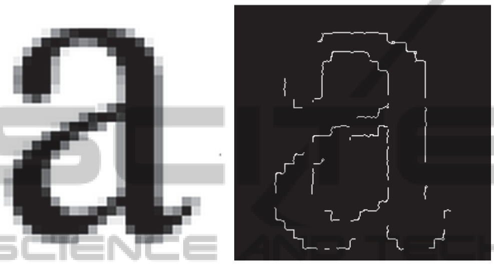

Figure 11 : (a) a traditionally anti-aliased and (b) edge-matted output images.

In this Figure 11 (a), it can be seen that the traditional antialiasing technique applied along the vertical and horizontal lines of the non-antialiasing image were no longer necessary. This is another advantage of the TAA technique as it means that TAA does not use more computational effort to calculate average pixel values to pad along such lines. In the experimental results, shown in Figure 12 (c) and Figure 12 (d), a blurring layer of the calculated average pixel values is added to the non-antialiasing image edge boundaries. Until it would be presented at a lower resolution, for example, at 64 x 64, such images would look smoother than images at 256 × 256 resolution – thereby tricking the human eyes through the appearance of the smoothed boundaries of edges. A disadvantage of this artificial-layer-adding based method is its dependence on the images for its operation. For example, see the results in Figure 12 (c) and Figure 12 (d). At the actual resolution, it can be observed that they do not demonstrate the same level of artefacts though processed by the same traditional antialiasing method. In case of TAA shown in Figure 12 (e) and Figure 12 (f), the boundaries of the antialiased image look quite different – especially, on the right-angled or vertices. In addition, the sharpness is increased with TAA as compared to the traditional technique which remains technically deficient. Further locally objective assessments using any image in various photo viewers or editing software programmes also display better performance of the proposed TAA compared to the Snowbound Software case presented –especially when the antialiased image size is reduced to 10% of its original size. Technically reducing an image to a particular percentage does not mean that should be the size at which that image should remain for further processing.

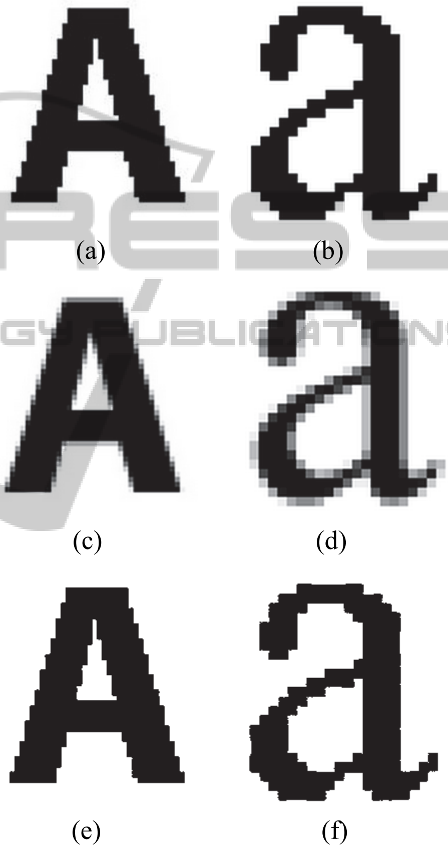

Figure 12: (a) and (b) - two non-antialiasing images, (c) and (d) - two traditionally-antialiased images, (e) and (f) - two TAA antialiased images.

This is usually done to increase the size of an aliased image to a higher resolution for antialiasing activities, and afterwards reduce it to a lower resolution (matching the desired size), as a method of concealing the visual artefacts induced and thus creating a visually pleasing perception of that output for a low resolution image. Further studies on this non-artificial pixels-based antialiasing concept are necessary given the increasing interest in achieving near-realism as well as preventing aliasing artefacts from occurring in images processed digitally. The following section further discusses the potential of the TAA approach and offers insights into the process of the *imfuse* function when it is applied between the edge-matted and non-antialiasing images (see Figure 13 (a) and Figure 13 (b)) just before the threshold is applied.

6 FURTHER DISCUSSIONS AND APPLICATIONS

There is the potential to further develop the proposed approach and extend its application to refining other categories of digital images which also require visual saliency, as well as many other cases in which effort to view the object's near-realism is needed.

Figure 13: (a) and (b) - overlay images obtained from the edge-matted and original images, in each case. The false color has been enabled by the 'falsecolor' command to clearly show the boundaries between the edge-matted and original images of the overlay images.

It should be noted that the proposed approach enables the exclusive use of original pixels information which is not possible with traditional antialiasing methods – and is particularly important when the blurriness artefacts are undesired. Figure 13 (a) and Figure 13 (b) show the overlay images of each of the two processed images (of the capital and small 'A' letter) – obtained by using only the *imfuse* command function with no threshold applied. However, the corresponding overlay images do not automatically produce the desired visual effects and therefore a threshold has been introduced and applied to ensure that the widest visible rift is output or obtained. Such a threshold has been applied to the results of the compositing operation Equation 1 according to the Equation (2) and Equation (3) commands. The average matlab lines execution/processing time required to output the composite image ($ACO_{average\ (four\ kernel\ functions\ with\ 30\ steps)}$ = 24.51856875 sec. or $ACO_{average\ (one\ kernel\ function\ with\ 30\ steps)}$ = 9.989709, thinning = 13.293638 sec. and compositing = 2.085868 sec.) as well as other computational complexity related issues will be discussed in the optimization effort in future papers.

7 CONCLUSIONS

The results and arguments presented in this paper concentrated on increasing or improving the visual saliency of an image processed digitally. The undesired phenomenon associated with digital image processing – aliasing – results from an unfaithful reproduction of a continuous signal or sample by a digital processor/software. Effort to tackle this phenomenon has led to the development of an antialiasing method that exclusively uses non-artificial pixels in its operations to avoid inducing other phenomena such as blurriness or haloness, (which also reduce the visual saliency). This TAA approach comprises of edge-matting and compositing strategic steps – which have been thoroughly examined and experimental results have been provided in the previous sections of this paper. The key differences between the two strategies have been explored and the preliminary experimental results demonstrate the lack of edge vertices errors via avoiding the averaged pixels' padding process and by achieving high quality edge refinement via thinning. The quality performance measure selected locally served as an 'objective assessment' as it is possible to use any photo viewer or editing software locally due to the simplicity of the latter. The sharpness achieved by TAA is very promising but the smoothness remained debatable to some extent compared to the other case studies mentioned. Further investigations and improvements are expected in the future research works, especially in terms of computational complexity and accuracy as well as comparative evaluations using more complicated images for testing more objects.

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