# A Haar Wavelet-based Multi-resolution Representation Method of Time Series Data

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Keywords: Dimensionality Reduction Techniques, Haar Wavelets, Multi-resolution, Similarity Search, Time Series Data Mining.

Abstract: Similarity search of time series can be efficiently handled through a multi-resolution representation scheme which offers the possibility to use pre-computed distances that are calculated and stored at indexing time and then utilized at query time together with filters in the form of exclusion conditions which speed up the search. In this paper we introduce a new multi-resolution representation and search framework of time series. Compared with our previous multi-resolution methods which use first degree polynomials to reduce the dimensionality of the time series at different resolution levels, the novelty of this work is that it applies Haar wavelets to represent the time series. This representation is particularly adapted to our multi-resolution approach as discrete wavelet transforms have the ability of reflecting the local and global information content at every resolution level thus enhancing the performance of the similarity search algorithm, which is what we have shown in this paper through extensive experiments on different datasets.

## 1 INTRODUCTION

A *time series* is an ordered collection of observations over a period of time. Time series data arises in many applications including medical, financial, and engineering. For this reason, time series data mining has received attention over the last years.

Time series data mining handles several tasks, the most important of which are query-by-content, clustering, and classification. Executing these tasks requires performing another fundamental task in data mining which is the *similarity search*.

A similarity search problem consists of a database $D$, a query or a pattern $q$, which does not necessarily belong to $D$, and a constraint that determines the extent of proximity that the data objects should satisfy to qualify as answers to that query.

The time series similarity search problem has many applications in computer science. Similarity between two time series can be depicted using a similarity measure, which is usually a costly operation compared with other tasks such as CPU time or even I/O time.

Direct *sequential scanning* compares every single time series in $D$ against $q$ to answer this query. Obviously this is not an efficient approach given that modern time series databases are usually very large.

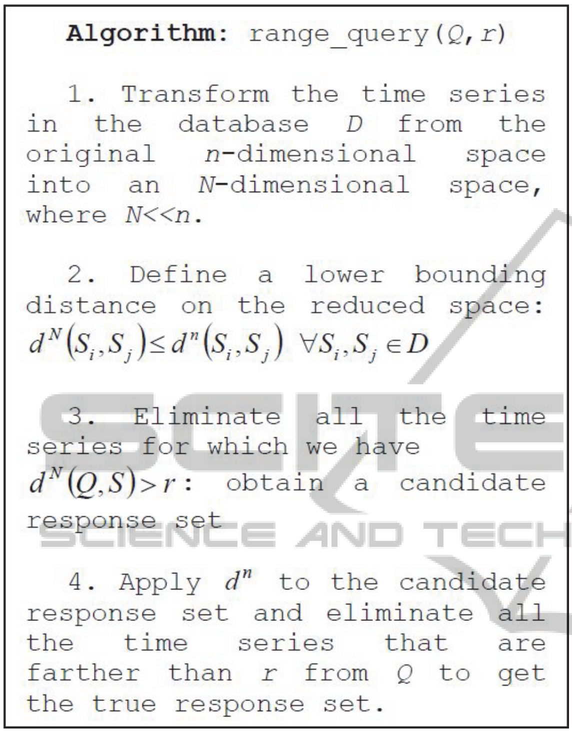
The main framework for reducing the computational cost of the similarity search problem is the *Generic Multimedia Indexing* (GEMINI) algorithm (Faloutsos *et al*, 1994). GEMINI reduces the dimensionality of the time series by converting them from a point in an $n$-dimensional space into a point in an $N$-dimensional space, where $N \ll n$. If the similarity measure defined on the reduced space is a lower bound of the original similarity measure then the similarity search returns no false dismissals in this case. A post-processing sequential scan on the candidate response set is performed to filter out all the false alarms and return the final response set. Figure 1 illustrates the GEMINI algorithm.

*Dimensionality Reduction Techniques*, also known as *Representation Methods*, follow the GEMININ framework to find a faster solution to the similarity search problem in time series databases.

This is achieved by mapping the time series to lower dimensional spaces, thus reducing their dimensionality, and then processing the query in

Muhammad Fuad M..
A Haar Wavelet-based Multi-resolution Representation Method of Time Series Data.
DOI: 10.5220/0005307006200626
In *Proceedings of the International Conference on Agents and Artificial Intelligence* (ICAART-2015), pages 620-626
ISBN: 978-989-758-074-1

those reduced spaces. The main objective of dimensionality reduction is to reveal data structure which is hard to obtain from a high-dimensional space (Yang, 2010).



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Algorithm: range_query(Q,r)

  1. Transform the time series
in the database D from the
original n-dimensional space
into an N-dimensional space,
where N<<n.

  2. Define a lower bounding
distance on the reduced space:
$d^N(S_i,S_j) \le d^n(S_i,S_j) \ \forall S_i,S_j \in D$

  3. Eliminate all the time
series for which we have
$d^N(Q,S) > r$: obtain a candidate
response set

  4. Apply $d^n$ to the candidate
response set and eliminate all
the time series that are
farther than r from Q to get
the true response set.
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Figure 1: The GEMINI algorithm for range queries.

Several dimensionality reduction techniques have been suggested in the literature, of those we mention: *Piecewise Aggregate Approximation* (PAA) (Keogh et al., 2000) and (Yi and Faloutsos, 2000), *Piecewise Linear Approximation* (PLA) (Morinaka et al., 2001), and *Adaptive Piecewise Constant Approximation* (APCA) (Keogh et al., 2001).

The problem with all these dimensionality reduction techniques is that they use a "one-resolution" approach. The dimension of the reduced space is decided at indexing time and the performance at query time depends completely on the choice made at indexing time. But in practice we do not necessarily know a priori the optimal dimension of the reduced space.

This was the motivation behind our multi-resolution approach which offers more control on the parameters that determine the effectiveness and efficiency of the dimensionality reduction methods. The basis of these multi-resolution methods is to map the time series to multiple spaces instead of one. In (Muhammad Fuad and Marteau, 2010b) we presented *Weak-MIR*: a multi-resolution indexing and retrieval method of time series. Weak-MIR is a standalone method that uses two filters to exclude non-qualifying time series. Later in (Muhammad Fuad and Marteau, 2010c) we introduced *MIR-X* which associates the multi-resolution approach with another dimensionality reduction technique. Our last multi-resolution method *Tight-MIR* was presented in (Muhammad Fuad and Marteau, 2010a). Tight-MIR has the advantages of the two previously mentioned methods. All these versions were validated through extensive experiments.

In this paper we introduce a new multi-resolution method of time series data which uses the *Discrete Wavelet Transform* (DWT), namely the Haar Wavelets, in conjunction with the multi-resolution approach. This combination boosts the performance of the multi-resolution approach.

In the following we first present the related background in Section 2. In Section 3, we introduce the new method which we validate in Section 4. Finally, the concluding discussion is presented in Section 5.

## 2 BACKGROUND

Multi-representation approaches store data at different scales called *resolution levels*. The principle of this representation is that a representation of a higher resolution contains all the data of the lower resolutions (Sun and Zhou, 2005). Multi-resolution methods are widely used in multimedia databases. In (Figueras et al., 2002) a multi-resolution *Matching Pursuit* is used to decompose images. Multi-resolution is also used for a color reduction algorithm in (Ramella and Sanniti di Baja, 2010). In (Vogiatzis and Tsapatsoulis, 2006) the authors use multi-resolution schemes to estimate missing values for DNA micro-arrays.

Multi-resolution methods have also been exploited in time series information retrieval and data mining. In (Bergeron and Foulks, 2006) a visualization application for very large multidimensional time series datasets is developed. The proposed data model supports multiple integrated spatial and temporal resolutions of the original data. Using multi-resolution techniques to effectively visualize large time series is also applied in (Hao et al., 2007) where the proposed framework uses multiple resolution levels. In (Castro and Azevedo, 2010) the authors propose a method based on the multi-resolution property of *i*SAX (Shieh and Keogh, 2008), (Shieh and Keogh, 2009) to derive motifs at different resolutions. In (Lin et al., 2005) the authors propose a multi-resolution PAA (Keogh *et al*, 2000), (Yi and Faloutsos, 2000); a well-known time series dimensionality reduction technique, to achieve an algorithm for iterative clustering. This clustering process is sped up by examining the time series at increasingly higher resolution levels of the PAA.

In (Vlachos et al., 2003) and (Lin et al., 2007) the authors propose a time series *k*-means clustering algorithm based on the multi-resolution property of wavelets. In (Megalooikonomou *et al*, 2005) and (Wang *et al*, 2010) a method of multi resolution representation of time series is presented.

In (Muhammad Fuad and Marteau, 2010b) we presented the *Multi-resolution Indexing and Retrieval Algorithm* (Weak-MIR). Weak-MIR involves a multi-resolution representation of time series. The indexing system stores different numbers of pre-computed distances, corresponding to the number of resolution levels. Lower resolution levels have lower dimensions, so distance computations at these levels are less costly than higher resolution levels where dimensions are higher, so distance evaluations are more expensive. But the computational complexity at any level is always less than that of sequential scanning because even at the highest level the dimension is still lower than that of the original space which is used in sequential scanning. The search algorithm of Weak-MIR starts at the lowest resolution level and tries to exclude the time series, which are not answers to the query, at that level where the distances are not costly to calculate, and the algorithm does not access a higher level until all the pre-computed distances of the lower level have been exploited.

Later in (Muhammad Fuad and Marteau, 2010c) we introduced another version of the multi-resolution method called MIR-X. MIR-X combines a representation method with a multi-resolution one, so we have two representations of each segment of the time series. We showed in (Muhammad Fuad and Marteau, 2010c) how MIR-X can boost the performance of Weak-MIR. MIR-X uses one of the two filters that Weak-MIR uses together with the lower-dimensional distance of a time series dimensionality reduction technique.

In (Muhammad Fuad and Marteau, 2010a) we presented Tight-MIR which is an improved multi-resolution indexing and retrieval algorithm. The principle of Tight-MIR is based on the remark that the two filters used in Weak-MIR can be applied separately, so the second filter in Tight-MIR is applied by directly accessing the raw data in the original space using a number of points that corresponds to the dimension of the reduced space at that resolution level. Tight-MIR has the advantages of both Weak-MIR and MIR-X in that it is a standalone method, like Weak-MIR, yet it has the same competitive performance of MIR-X. This fact has been shown through extensive experiments.

## 3 THE HAAR WAVELET-BASED MULTI-RESOLUTION METHOD (H-MIR)

Despite the improvement that Weak-MIR, MIR-X and Tight-MIR offer, they all share a drawback that hinders their performance; the dimensionality reduction technique they use, which is first degree polynomials linking the two end points of each segment of the time series, is too basic. A more sophisticated dimensionality reduction technique that better reflects the local and global information content of the whole time series at every resolution level will give better results.

Of all the dimensionality reduction techniques known in the literature one is particularly adapted for this purpose. It is Discrete Wavelet Transform

### 3.1 Discrete Wavelet Transform (DWT)

Wavelets are mathematical tools for hierarchically decomposing functions. Regardless of whether the function of interest is an image, a curve, or a surface, wavelets offer an elegant technique for representing the levels of details present (Stollnitz et al., 1995). Wavelets have successfully been used in many fields of computer science such as image compression (DeVore et al., 1992), image querying and many others. DWT has also been used in time series information retrieval as a dimensionality reduction technique (Chan and Wai-chee Fu, 1999), (Popivanov and Miller, 2002), (Wu et al., 2000). The advantage that DWT has over other methods in indexing time series data is that DWT is a multi-resolution representation method and it can represent local information in addition to global information.

Table 1: Example of the Haar wavelet decomposition.

| Resolution | Averages | DTW Coefficients |
|---|---|---|
| 4 | [8,4,3,5] | |
| 2 | [6,4] | [2,-1] |
| 1 | [5] | [1] |

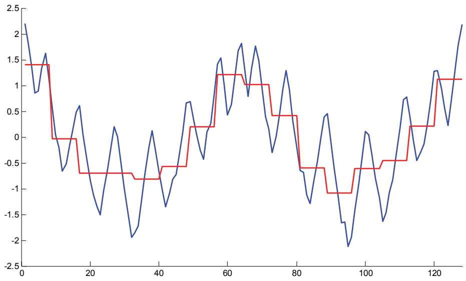

Figure 2: DWT representation of a time series.

*Haar wavelets* are the simplest form of wavelets. Haar wavelet transform is a series of averaging and differentiating operations. To get an idea of how 1-dimensional Haar wavelets work, let us consider the following 4-dimensional time series: $s = [8,4,3,5]$. By taking the average of each two successive values we get the following 2-dimenisonal time series: $s' = [6,4]$. Recursively repeating this process we get the full decomposition of $s$ as shown in the Table 1.

So the wavelet transform of $s$ is $[5,1,2,-1]$.

The principal idea behind using DWT as a dimensionality reduction technique is that a time series can uniquely be represented by a wavelet transform, but by keeping only the first *N* coefficients we can reduce the dimensionality and keep much of the information that is in the original time series. For instance, Figure 2 shows the DWT decomposition at level 7 of a 128-dimension time series.

A lower bounding distance to the Euclidean distance was presented in (Chan and Wai-chee Fu, 1999) and it was proven that this lower bound guarantees no false alarms. It is important to mention that DWT requires that the length of the time series be a power of 2.

## 3.2 The Proposed H-MIR Algorithm

The basis of our new H-MIR representation method is as follows: let $S$ be the original $n$-dimensional space where the time series are embedded. At each resolution level *k* each time series is represented by DWT (Haar wavelets) keeping the first $2^k$ coefficients. We refer to this reduced space by $R^{(k)}$. The distance between this DWT representation and the time series is minimal thus this representation is the best approximation at level *k*. The image of all the points of the time series on DWT is an $n$-dimensional vector which we call the *image vector* and denote by $\bar{s}^{(k)}$. The DWT representation at every resolution level is denoted by $s^{R(k)}$. We define two distances, the first is $d^n$ : an *n*-dimensional distance metric (so it is the distance between two time series in $S$, or the distance between a time series and its image vector). The second distance denoted by $d^{R(k)}$ is the distance between two DWT representations of two time series at level *k*. As mentioned in Section 3.1, this distance is proven to lower bound the Euclidean distance.

The principle of H-MIR is to speed up the search by establishing exclusion conditions that filter out non-qualifying time series using pre-computed distances.

Given a query $(q,\varepsilon)$, let $\bar{s}^{(k)}$, $\bar{q}^{(k)}$ be the image vectors of $s$, $q$, respectively, on their DTW representation at level *k*. Given that $d^n$ is metric and by applying the triangle inequality we get:

$$d^n\left(\bar{q}^{(k)}, s\right) \le d^n(q,s) + d^n\left(q,\bar{q}^{(k)}\right) \quad \forall s \in S \qquad (1)$$

The range query can thus be expressed as:

$$d^n\left(\bar{q}^{(k)}, s\right) \le \varepsilon + d^n\left(q,\bar{q}^{(k)}\right) \qquad (2)$$

Since the distance between $\bar{s}^{(k)}$ and $s$ at level $k$ is minimal we get:

$$d^n\left(\bar{q}^{(k)}, s\right) \ge d^n\left(s,\bar{s}^{(k)}\right) \qquad (3)$$

So equation (2) can be written as:

$$d^n\left(s,\bar{s}^{(k)}\right) \le \varepsilon + d^n\left(q,\bar{q}^{(k)}\right) \qquad (4)$$

So all the time series that satisfy:

$$d^n\left(s,\bar{s}^{(k)}\right) > \varepsilon + d^n\left(q,\bar{q}^{(k)}\right) \qquad (5)$$

are non-qualifying and can be safely excluded.

In a similar manner, and by applying the triangle inequality again, we get:

$$d^n\left(q,\bar{q}^{(k)}\right) \le \varepsilon + d^n\left(s,\bar{s}^{(k)}\right) \qquad (6)$$

Equation (6) implies that all the time series that satisfy:

$$d^n\left(q,\bar{q}^{(k)}\right) > \varepsilon + d^n\left(s,\bar{s}^{(k)}\right) \qquad (7)$$

are non-qualifying and can also be excluded.

From equations (5) and (7) we get the first filter of H-MIR which is:

$$\left| d^n\left(q,\bar{q}^{(k)}\right) - d^n\left(s,\bar{s}^{(k)}\right) \right| > \varepsilon \qquad (8)$$

In addition to the above filter H-MIR, and at

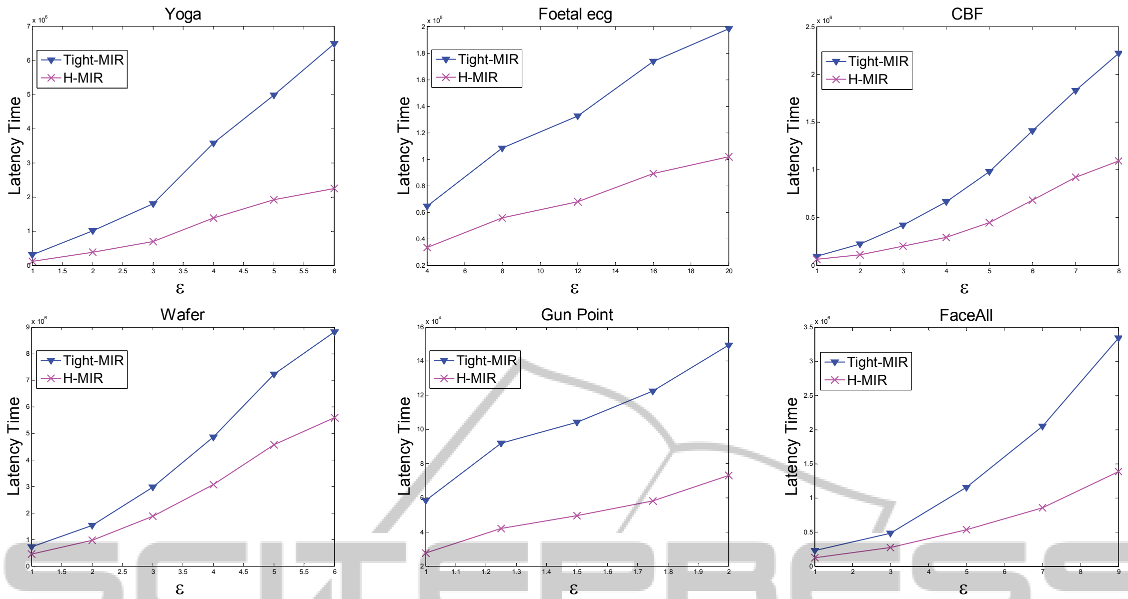

Figure 3: Comparison of the latency time between MIR-Tight and H-MIR on datasets (Foetal ecg), (CBF), (Yoga), (Wafer), (GunPoint) and (FaceAll).

each resolution level *k*, uses the following exclusion condition:

$$d^{R(k)}\left(q^{R(k)},s^{R(k)}\right)>\varepsilon \qquad (9)$$

Equation (9) is the second filter of H-MIR.

**Indexing Time:** At each resolution level *k* the time series are mapped to a $2^k$ –dimension space using a DWT transform and keeping the first $2^k$ coefficients. We compute and store all the distances $d^n\left(s,\bar{s}^{(k)}\right)$ , $\forall s \in S$ .

**Query Time:** The query is also mapped to a $2^k$ – dimension space using a DWT transform and keeping the first $2^k$ coefficients.

The first filter has a much lower computational cost than the second filter as it does not include any online distance evaluation. The computational cost of the second filter also increases as the resolution level gets higher.

After applying the first filter to all time series at all resolution levels, we apply the second filter.

This filter is applied starting the lowest level first before moving to the higher level because this filter requires computing $d^R$ whose computational cost increases as the resolution level gets higher, but the pruning power of the second filter also rises as we move to higher resolution levels.

# 4 EXPERIMENTS

In order to evaluate the performance of our new method H-MIR we conducted several similarity search experiments on different time series datasets from different time series repositories (Povinelli), (SISTA's Identification Database), (StatLib - Datasets Archive), (Keogh et al., 2011) using different threshold values. In our experiments we compared H-MIR against Tight-MIR since it was shown in (Muhammad Fuad and Marteau, 2010a) that Tight-MIR outperforms both Weak-MIR and MIR-X.

As mentioned in Section 3.1 DWT is applicable to time series whose lengths are of the power of 2, so when this was not the case for the dataset tested we truncated the time series.

As in (Muhammad Fuad and Marteau, 2010a), the comparison criteria was based on the latency time concept presented in (Schulte et al., 2005) which calculates the number of cycles the processor takes to perform different arithmetic operations (>,+ - ,*,abs, sqrt) to execute the similarity search. This number for each operation is multiplied by the latency time of that operation to get the total latency time of the similarity search. The latency time is 5 cycles for (>, + -), 1 cycle for (abs), 24 cycles for (*), and 209 cycles for (sqrt) (Schulte et al., 2005).

In the first set of experiments we compared H-MIR against Tight-MIR on different datasets of

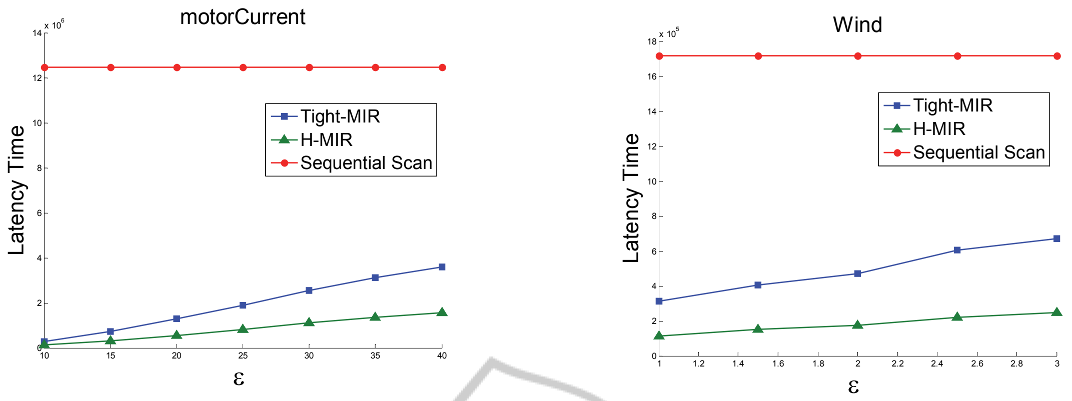

Figure 4: The latency time of H-MIR, Tight-MIR, and sequential scanning on datasets (motoCurrent) and (Wind).

different dimensionalities and sizes and for different threshold values. In Figure 3 we present some of the results obtained.

The results obtained show that H-MIR outperforms Tight-MIR on all these datasets and for all values of the threshold ε.

As in (Muhammad Fuad and Marteau, 2010a), we also tested the stability of H-MIR on a wide range of time series dimension. We report in Figure 4 the results of the similarity search on two time series with very different dimensions, the first is (Wind) whose dimension is 12, and the second is (motoCurrent) whose dimension is 1500. These experiments were conducted for different values of the threshold ε. For comparison, we also report the results obtained by using Tight-MIR and sequential scanning. This latter method represents the baseline performance.

As we can see, H-MIR has a stable performance on time series of different dimensions, which is the same advantage that Tight-MIR has.

## 5 CONCLUSIONS

We presented in this paper a new representation method of time series data which, in contrast to other time series representation methods, uses multiple spaces to represent the data. We also proposed a framework for performing the similarity search using the new method. This framework reduces the number of online distance evaluations by using pre-computed distances and exclusion conditions. The particularity of the new method over other multi-resolution representation methods is that the new method uses a dimensionality reduction technique, DWT, which is especially adapted for our method owing to its multi-resolution nature. We validated our new method through experiments on datasets from different time series archives.

We believe our new method can be extended to handle other data types, especially to process image querying where the concept of multi-resolution levels is pertinent and where DWT is widely used.

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