Competency Mining in Large Data Sets
Preparing Large Scale Investigations in Computer Science Education

Peter Hubwieser and Andreas Mühling

TUM School of Education, Technische Universität München, Arcisstr. 21, 80333 München, Germany

Keywords: Large Scale Studies, Competencies, Item Response Theory, Rasch Model, Computational Thinking.

Abstract: In preparation of large scale surveys on computer science competencies, we are developing proper competency models and evaluation methodologies, aiming to define competencies by sets of exiting questions that are testing congruent abilities. For this purpose, we have to look for sets of test questions that are measuring joint psychometric constructs (competencies) according to the responses of the test persons. We have developed a methodology for this goal by applying latent trait analysis on all combinations of questions of a certain test. After identifying suitable sets of questions, we test the fit of the mono-parametric Rasch Model and evaluate the distribution of person parameters. As a test bed for first feasibility studies, we have utilized the large scale Bebras Contest in Germany 2009. The results show that this methodology works and might result one day in a set of empirically founded competencies in the field of Computational Thinking.

1 INTRODUCTION

Since 2000 the Organisation for Economic Co-operation and Development (OECD) is conducting the well-known international PISA (Programme for International Student Assessment) studies. Most member states of the OECD and a growing number of partner countries are conducting these studies in 3-year cycles. While some of the political implications are under discussion, the scientific community had to acknowledge that the PISA studies follow a sophisticated, well-founded methodology that had a ground-breaking impact on the whole field of empirical educational research.

So far, the focus of PISA has been on mathematics, natural science and language understanding. Yet, in our opinion, a PISA survey of computer science (CS) competencies would advance the research methodologies of Computer Science Education (CSE) in a pioneering way, lifting this field on the level of educational research in traditional subjects. However, this would require considerable prerequisite work. First, we have to agree on a normative grounding of computer science abilities that is commonly accepted. Second, we need a properly defined competency model, derived from this grounding, which would provide a framework for measurements. Some of this work is already done (see e.g. (Magenheim et al. 2010), but substantial research efforts are still required. Finally, we need a test field of sufficiently large scale to develop and explore competency definitions and test methodologies in our subject domain. All in all, this would take several more years before we can even start with the design of large scale investigations according to such a purely sequential strategy.

Consequently, we considered a possible parallelization of these steps. As the definition of competency models according to the usually applied methodology (see e.g. Klieme et al. 2004) is requiring a careful study of literature and many expert interviews, it represents the most time-consuming step. Yet, it is generally accepted in educational research to alternatively define a competency by a set of test items that require exactly this competency to be solved (Schott & Azizi Ghanbari 2009). However, a necessary precondition for such a definition is that such a set of test items is homogenous in the sense that all items are measuring a common psychometric construct, which could be identified with the respecting competency. Therefore, we were looking for tests or examination results in CS that might comprise such homogenous sets of items. Once we have found such sets, we could assume that each of those would represent a certain competency. To measure these competencies, we could simply use sets of questions that are very similar to those who had defined the respective competency. Compared to
the usual methodology, this would be much easier to validate. These sets of test questions could be regarded as a first draft of a large scale survey on computer science competencies that could be utilized to develop and apply proper test and evaluation methodologies.

Unfortunately, quite large data sets are required to assess the homogeneity of item sets, e.g. about 5,000 data sets for a set of 10 questions (Bartholomew et al. 2008). Taking into account the typical rates of non-responses or partial responses in such tests, we assume that we need tests with at least 10,000 participants in order to get workable results. Given the large amount of data and the explorative approach, this is a typical data-mining scenario.

As far as we know, the only event of sufficient large scale in computer science is the annual Bebras online contest (www.bebras.org), see section 3.2. We investigated by explorative statistical methods, if there are subsets of questions among the question sets of a certain Bebras contest that “measure” a common psychometric construct (or competence). For this purpose, we have programmed several R scripts that perform an exhaustive search of the smallest suitable sets of 4 questions and use the results to investigate larger sets.

In this paper, we will describe this methodology, using the German Bebras contest of 2009 as an example. This contest was attended by about 120,000 students. Additionally, we will present some interesting results regarding the difference in the outcomes for certain groups of participants, discriminated by gender, number of team members, and states.

2 BACKGROUND

2.1 Competencies

Stimulated by the detection of the phenomenon of “tacit knowledge” and the upsetting results of the first large scale studies of learning outcomes TIMSS (Trends in International Mathematics and Science Study) and PISA, during the first years of this century the focus of education has shifted from knowledge and learning outcomes towards competencies.

In this paper, we will refer to the well-known definition of competence by Weinert (Weinert 2001), who defined competencies as “the cognitive abilities and skills possessed by or able to be learned by individuals that enable them to solve particular problems, as well as the motivational, volitional and social readiness and capacity to use the solutions successfully and responsibly in variable situations.” Furthermore, he stressed that competencies may be composed of several facets: ability, knowledge, understanding, skills, action, experience, and motivation.

Due to their complex structure, it is apparent that the definition and the measurement of competencies are not an easy matter. According to Klieme et al. (Klieme et al. 2004), “competence
\- can only be assessed and measured in terms of performance.
\- forms the link between knowledge and skills and can be seen as the ability to deal with situations or tasks. Any illustration or operationalization of a competence must therefore relate directly to a concrete situation.”

Additionally, Klieme et al. stress that “Competencies cannot be reflected by or assessed in terms of a single, isolated performance. Rather, the range of situations in which a specific competence takes effect always spans a certain spectrum of performance. Narrow assessments cannot meet the requirements of competency models. The seven facets of competence listed above make it quite clear that competence must be assessed by an array of tasks and tests that do more than simply tap factual knowledge”. Even more, some authors identify the competency with a set of tasks, e.g. (Schott & Azizi Ghanbari 2009) p. 15 (translated by the authors): “A competency consists of certain sets of tasks that can be performed by those who have this competency”.

In the light of these statements, it seems possible to define a competency by a set of tasks.

2.2 Item Response Theory

Most surveys of competencies are currently evaluated according to the Item Response Theory (IRT), which treats the constructs of interest (e.g. competencies) as latent psychometric constructs that can’t be measured directly. Yet, the probability of correct answers depends on those constructs in a certain way:

\[ P(X_{ik} = 1 | \theta_i, \beta_k) = f(\theta_i, \beta_k) \] (1)

\( \theta_i \) is the parameter of person \( i \), representing the manifestation of the psychometric construct, \( \beta_k \) the parameter of Item \( k \), representing its difficulty, and \( f(\theta_i, \beta_k) \) a function that is determined by the psychometric model (e.g. the Rasch Model (RM), see below) that is assumed to fit the observations. In most
cases these parameters have to be estimated by effortful numerical calculations.

Depending on the structure of the psychometric constructs that are to be measured, several different models may be considered, e.g. unidimensional models that cover only one single latent variable or alternatively multidimensional models. One of the simplest and most widely used ones is the basic unidimensional (monofactorial) RM with one parameter (1F1P):

\[
p(X_{ik} = 1 | \theta_p, \beta_i) = \frac{\exp(\theta_i - \beta_k)}{1 + \exp(\theta_p - \beta_k)}
\]  

(2)

The graph of this function looks like the ones displayed in figure 1 for a set of Bebras questions. These graphs are called Item Characteristic Curves (ICCs).

Provided that this model is applicable, some very convenient simplifications can be made. For example, the sum over the scores of all individual items is a sufficient statistics, which means that the (estimated) person parameter depends only on the total number of correct answers of this person. It does not matter, which questions the person had responded to correctly. Yet, this model is applicable only if the ICCs have (at least nearly) the same slope. This slope is represented by an additional Discrimination Parameter \( \delta_k \) in the 2-parametric RM (1F2P):

\[
p(X_{ik} = 1 | \theta_p, \beta_i) = \frac{\exp(\delta_k(\theta_i - \beta_k))}{1 + \exp(\delta_k(\theta_p - \beta_k))}
\]  

(3)

In both cases, three general preconditions have to be met for the application of the RMs:

1) Homogeneity of items: All items are measuring the same psychometric construct.
2) Local stochastic independence: the underlying psychometric construct is the only coupling factor between items.
3) Specific objectivity: for all samples from the population, the item parameters are independent from the specific sample; the same holds for all samples of questions and person parameters.

3 THE CONTEXT

3.1 The PISA Surveys

The PISA studies are of very large scale and investigate how well an educational system prepares children for their adult life regarding certain abilities, for example in mathematics. According to (OECD 2013), the 5th survey in 2012 “assessed the competencies of 15-year-olds in reading, mathematics and science (with a focus on mathematics) in 65 countries and economies. [...] Around 510 000 students between the ages of 15 years 3 months and 16 years 2 months participated in PISA 2012 representing about 28 million 15-year-olds globally. The students took a paper-based test that lasted 2 hours. The tests were a mixture of open-ended and multiple-choice questions that were organized in groups based on a passage setting out a real-life situation.”

The PISA surveys follow a carefully worked out process model (Seidel & Prenzel 2008). The first step is to define the research objectives. In the second step, the framework for the assessment of competencies has to be developed (see above). The next step is test development. For this, proposals for the questions are collected from experts. The questions should test complex abilities that are required to solve real-world problems. Each question may comprise one or more items, which will represent the units of measurement at the end. Exemplary questions can be found at http://pisa-sq.acer.edu.au.

The proposals are evaluated and validated regarding the competencies that are intended to be measured. The tasks that are selected according to certain criteria are translated to all languages of the participating countries. Following this, a field trial is conducted by all participating countries of PISA one year before the main study. The items to be included in the main study are selected according to the results of the field trial. Finally, the main study is carried out identically to the field trial, except for a much larger sample. The complicated research design of PISA is well-founded in theory. It encompasses cross- and longitudinal-sections, which are supported and combined by diagonal sections (Seidel & Prenzel 2008). Following this, open format test items are coded according to the manuals and data are cleaned. The item and person parameter are estimated according to IRT, applying multidimensional, mixed models. To be able to interpret the results more easily, the item difficulties and person estimates are normalized in such a way that the estimates have a mean of 500 and standard deviation of 100.

3.2 The Bebras Contest

The Bebras contest was founded by V. Dagiene, see (Dagiene 2008), who named it according to the Lithuanian word for (Busy) “Beaver”. According to
the founders (Dagiene & Futschek 2008), the Bebras Contest aims to interest children and adolescents in typical problems of computer science and does not require prerequisite knowledge (at least officially). In consequence, the Bebras Contest does not intend to be a test originally. Nevertheless, in absence of other test fields, we decided to investigate this contest and find out what it would measure if it were a test.

Similarly to PISA, the tasks are proposed by the members of an international board of experts. Following this, the tasks are discussed, assessed and selected by this board according to their fitting to the goals of the contest. In contrary to PISA, the tasks are not pre-tested. Exemplary questions are paraphrased in table 3.

The contest started in 2004 in Lithuania with 3470 participants and has grown to 523,319 participants in 21 countries in 2013, which represents a scale very similar to PISA (see section 3).

The German issue of Bebras (called Informatik-Biber, see www.informatik-biber.de) is performed in all German federal states and in all types of secondary schools. It is the largest of all national Bebras contests (206,430 participants in 2013), followed by France (171,932).

In Germany, the contest comprises 18 multiple choice questions for each of the 4 age groups (see table 1). The difficulty of the questions is assessed a priori by the board of experts that selects the questions for the national contests. The students have the choice to take the test alone or together with a partner. The test is performed online. In each age group a different set of questions - in total 18 - has to be answered, out of a pool of 39. Yet, some of the 39 different questions of the contest were presented to several age groups. If the same question is posed to more than one group, a different degree of difficulty can be applied for each group.

Table 1: German Bebras age groups since 2009.

<table>
<thead>
<tr>
<th>Group</th>
<th>Grades</th>
<th>Age approx.</th>
</tr>
</thead>
<tbody>
<tr>
<td>AG1</td>
<td>5-6</td>
<td>10-12</td>
</tr>
<tr>
<td>AG2</td>
<td>7-8</td>
<td>12-14</td>
</tr>
<tr>
<td>AG3</td>
<td>9-10</td>
<td>14-16</td>
</tr>
<tr>
<td>AG4</td>
<td>11-13</td>
<td>16-19</td>
</tr>
</tbody>
</table>

4 THE DATA

To date, we have acquired the German Bebras data of the years 2007-2010, 2011 and 2013. Due to technical reasons, we chose the data of competition No 33 (October 2009) for this feasibility study.

The relational data base of Bebras 2009 was composed of 18 tables. At first the raw data was read, verified and put into suitable format by several SQL statements. Basically two types of tables were produced: result-tables (one for each age group) for the responses of the participants to the questions and one participant-table for the personal attributes of the participants, e.g. gender, grade or school type. All analysis steps were performed in GNU R.

In a second step, we produced the pattern-tables from the result-tables, having 18 columns, each representing one question, and one row for each participant. The original score values of the questions cover a range from -3 to +12, depending on the difficulty by the experts. For our purpose, we had to transform these values to a dichotomous scale. For this, we represented the correct answers by 1 and the incorrect ones by 0. As the original 0-values (meaning “no answer”) could have been caused by many reasons, e.g. running out of time or laziness, we decided to delete all data sets with any “no answer” values. Due to the large scale of the contest, a quite satisfying numbers of 38,873 participants remained. The distribution over the four age groups was as follows: 8221 in AG1, 15,547 in AG2, 11,672 in AG3, and 3,433 in AG4.

Additionally, we had to distinguish in the participant-tables between persons who worked alone and those who worked in pairs.

5 METHODOLOGY

As already explained, we assumed that certain sets of Bebras questions represent some kind of psychometric test that measures certain joint psychometric constructs (or competencies). Hence, our research question was whether there are subsets of questions that are measuring such joint (combinations of) psychometric constructs (competencies) and if so, which construct(s) this might be. We will call such sets of questions homogenous from now on. For this purpose, we explored all possible subsets of questions. This process was automatically performed by a set of R-scripts.

Traditionally, classical explorative factor analysis is applied for the purpose of detecting subsets of questions that measure joint personal abilities. Yet, as our score format is dichotomous, this is not applicable, as explained in (Bartholomew et al. 2008). Additionally, we were looking for a method that is more suitable to the IRT principles. Hence, we chose the methodology of latent trait analysis (LTA) as presented in Chapter 8 of (Bartholomew et al. 2008).
5.1 Latent Trait Analysis

According to this methodology, it is assumed that the responses of the students to a certain set of questions can be described by a certain psychometric model, for example by the monofactorial Rasch Model (Rost & Carstensen 2002) with one parameter (1F1P), which is explained in section 2.2.

The outcome of our contest is a set of dichotomous response patterns (one pattern per participant) that was recorded by the Bebras online system. For p questions, we have \(2^p\) possible response patterns. From this outcome, one can estimate both the person and item parameters from the results of the contest using an expectation-maximization algorithm. Based on this estimation, by calculating the probability \(P\) in equation 1 of section 2.2, the expected number of occurrences \(E(r)\) of all possible response patterns \(r\) can be calculated.

In the next step these expected frequencies \(E(r)\) are compared to the actually measured pattern frequencies \(O(r)\). Based on the differences, two different test statistics are calculated that describe the deviation of the expected from the measured values: the log-likelihood statistic \(G^2\) (see Equation 4) and a \(X^2\) statistic (See Equation 5).

\[
G^2 = 2 \sum_{r=1}^{2^p} O(r) \ln \frac{O(r)}{E(r)}
\]

\[
X^2 = \sum_{r=1}^{2^p} \frac{(O(r) - E(r))^2}{E(r)}
\]

As both statistics are approximately \(\chi^2\) distributed, we can estimate the quality of the model-fit with \(df\) degrees of freedom, where

\[
df = 2^p - p(q + 1) - 1
\]

As a precondition for this calculation, there has to be a sufficient number of datasets. According to (Bartholomew et al. 2008), it has to be large enough to ensure that the frequency of each pattern has an expectation value of more than 5. In the case of 6 questions for example, this results in a minimum of 320 data sets. For testing all 18 questions of an age group, we would need more than 1.3 Mio participants.

Unfortunately, this method is confirmatory in nature and therefore requires an a priori defined set of questions that is to be tested. We applied a brute force approach, calculating both statistics \(G^2\) and \(X^2\) for all possible combinations of \(p = 3, 4, 5, 6\) out of the total set of 18 questions per age group.

Finally, we selected those combinations where the RM has shown a sufficiently good prediction of the observed results. More precisely, we have selected all combinations of the \(p\) questions where both \(G^2\) and \(X^2\) did not exceed the \(\chi^2\) limits for the respecting values of \(df\) (see equation 6), which are:

\[
\chi^2 = \begin{cases} 
3.8 & (p=3), \\
14.1 & (p=4), \\
32.7 & (p=5), \\
68.7 & (p=6).
\end{cases}
\]

The computing was executed applying the \texttt{ltm} package of \texttt{R} (Rizopoulos 2006).

It turned out that a lot of 3-question combinations (more than 30), many 4-question combinations (10-20), only a few (0-4) 5-question combinations and no 6-question combinations meet the requirements of this Likelihood analysis. Driven by the goal to find preferably large combinations, we decided to focus on the 5-question combinations from this point on. In AG1, we found 3 combinations (see table 2), in AG2 four, in AG3 none and in AG4 three.

### Table 2: Results of latent trait analysis in AG1.

<table>
<thead>
<tr>
<th>No</th>
<th>Combination (questions X…)</th>
<th>(G^2)</th>
<th>(X^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>X156 X162 X164 X184 X187</td>
<td>16.33</td>
<td>16.47</td>
</tr>
<tr>
<td>2</td>
<td>X156 X164 X184 X187 X189</td>
<td>30.72</td>
<td>30.68</td>
</tr>
<tr>
<td>3</td>
<td>X156 X164 X184 X187 X194</td>
<td>28.60</td>
<td>29.09</td>
</tr>
</tbody>
</table>

5.2 Rasch Model Tests

Although LTA already suggests that the monoparametric and mono-factorial RM will fit quite well on our data, there remain some uncertainties. Most important, LTA is focused solely on the item difficulties and parameters, neglecting the distribution of person parameters, as demanded by the precondition of specific objectivity (see section 2.2). Although we have apparently found a good model, there may be an even better fitting one (e.g. the RM with two parameters). Therefore, we have performed a set of standard tests for the fit of the RM, which are presented in the following for one exemplary combination of questions selected from AG1.

First, we applied different latent trait models on the pattern matrix, using the packages \texttt{ltm} and \texttt{eRm} in \texttt{R}: We applied the RMs with 1 factor and 1 parameter (1F1P), 1 factor and 2 parameters (1F2P), 2 factors (2F), and two factors with interaction parameter (2FI).

Next, we performed an ANOVA comparison of all applied models, comparing the values for AIC, BIC and Log-Likelihood. The result was quite acceptable in all cases, indicating the 1F1P model was not fitting significantly worse than 1F2P or the two-factor models.

The following tests of specific objectivity (see section 2.2) follow the joint assumption that the
Likelihood of a well-fitting model should be nearly the same for any subgroup of participants. In other words, the predictive power of the model should be independent of the particular set of participants that was chosen to estimate it. For this purpose the persons are split in subgroups according different criteria. We applied the splitting criteria median (respectively mean), values of combination score and gender. For these subgroups a test-specific statistic, basically representing the Likelihood of this model given the estimated parameters, is calculated. Finally, the p-value for the hypothesis that the statistic would be equal for all subgroups is calculated. The hypothesis (and thus the model) is rejected if \( p < \alpha = 0.05 \). We have applied three different tests, again using the eRm package in R: the Likelihood-Ratio Test according to Andersen (Andersen 1973) the Martin-Löf-Test (see Martin-Löf 1974) and the Wald-Test (see Wald 1943). While the Martin-Löf-Test and the LR tests regarding median/mean and score were passed by all question combinations, only the two combinations AG1-1 and AG1-3 passed the LR-Test regarding gender.

On the question level, the Wald test demonstrated the same problematic nature of the gender splitting, because all but two combinations (again AG1-1 and AG1-3) included questions that produced p-values below 0.05. According to the Wald test on median/mean, there were questions in the combinations AG2-1, AG2-3, and AG2-4 that would have to be excluded.

In summary, over all age groups only the two combinations AG1-1 and AG1-3 (of the originally 10) passed all tests without any problems. Thus, when looking for a suitable set of homogenous test questions, those would be the ones to consider.

Interestingly, both combinations are very strongly correlated with the total score over all 18 questions (0.74 vs. 0.77). In figure 1, the ICCs of AG1-1 of the 1F1P model are displayed.

Aiming to assess the suitability for test application, we calculated the standard deviation of the difficulty parameters by applying 1F1P and the discrimination parameter according to 1F2P. In order to represent a good set of Rasch test items, the former would have to be large, allowing to measure the person parameters over a large scale, while the latter would have to be low, avoiding cross-overs of the Item Characteristic Curves (ICC), see figure 1. It turned out that AG1-1 was clearly better than AG1-3, due to its higher variation in difficulty (1.43 vs. 0.62) and its lower variation in discrimination (0.04 vs. 0.07).

5.3 Evaluation of Person Parameters

To illustrate our methodology, we have conducted an exemplary evaluation of the most homogenous and suitable question combination AG1-1. First we compared the mean scores of different groups of participants: singles and pairs, girls and boys (see table 4.). Considering the scale properties, the proper significance test for the differences is the 2-side approximate Gauss Test (Bamberg, Baur & Krapp 2011). The theta-values of person parameters were normalized according to the PISA scale, which
results per definition in a mean of 500 and a standard deviation of 100 points (over all participants).

Table 4: Differences in score means.

<table>
<thead>
<tr>
<th>Total scores 18 questions</th>
<th>AG1-1 PISA score</th>
</tr>
</thead>
<tbody>
<tr>
<td>All boys – all girls</td>
<td>0.58*</td>
</tr>
<tr>
<td>Singles – pairs</td>
<td>-0.90*</td>
</tr>
<tr>
<td>Single boys – single girls</td>
<td>0.62*</td>
</tr>
<tr>
<td>Male pairs – fem. pairs</td>
<td>0.47*</td>
</tr>
</tbody>
</table>

*Significant difference for \( \alpha = 0.05 \).

Apparently, the boys show a significant better performance compared to the girls. Also, the pairs performed better than the singles. As the difference in the mathematical competence between boys and girls in PISA 2012 was only 11 points in the OECD average (OECD 2013), these results seem quite considerable.

Second, we have ranked the German federal states ("Länder") according to the performance of their students in grade 5 and 6 (see figure 2).

![Figure 2: Performance of the German Länder in grade 5-6.](image)

On the other hand, despite all these deficits compared to a proper large scale study, our methodology has produced some remarkable results. First, it is amazing that there is a coherence between 17 questions that are contained in any of the combinations selected by the LTA over all age groups, while none of the remaining 12 shows up in any of these combinations. This suggests that those 17 questions are measuring some joint construct, while the remaining 12 don’t show any commonality. Yet, the quality of the 17 questions seems not high enough to represent good test items which is easy to explain considering the goals of the Bebras contest.

7 CONCLUSIONS

In conclusion, we have presented an exploratory analysis of the questions of the German Bebras contest of 2009, regarding the homogeneity of the measured competencies. While Brandenburg (540) would be in second place, Hamburg (430) would be ranked last but two among all OECD countries.

6 DISCUSSION

First, we have to stress once again that the Bebras Contest is not a PISA like study. This might be the most probable reason for the low number of question combinations that are measuring some homogenous psychometric constructs in a suitable way. Additionally, as pointed out in the introduction, the Bebras contest does (at least officially) not require any prerequisite knowledge. In view of the postulated cognitive component of competencies, this puts into question whether or not the questions are measuring competencies at all. Further, compared to PISA, the participation is nearly totally uncontrolled. Thus, the sampling might provide serious biases, because at least in some regions only classes of very interested and motivated teachers might participate. Regarding the work on the questions, it is not clear which assistance the students had, e.g. by the teacher or other peers.

On the other hand, despite all these deficits compared to a proper large scale study, our methodology has produced some remarkable results. First, it is amazing that there is a coherence between 17 questions that are contained in any of the combinations selected by the LTA over all age groups, while none of the remaining 12 shows up in any of these combinations. This suggests that those 17 questions are measuring some joint construct, while the remaining 12 don’t show any commonality. Yet, the quality of the 17 questions seems not high enough to represent good test items which is easy to explain considering the goals of the Bebras contest.

Additionally, the results we have found when using the most homogenous set of questions AG1-1 and analyzing student performance, seem very reasonable compared to the PISA results in Mathematics. Although basic skills of computer science are not part of mathematical competencies, they seem quite close and related to those. Therefore the results can be expected to be similar in nature.
this will allow us to describe the psychometric constructs that we have in terms of Computational Thinking (Wing 2006). In the long run, we hope to identify several competency components of Computational Thinking in this way. At the end, these might be combined to construct a structural competency model, suitable to serve as a framework for a multidimensional test in large scale, e.g. in the context of PISA.

REFERENCES


Schott, F & Azizi Ghanbari, S 2009, 'Modellierung, Vermittlung und Diagnostik der Kompetenz


Wald, A 1943, 'Tests of Statistical Hypotheses Concerning Several Parameters When the Number of Observations is Large', Transactions of the American Mathematical Society 1943, pp. 426–482.
