Using Bitmaps for Executing Range Queries in Encrypted Databases

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Abstract: Privacy of data stored at un-trusted servers is an important problem of today. A solution to this problem can be achieved by encrypting the outsourced data, but simple encryption does not allow efficient query processing. In this paper we propose a novel scheme for encrypting relational databases so that range queries can be efficiently executed on the encrypted data. We formally define the syntax and security of the problem and specify a scheme called ESRQ1. ESRQ1 uses a deterministic encryption scheme along with bitmap indices to encrypt a relational database. We provide details of the functionality of ESRQ1 and prove its security in the specified model.

1 INTRODUCTION

Encryption, if applied to relational databases, should be done in such a way that a large class of queries can be executed on the encrypted tables. For example, if we consider the relation R1 shown in Table 1 some representative queries on this relation can be:

(1) What are the names of the employees whose age is 18 years? (2) What are the names of the employees whose age is more than 30 years? (3) What is the average age of an employee?

If the relation R1 is encrypted normally, i.e., encrypting separately each cell of the relation with a strong (say a IND-CPA secure) encryption, then none of the above queries can be executed on the encrypted table.

Regarding the queries that we posed above, query (1) can be addressed if the relation is encrypted using a deterministic encryption scheme, such encryption preserves the equality relation of plaintexts. For query (2), it is required that the ciphertexts maintains the order of the plaintexts, this can be achieved by order preserving encryption. Finally, for query (3) it is required that meaningful computations can be performed on the ciphertexts, homomorphic encryption schemes can support such computations on encrypted data. Generally to treat a large class of queries different encryption mechanisms on the same database is used. Among others, this paradigm is followed in a recent work (Popa et al., 2011). In this work we will mainly focus on processing a class of range queries (query (2) in the example above).

To enable range queries in an encrypted database the ciphertext values should provide order information of the plaintexts. This can be achieved by an order preserving encryption (OPE) scheme. An OPE scheme E is such a scheme where \( E(x) \geq E(y) \), iff \( x \geq y \). This interesting primitive has received lot of attention in the current years. The first concepts appeared in the paper (Agrawal et al., 2004), where the main aim was to design a scheme where efficient range queries can be executed on encrypted data. This work does not delve into formal definitional and security perspectives of OPE. The first work which formally deals with OPE is (Boldyreva et al., 2009), where the ideal security notion for an OPE scheme, IND-OCPA, was introduced. The IND-OCPA definition specifies that the main goal for an OPE is to reveal no additional information about the plaintext values besides their order. In (Boldyreva et al., 2009) it was shown that it is infeasible to achieve IND-OCPA security with a stateless encryption scheme. As a result, they settled on a weaker security guarantee that was later shown in (Boldyreva et al., 2011) to leak at least half of the plaintext bits. Later, several other order preserving schemes were proposed (Boldyreva et al., 2011; Lee et al., 2009; Yum et al., 2012), but as it is suggested in (Popa et al., 2013) none of them has achieved the ideal IND-OCPA security.

Recently, in (Popa et al., 2013), the first ideal-security order-preserving encoding scheme was proposed, where the ciphertexts reveal nothing except the order of the plaintext values. The insight that allow them to avoid the infeasibility result in (Boldyreva
et al., 2009) is that the encryption protocol proposed is interactive, and a small number of ciphertexts of already-encrypted values change, as new plaintext values are encrypted. A property which the authors terms as “mutable”.

Our Contributions. We see the problem of executing range queries in an encrypted database from a different direction. The main component of our solution is not to use any special type of encryption scheme, i.e., the encryption scheme we use is not an OPE scheme. We encrypt the tables with a deterministic encryption scheme, additionally we maintain another table which contains just the order information of the various attribute values. Both of these tables are kept with the server, and the server can respond to range queries without the knowledge of the real attribute values in the table. We propose a generic framework for a database encryption scheme supporting range queries (ESRQ), and define its syntax and security notion. We also provide a complete scheme called ESRQ1. A novel component of ESRQ1 is that it uses bitmap indices to encode the order information of the attributes. Bitmap indices are gaining popularity in the current days for accelerated query processing, and many commercial databases now implement them. Thus it is likely that our solution can be adopted in an existing database easily and efficiently. To our knowledge bitmaps have not been used previously to ensure privacy of databases.

2 PRELIMINARIES

Relations. In what follows, by \( R(A) \) we would denote a relation over a set of attributes \( A \). If \( A = \{a_1, a_2, \ldots, a_m\} \), we shall sometimes write \( R(a_1, a_2, \ldots, a_m) \). Given an attribute \( a \), \( \text{Dom}_R(a) \) would represent the distinct values of the attribute \( a \) in the relation \( R \).

By cardinality of an attribute \( a \) in \( R \) we shall mean the cardinality of \( \text{Dom}_R(a) \), i.e. \( \text{Card}_R(a) = |\text{Dom}_R(a)| \).

A tuple \( t \) in a relation over a set of attributes is a function that associates with each attribute a value in its specific domain. Specifically if \( A = \{a_1, a_2, \ldots, a_m\} \) and \( R(A) \) be a relation, then the \( j^{th} \) tuple of \( R(A) \) would be denoted by \( t_j^R \). And for \( a_i \in A \), by \( t_j^R[a_i] \) we shall denote the value of attribute \( a_i \) in the \( j^{th} \) tuple in \( R \). For \( B \subseteq A \), \( t_j^R[B] \) will denote the set of values of the attributes in \( B \) in the \( j^{th} \) tuple. We shall sometimes omit the subscripts and superscripts from \( t_j^R \) and denote the tuple by \( t \) if the concerned relation is clear from the context and the tuple number is irrelevant.

Binary Strings. The set of all binary strings would be denoted by \{0, 1\}*, and the set of \( n \) bit strings by \{0, 1\}n. For \( X_1, X_2 \in \{0, 1\}^* \), by \( X_1 || X_2 \) we shall mean the concatenation of \( X_1 \) and \( X_2 \); \( |X| \) will denote the length of \( X \) in bits; and \( \mathbb{X}_i \) will denote the bit-wise complement of \( X_i \). If \( |X_1| = |X_2| \), and \( \oplus \) be a Boolean operator then by \( X_1 \oplus X_2 \) we will mean that the operation \( \oplus \) is applied bitwise. For \( X \in \{0, 1\}^n \) and \( 1 \leq k \leq n \), \( \text{bit}_k(X) \) will denote the \( k^{th} \) bit of \( X \).

We shall always consider that the domains of all attributes in the relations and attribute names are subsets of \{0, 1\}*, this convention would allow us to apply transformations and functions on the values of the tuples in a relation without describing explicit encoding schemes.

Bitmaps. Consider a relation \( R(a_1, \ldots, a_m) \) with \( nT \) many rows. Consider that for each attribute \( a_i \), \( \text{Dom}_R(a_i) = \{v_1, v_2, \ldots, v_{m_i}\} \), thus \( \text{Card}_R(a_i) = \lambda_i \) for \( 1 \leq i \leq m \). We define the bitmap of an attribute \( a_i \) corresponding to its value \( v_j \) in the relation \( R \) as \( \text{BitMap}_R(a_i, v_j) = X \), where \( X \) is a binary string, such that \( |X| = nT \) and for \( 1 \leq k \leq nT \),

\[
\text{bit}_k(X) = \begin{cases} 1 & \text{if } t_k^R[a_i] = v_j \\ 0 & \text{otherwise.} \end{cases}
\]

The encoding for the previous definition is known as equality encoding, we shall call such bitmaps as e-encoded bitmaps. In the literature there are other kinds of bitmap encodings to allow different kinds of queries. We would be interested in two types of range bitmaps BitMap< and BitMap> which we will further call as l-encoded and g-encoded bitmaps respectively. For the relation \( R \) described above, we have

\[
\text{BitMap}_R(a_i, v_j) = Y, \text{ where}
\]

\[
\text{bit}_k(Y) = \begin{cases} 1 & \text{if } t_k^R[a_i] < v_j \\ 0 & \text{otherwise.} \end{cases}
\]

From the l-encoding and e-encoding other bitmap encodings can be easily derived. For example, we define

\[
\begin{align*}
\text{BitMap}_R^< (a_i, v_j) &= \text{BitMap}_R (a_i, v_j) \circ \text{BitMap}_R (a_i, v_j) \\
\text{BitMap}_R^> (a_i, v_j) &= \text{BitMap}_R (a_i, v_j) \cup \text{BitMap}_R (a_i, v_j) \\
\text{BitMap}_R^\sim (a_i, v_j) &= \text{BitMap}_R^< (a_i, v_j)
\end{align*}
\]
In what follows we will sometimes use two procedures related to bitmaps. Let \( nT \) be a positive integer and \( S \subseteq \{1, 2, \ldots, nT\} \). MakeBitMap(\( nT, S \)) outputs a string \( X \) such that \( |X| = nT \) and for \( 1 \leq k \leq nT \), \( \text{bit}_k(X) = 1 \) if \( k \in S \) and \( \text{bit}_k(X) = 0 \), otherwise. If \( Y \in \{0, 1\}^{nT} \), then MakeSet(\( Y \)) returns the set \( S = \{i : 1 \leq i \leq nT \text{ and } \text{bit}_i(Y) = 1\} \).

**Deterministic Encryption Schemes:** A deterministic encryption scheme with associated data (DEAD) is a deterministic function \( E : \mathcal{K} \times T \times M \rightarrow C \), where \( \mathcal{K}, T, M \) and \( C \) are the key space, tweak (associated data) space, message space and cipher space, respectively. Thus, \( E \) receives as input a key \( K \in \mathcal{K} \), a tweak \( T \in T \) and a message \( M \in M \) and produces as output a cipher \( C \in C \). We shall often write \( E_K(T, M) \) instead of \( E(K, T, M) \). \( E \) also has an inverse function \( D : \mathcal{K} \times T \times C \rightarrow M \), such that for every \( K \in \mathcal{K} \), every \( T \in T \) and every \( M \in M \), \( D_K(E_K(T, M)) = M \). This implies that for every \( T \in T \) and every \( K \in \mathcal{K} \), \( E_K(T, -) : C \rightarrow \mathcal{C} \) is an injective function.

To define security of \( E \) we consider an adversary \( A \) which is given an oracle \( O \). The oracle \( O \) can either be \( E_K(\cdot, \cdot) \), i.e., the encryption function which is instantiated with a uniform random key \( K \) selected from \( \mathcal{K} \) or it can be \( \mathcal{S}(\cdot, \cdot) \) which is an oracle which on input \( (T, M) \in T \times M \) outputs a random string of size \( |E_K(T, M)| \). One important restriction of \( A \) is that it is not allowed to repeat any query. We formally define the advantage of \( A \) as

\[
\text{Adv}_{E}^\text{det-cpa}(A) = \left| \Pr[K \xleftarrow{\mathcal{K}}, \sigma \xleftarrow{\mathcal{S}(\cdot, \cdot)} \Rightarrow 1] - \Pr[K \xleftarrow{\mathcal{K}}, \sigma \xleftarrow{\mathcal{S}(\cdot, \cdot)} \Rightarrow 0] \right|
\]

We define \( \text{Adv}_{E}^\text{det-cpa}(q, \sigma_n, t) \) by \( \max_A \text{Adv}_{E}^\text{det-cpa}(q, \sigma_n, t) \) where maximum is taken over all adversaries which makes at most \( q \) queries having at most \( \sigma_n \) many blocks (of \( n \) bits) and runs for time at most \( t \). We consider an encryption scheme to be \((\epsilon, q, t)\) det-cpa secure if \( \text{Adv}_{E}^\text{det-cpa}(q, \sigma_n, t) \leq \epsilon \).

**3 ESRQ: BASIC NOTIONS**

An encryption scheme supporting range queries (ESRQ) consists of a tuple of algorithms \((\mathcal{K}, \text{Enc}, \Phi, \Psi, \text{Dec})\), which work as follows.

The key Generation Algorithm \( \mathcal{K} \) runs at the client side and outputs a set of keys that are randomly selected from a pre-specified key space. We will sometimes denote the key space also by \( \mathcal{K} \).

![Game ESRQ](image)

**Figure 1:** Game used to define security of ESRQ.

The Privacy Transform \( \text{Enc} \) receives as an input a relation called \( R \) and the set of keys generated by \( \mathcal{K} \). This transform generates a new set of encrypted relations called \( R' \). We denote this operation as \( R' \xleftarrow{\text{Enc}} R \). This transform is executed in the client side.

The Query Translator \( \Phi \) is a keyed transform that runs at the client. It takes in the key \( K \) and a query \( q \) and converts it to a query \( q' \) which can be executed in \( R' \). For convenience, we shall sometimes refer to a query meant for \( R \) to be a \( R \)-query and a query meant for \( R' \) as \( R' \)-query.

The Response Procedure \( \Psi \) runs at the server side. \( \Psi \) allows the server to answer a query \( \Phi(q) \) on \( R' \). The output of \( \Psi \) for a query is the response of the server, called \( \rho \). \( \rho \) contains the encrypted answer, the server returns it to the client.

The Decryption Procedure is a keyed transform \( \text{Dec}_K \) which runs in the client. It receives as input the server response \( \rho \) and outputs the answer ans for the query \( q \).

**3.1 Security Notion**

**Definition 1.** Given a relation \( R \) over a set of attributes \( \mathcal{A} \), we classify the attributes in two different classes: the ones where a range query is valid (range attributes) and the ones where such queries are not valid. Define \( ty : \mathcal{A} \rightarrow \{0, 1\} \), where for any \( a \in \mathcal{A} \), \( ty(a) = 0 \), if range queries on the attribute \( a \) is not applicable and \( ty(a) = 1 \), otherwise.

**Definition 2.** Two relations \( R \) and \( S \) are said to be equivalent (denoted by \( R \approx S \)) if the following conditions hold.

1. \( R \) and \( S \) are defined over the same set of attributes \( \mathcal{A} \).
2. \( R \) and \( S \) contains the same number of tuples \( nT \).
3. For every \( i, j \in \{1, 2, \ldots, nT\} \), and every \( a \in A, t_i^a[a] = t_j^a[a] \iff t_i^a[a] = t_j^a[a] \).
4. For every \(i, j \in \{1, 2, \ldots, nT\}\), and every \(a \in A\), such that \(ty(a) = 1\),
   \[t^i|a| \geq t^j|a| \iff t^i|a| \geq t^j|a|\].
5. For every \(i \in \{1, 2, \ldots, nT\}\), and every \(a \in A\), \(|t^i|a|| = |t^i|a|\).

The basic goal of ESRQ is to transform \(R\) to \(R^{'1}\) in such a way that \(R^{'1}\) should not contain any information beyond the order relation between the attribute values. We formally define security of an ESRQ \(Y\) as a game between an adversary and a challenger, see Figure 1.

**Definition 3.** The advantage of an adversary \(A\) in attacking an ESRQ \(Y\) is defined as

\[
\text{Adv}_{Y}^{\text{esrq}}(A) = \Pr[\text{ESRQ}^A \Rightarrow 1] - \frac{1}{2}
\]

The relevant resources of an adversary attacking an ESRQ scheme is its running time and the size of the relations that it chooses and submits to its challenger. We measure the size of the relation in two different ways, we denote by \(q\) the number of cells. Let \(s_i\) be the size of each cell, then we define \(s_q = \sum_{i=1}^{q} \left\lfloor \frac{s_i}{n} \right\rfloor\). We call \(q\) as the cell complexity of \(A\) and \(s_q\) as the query complexity of \(A\). We define \(\text{Adv}_{Y}^{\text{esrq}}(q, s_q, t) = \max \text{Adv}_{Y}^{\text{esrq}}(A)\), where the maximum is taken over all adversaries \(A\) which runs for time at most \(t\) and has cell complexity and query complexity of at most \(q\) and \(s_q\), respectively. Moreover, we say that an ESRQ scheme \(Y\) is \((\varepsilon, q, s_q, t)\) secure, if \(\text{Adv}_{Y}^{\text{esrq}}(q, s_q, t) \leq \varepsilon\).

### 3.2 ESRQ1

Here we discuss a specific scheme ESRQ1 for encrypting relations such that simple select and range queries can be executed in the encrypted relations. Consider a relation \(R(A)\) where \(A = \{a_1, a_2, \ldots, a_A\}\), and the function \(ty : A \rightarrow \{0, 1\}\) defined on \(A\). We consider that a client wants to outsource this generic relation \(R(A)\) to a server. To ensure privacy, the client, encrypts the relation \(R(A)\) using ESRQ1 and delegates this encrypted relation instead of the original one. The client will pose queries to the server and expects that the server to execute these queries on his/her behalf without knowing the real contents of the relation \(R(A)\).

In what follows, we present a generic description of the scheme, also throughout we discuss a specific example based on the relation shown in Table 1. The only cryptographic object used by ESRQ1 is a deterministic encryption scheme \(E\) which is required to be det-cpa secure. We assume that \(E : X \times \{0, 1\}^n \times \{0, 1\}^n \rightarrow \{0, 1\}^n\). Other than the deterministic encryption scheme \(E\), ESRQ1 uses bitmap indices.

The various algorithms involved in ESRQ1 are discussed next in order:

**ESRQ1.Enc:** Given \(R(A)\) and the key \(K\) as input, ESRQ1.Enc outputs two relations \(R_a\) and \(R_B\). We assume that \(R(A)\) contains \(nT\) many tuples and \(A = \{a_1, a_2, \ldots, a_A\}\). To each attribute \(a_i \in A\) we associate an unique identifier \(id_i \in \{0, 1\}_\lambda\). Among other possibilities, this identifier can be the (appropriately encoded) name of the attribute or a counter.

- \(R_a\) contains \(nT\) tuples and is defined over the attributes \(B = \{\text{Row}\} \cup \{b_1, b_2, \ldots, b_A\}\). Where \(b_i = E_K(id^*, a_i)\) for some \(id^* \in \{0, 1\}_\lambda\) such that \(id^* \notin \{id_{1}, id_{2}, \ldots, id_{A}\}\). Hence, \(R_a\) has one attribute more than in \(R\), the extra attribute is \(Row\). The other attributes of \(R_a\) are the encryption of the attribute names in \(A\). The specific way in which \(R_a\) is created from \(R\) is shown in Figure 2, which shows that \(R_a\) contains the encryption of the values present in \(R\).

- The relation \(R_B\) contains the attributes \(\{\text{Name}, \text{SearchKey}, \text{Bitmap}\}\), irrespective of the attributes in relation \(R\). The way the relation \(R_B\) is populated is shown in Figure 2. \(R_B\) stores information regarding each range attribute in \(A\). For a range attribute \(a_i\) all its values occurring in \(R\) are encrypted. These encrypted values along with the corresponding attribute name and the l-encoded bitmap are stored in \(R_B\).

For a concrete example, consider that ESRQ1.Enc has as input the relation \(R_1\) as shown in Table 1. The only attribute in \(R_1\), where range queries are meaningful is the attribute \(a_2\). Then ESRQ1.Enc(\(R_1\)) would produce as output the relations \(R_{a_1}\) and \(R_{a_2}\) as shown in Table 2. While applying encryption, the unique identifier of each column is used as the associated data. The attribute names of the original relation \(R_1\) occur in \(R_{a_1}\) in the encrypted form.

The relation \(R_{a_2}\) is created as described in Figure 2. The basic idea is to store the order information of all the values corresponding to the range attributes in a suitable manner. The table stores the encrypted values along with the encrypted attribute names. The order information is stored in the form of l-encoded bitmaps. Note that, for encrypting a specific value the identifier of the attribute is used as the associated data. ESRQ1.Φ: The transform \(Φ\) receives as input a query meant for \(R\) and converts it to a query which can be executed in \((R_{a_1}, R_B)\). The allowed set of queries are simple select queries and range queries. The generic format of an allowed query is

\[Q: SELECT * FROM R WHERE (a_1 < v_1) \circ_1 (a_1 < v_2) \circ_2 \ldots \ldots \circ_{l-1} (a_1 < v_l),\]
where $a_i$ represent an attribute name and $v_i$ a value of the attribute. $\in, \notin, <, \leq, >, \geq$, and $\oplus_i$ can be an arbitrary Boolean connective, say $\lor, \land$ etc.

Given as input a valid query $Q$, $\Phi(Q)$ is the translation of the original query which just hides the attribute names and the values assigned to the attributes by encryption. As the input query has a specific structure, hence the translated query $\Phi(Q)$ is just $Q': (c_1 \leq a_1) \oplus (c_2 \leq a_2) \oplus \ldots \oplus (c_{18} \leq a_{18})$,

where $c'_{a_i} = E_{K}(id'_{a_i})$ and $c_i = E_{K}(id_i, v_i)$.

Going back to the concrete example, consider the following query $Q1$ for the relation $R1$:

\[ Q1: \text{SELECT * FROM } R \text{ WHERE Age } \geq 18 \text{ AND Age } \leq 36 \]

The transformation $\Phi(Q1)$, will output

\[ Q1': (E_{K}(id', Age) \geq E_{K}(id_{18}, 18)) \land (E_{K}(id', Age) \leq E_{K}(id_{36}, 36)) \]

This transformed query is sent to the server.

ESRQ1.$\Psi$: As discussed, $\Psi$ is the transform that runs in the server to execute the client’s instructions. The instruction from the client comes to the server encoded as $Q'$, and it executes the procedure $\Psi(Q')$ as described in Figure 3. $\Psi$ treats the equality/inequality conditions in $Q'$ separately, and constructs a bitmap for each of these conditions. Once the bitmaps for the individual conditions are constructed, it aggregates the bitmaps using the given Boolean connectives. The final bitmap $B$ constructed in line 13 contains the information regarding the tuples in $R_{\alpha}$, which satisfies the client’s query.

Going back to our example, the query $Q1'$ consists of two inequality conditions. The first one is $(E_{K}(id', Age) \geq E_{K}(id_{18}, 18))$, the bitmap for this condition is constructed by checking the relation $R_{\delta}$, the bitmap corresponding to $\text{Name} = E_{K}(id', Age)$ AND $\text{SearchKey} = E_{K}(id_{18}, 18)$ is 000000. But recall that the bitmap stored in $R_{\delta}$ is the l-encoded bitmap. Hence, the bitmap for the condition $(E_{K}(id', Age) \leq E_{K}(id_{36}, 36))$ would be the complement of the stored bitmap, i.e., 101111. The bitmap for the second condition $(E_{K}(id', Age) \leq E_{K}(id_{36}, 36))$ is computed by retrieving the l-encoded bitmap for $\text{Name} = E_{K}(id', Age)$ AND $\text{SearchKey} = E_{K}(id_{36}, 36)$ that is equal to 101110. Then the e-encoded bitmap is computed from $R_{\alpha}$, i.e., 000001. Finally these two bitmaps are operated by an OR bitwise, giving as result 101111. As in the query the two conditions are connected by an AND, hence the bitmap for the query is $101111 \land 110111 = 100111$. This bitmap corresponds to the rows 1, 4, 5 and 6 of the table $R_{\alpha}$. Hence the server sends the tuples in $R_{\alpha}$ corresponding to these row numbers.

ESRQ1.Dec: The decryption procedure receives as input the response $S$ from the server and the keys. This procedure uses the inverse of the encryption $E^{-1}_{K}$ to decrypt the server response.

### Table 2: Relations $R_{1\alpha}$ and $R_{1\beta}$.

<table>
<thead>
<tr>
<th>RowNo</th>
<th>$E_{K}(id', \text{EmpId})$</th>
<th>$E_{K}(id', \text{Name})$</th>
<th>$E_{K}(id', \text{Age})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$E_{K}(id_1, \text{TRW})$</td>
<td>$E_{K}(id_2, \text{Tom})$</td>
<td>$E_{K}(id_1, 18)$</td>
</tr>
<tr>
<td>2</td>
<td>$E_{K}(id_1, \text{MST})$</td>
<td>$E_{K}(id_2, \text{Mary})$</td>
<td>$E_{K}(id_1, 17)$</td>
</tr>
<tr>
<td>3</td>
<td>$E_{K}(id_1, \text{JOH})$</td>
<td>$E_{K}(id_2, \text{John})$</td>
<td>$E_{K}(id_1, 52)$</td>
</tr>
<tr>
<td>4</td>
<td>$E_{K}(id_1, \text{MRH})$</td>
<td>$E_{K}(id_2, \text{Mary})$</td>
<td>$E_{K}(id_1, 33)$</td>
</tr>
<tr>
<td>5</td>
<td>$E_{K}(id_1, \text{ASY})$</td>
<td>$E_{K}(id_2, \text{Anne})$</td>
<td>$E_{K}(id_1, 18)$</td>
</tr>
<tr>
<td>6</td>
<td>$E_{K}(id_1, \text{RZT})$</td>
<td>$E_{K}(id_2, \text{Rosa})$</td>
<td>$E_{K}(id_1, 36)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>SearchKey</th>
<th>Bitmap</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{K}(id', \text{Age})$</td>
<td>$E_{K}(id_{18})$</td>
<td>000000</td>
</tr>
<tr>
<td>$E_{K}(id', \text{Age})$</td>
<td>$E_{K}(id_{36})$</td>
<td>101110</td>
</tr>
<tr>
<td>$E_{K}(id', \text{Age})$</td>
<td>$E_{K}(id_{36})$</td>
<td>110111</td>
</tr>
<tr>
<td>$E_{K}(id', \text{Age})$</td>
<td>$E_{K}(id_{52})$</td>
<td>110111</td>
</tr>
</tbody>
</table>
Response Procedure $\Psi(Q)$

(assume $Q'=(c_1\leq c'_1)\land (c_2\leq c'_2)\land \ldots \land (c_n\leq c'_n)$)

1. for $i\leftarrow 1$ to $t$
2. if $c_i < \alpha$ then
3. $B_i \leftarrow \text{Equal}(c_i,c'_i)$
4. if $c_i = \alpha$ then
5. $B_i \leftarrow \text{Lesser}(c_i,c'_i)$
6. if $c_i > \alpha$ then
7. $B_i \leftarrow \text{Greater}(c_i,c'_i)$
8. end if
9. $B_i \leftarrow \text{Lesser}(c_i,c'_i) \lor \text{Equal}(c_i,c'_i)$
10. end if
11. end for
12. $B = \text{MakeSet}(B_1, B_2, \ldots, B_t)$
13. $\Sigma \leftarrow \text{MakeSet}(B, nT)$
14. $\bar{\Sigma} \leftarrow \text{MakeSet}(\Sigma, nT)$
15. Run the query:

   \begin{align*}
   \text{SELECT } & \\
   & \text{BitMap FROM } R_0 \text{ WHERE } \\
   & \text{Name = a AND SearchKey = b}
   \end{align*}

16. let $S$ be the response of the above query.
17. return $S$

Figure 3: The response procedure $\Psi$.

Security of ESRQ1: Let ESRQ1[$E$] denote a ESRQ scheme where the deterministic encryption used is $E$.

**Theorem 1.** Fix natural numbers $q,\sigma_n,t$, and a deterministic encryption scheme $E: \mathcal{K} \times \mathcal{T} \times \mathcal{M} \rightarrow \mathcal{C}$, such that the smallest ciphertext in $\mathcal{C}$ has size $s$. Let $Y = \text{ESRQ1}[E]$. Then

$$\text{Adv}_Y^{\text{esrq}}(q,\sigma_n,t) \leq \text{Adv}_E^{\text{det-cpa}}(q',\sigma'_n,t') + \frac{q'^2}{2^\sigma T},$$

where $q' \leq q$, $\sigma'_n \leq \sigma_n$ and $t' = O(t)$.

The above Theorem relates the esrq security of ESRQ1[$E$], with the det-cpa security of $E$, and it implies that the ESRQ1 scheme provides almost the same security as that of $E$ with a small degradation.

4 CONCLUSION

In this paper we described a generic framework for database encryptions which enables range queries. We also specified a novel scheme called ESRQ1 which uses deterministic encryption and bitmap indices. This initial proposal is being extended in several ways: management for dynamic databases, a new scheme which would provide just det-cpa security, and an efficient implementation of this scheme on a postgres SQL database. These results would be soon appear in an extended version of this paper.

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